Differential Computation Analysis against Internally-Encoded White-Box Implementations

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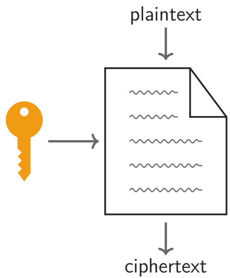
Overview

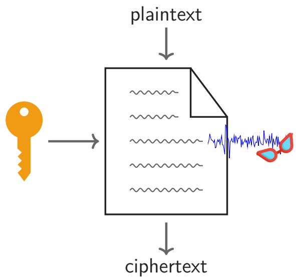
1. White-Box Context
2. DCA against Internal Encodings
3. Collision Attack against Internal Encodings
4. Can We Do Better?

White-Box Threat Model



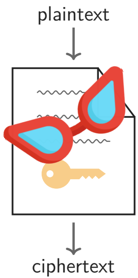
black-box model

knowing the cipher
observing I/O behavior
e.g. linear/differential cryptanalysis



gray-box model

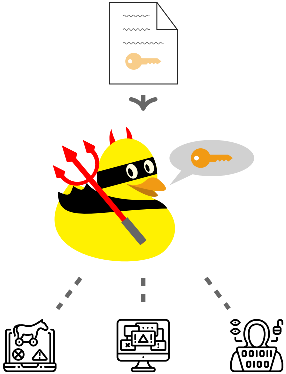
+ side-channel leakages
(power/EM/time/$\cdots$)
e.g. differential power analysis



white-box model [SAC02]

owning the binary
controlling the environment

White-Box Threat Model



- **Goal:** to extract a cryptographic key, $\cdots$
- **Where:** from a software impl. of cipher
- **Who:** malwares, co-hosted applications, user themselves, $\cdots$
- **How:** (*by all kinds of means*)
  - analyze the code
  - spy on the memory
  - interfere the execution
  - $\cdots$

No provably secure white-box scheme for standard block ciphers.

Typical Applications

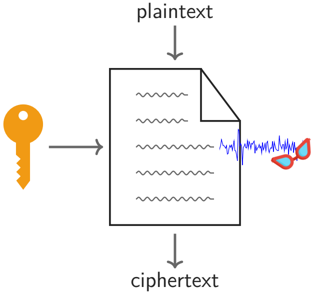
Digital Content Distribution

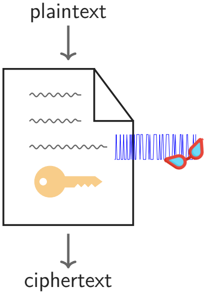
videos, music, games, e-books, $\cdots$

Host Card Emulation

mobile payment without a *secure element*

Differential Computation Analysis [CHES16]





gray-box model

side-channel leakages (*noisy*)

e.g. power/EM/time/···

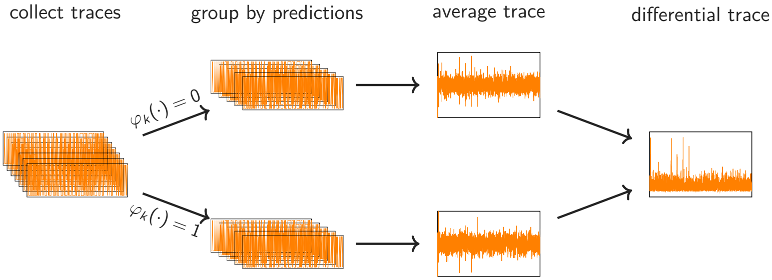
white-box model

computational leakage (*perfect*)

e.g. registers/accessed memory/···

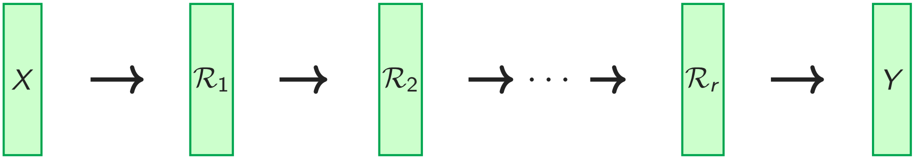
Differential Computation Analysis [CHES16]

Differential power analysis techniques on computational leakages



Implying strong *linear correlation* between the sensitive variables φ_k and the leaked samples in the computational traces.

Internal Encoding Countermeasure [SAC02]



1. Represent the cipher into a *network* of transformations
2. Obfuscate the network by encoding *adjacent* transformations
3. Store the encoded transformations into look-up tables

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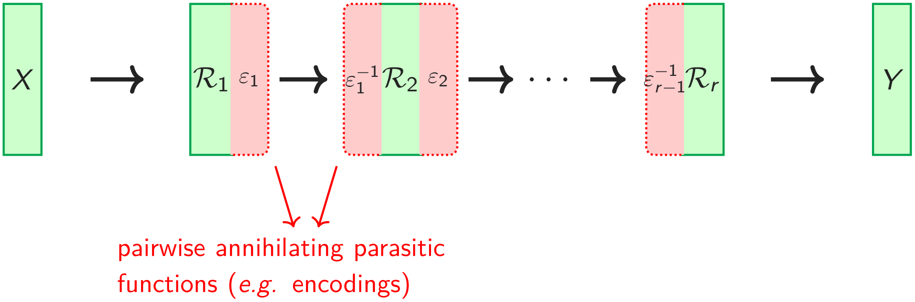

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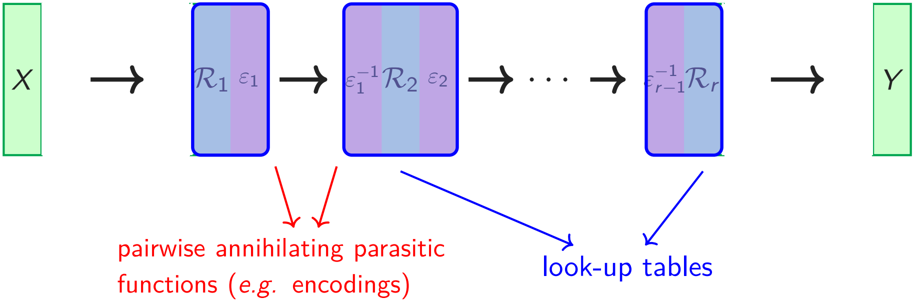

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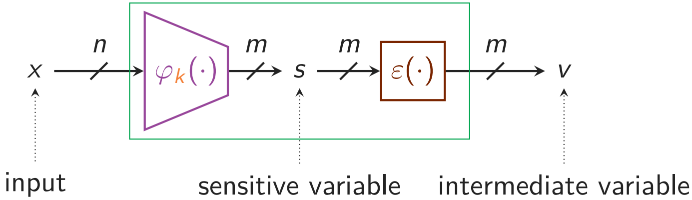
Internal Encoding Leakage



- A *key-dependent* (n, m) selection function φ_k in a block cipher
- A *random* selected m-bit bijection ε
- $\varepsilon \circ \varphi_k$, as a result of some **table look-ups**, is **leaked in the memory**

- To exploit the leakage of $\varepsilon \circ \varphi_k$, it is necessary that $n > m$

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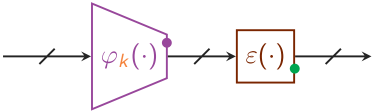
Understanding of DCA

1. The seminal work [CHES16] lacks in-depth understanding of DCA
2. The follow-up analysis [ACNS18] is
   - *partly* experimental (in particular for wrong key guesses)
   - *Only* known to work on **nibble** encodings
   - *Only* known to work on the **first** and **last** rounds
   - Success probability is unknown
3. The computational traces are only sub-optimally exploited

DCA Analysis against Internal Encoding

Based on well-established theory – *Boolean correlation*, instead of *difference of means*: for any key guess k

$$\rho_k = \mathrm{Cor}\Big(\varphi_k(\cdot)[i] \quad , \quad \varepsilon \circ \varphi_{k^*}(\cdot)[j] \Big)$$



DCA success (roughly) requires:

$$|\rho_{k^*}| \geq \max_{k^\times} |\rho_{k^\times}|$$

ρ_{k^*} and $\rho_{k^\times}$: Distributions

- **Ideal** assumption: $\left(\varphi_k\right)_k$ are mutually independent random (n, m) functions

Correct key guess k^*,

$$\rho_{k^*} = 2^{2-m} N^* - 1$$

where

$$N^* \sim \mathcal{HG}(2^m, 2^{m-1}, 2^{m-1}) \ .$$

Only depends on m.

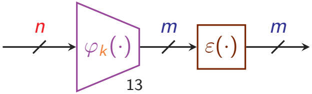
Incorrect key guess $k^\times$,

$$\rho_{k^\times} = 2^{2-n} N^\times - 1$$

where

$$N^\times \sim \mathcal{HG}(2^n, 2^{n-1}, 2^{n-1}) \ .$$

Only depends on n.



Lemma

Lemma

Let $\mathcal{B}(n)$ be the set of balanced n-bit Boolean function. If $f \in \mathcal{B}(n)$ and $g \xleftarrow{\$} \mathcal{B}(n)$ independent of f, then the balanceness of $f+g$ is $\mathrm{B}(f+g) = 4 \cdot N - 2^n$ where $N \sim \mathcal{HG}(2^n, 2^{n-1}, 2^{n-1})$ denotes the size of $\{x : f(x) = g(x) = 0\}$.

With

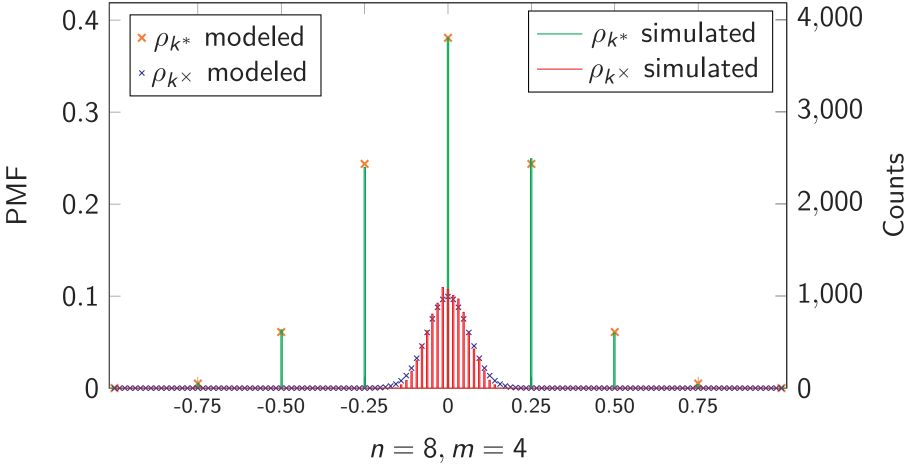
$$\mathrm{Cor}(f\ ,\ g) = \frac{1}{2^n}\mathrm{B}(f+g)$$

$\Rightarrow$

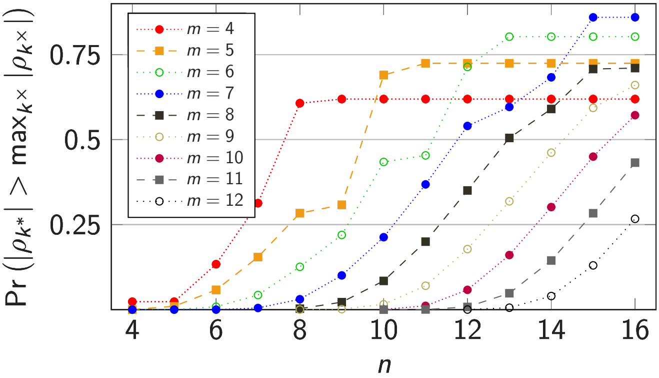
$$\rho_{k^*} = 2^{2-m} N^* - 1 \quad \text{and} \quad \rho_{k^\times} = 2^{2-n} N^\times - 1$$

where $N^* \sim \mathcal{HG}(2^m, 2^{m-1}, 2^{m-1})$ and $N^\times \sim \mathcal{HG}(2^n, 2^{n-1}, 2^{n-1})$.

ρ_{k^*} and $\rho_{k^\times}$: Distributions



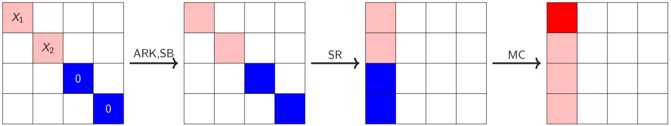
DCA Success Rate: $|\rho_{k^*}| > \max_{k^\times} |\rho_{k^\times}|$



DCA success probability converges towards $\approx 1 - \Pr_{N^*}(2^{m-2})$ for $n \geq 2m + 2$.

Attack a NSC Variant: a White-Box AES

- *Byte* encoding protected
- DCA has failed to break it *before this work*
- Our approach: target a output byte of MixColumn in the first round



$$\varphi_{k_1||k_2}(x_1||x_2) = 2 \cdot \textbf{Sbox}(x_1 \oplus k_1) \oplus 3 \cdot \textbf{Sbox}(x_2 \oplus k_2) \oplus \qquad \textbf{Sbox}(k_3) \quad \oplus \textbf{Sbox}(k_4) \qquad c$$

$$\varepsilon' = \varepsilon \circ \oplus_c \ ,$$

$$n = 16, m = 8 \ , |\mathcal{K}| = 2^{16}.$$

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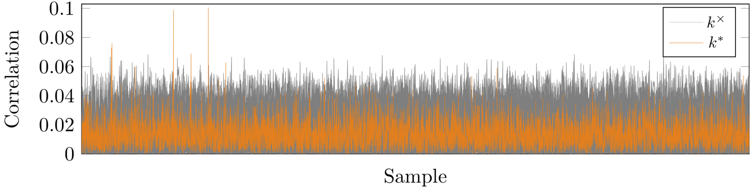
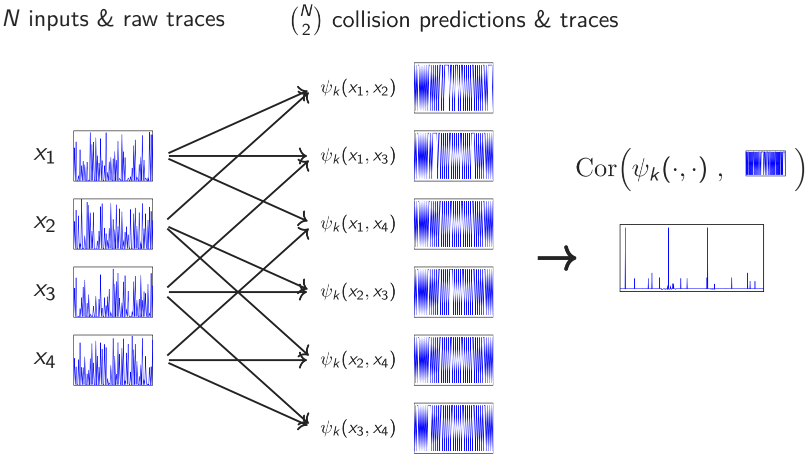
Attack a NSC Variant: a White-Box AES

- Attack results: $\sim$ 1800 traces



- Similar attack can be applied to a "masked" white-box implementation, which intends to resist DCA.

Collision Attack



$$\psi_k(x_1, x_2) := \Big(\varphi_k(x_1) = \varphi_k(x_2)\Big)$$

Collision Attack: Explanation

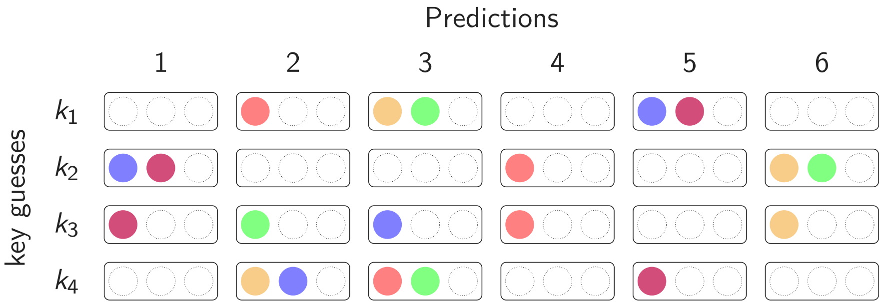
Based on the principle:

$$\varphi_k(x_1) = \varphi_k(x_2) \quad \Leftrightarrow \quad \varepsilon \circ \varphi_k(x_1) = \varepsilon \circ \varphi_k(x_2)$$

Trace Complexity:

$$N = O\left(2^{\frac{m}{2}}\right)$$

Collision Attack: Explanation

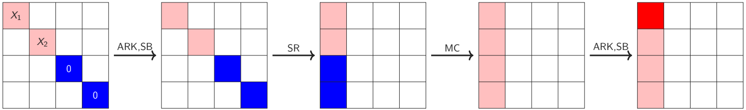


k^* "**collides**" $\bigwedge$ $\forall k^\times$, k^* and $k^\times$ are not "**isomorphic**"

$$\Rightarrow N = O\left(2^{\frac{m}{2}}\right)$$

Attack the NSC Variant

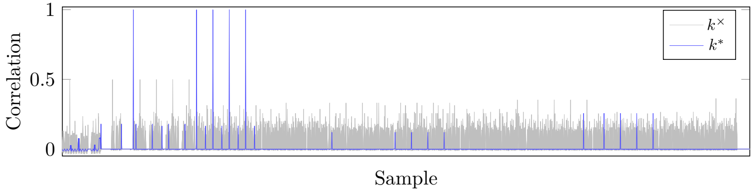
- Same target: a first round MixColumn output byte



$$\varphi_{k_1||k_2}(x_1||x_2) = 2 \cdot \textbf{Sbox}(x_1 \oplus k_1) \oplus 3 \cdot \textbf{Sbox}(x_2 \oplus k_2)$$

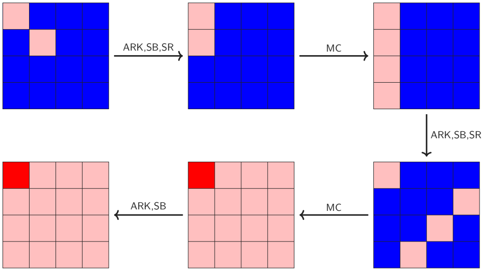
$$\varepsilon' = \varepsilon \circ \oplus_c \qquad \text{or} \qquad \varepsilon'' = \varepsilon \circ \textbf{Sbox} \circ \oplus_{c \oplus k_1'}$$

- Attack results: 60 traces



Can We Do Better?

- YES, WE CAN !!!



$$\varphi_{k_1||k_2||c}(x_1||x_2) = 2 \cdot \textbf{Sbox}\Big(2 \cdot \textbf{Sbox}(x_1 \oplus k_1) \oplus 3 \cdot \textbf{Sbox}(x_2 \oplus k_2) \oplus c\Big)$$

with $\varepsilon' = \varepsilon \circ \oplus_{c'}$ and $n = 16, m = 8, |\mathcal{K}| = 2^{24}$ where

$c = \textbf{Sbox}(k_3) \oplus \textbf{Sbox}(k_4) \oplus k_1'$ and $c' = 3 \cdot \textbf{Sbox}(\cdots) \cdot \textbf{Sbox}(\cdots) \cdot \textbf{Sbox}(\cdots)$.

Conclusion

- DCA against internal encodings has been analysed in depth
  - Allows to attack wider encodings
- Computation traces have been further exploited
  - Showcase to attack variables beyond the first round of the cipher
  - New class of collision attack with very low trace complexity
- Hence, protecting AES with internal encodings in the beginning rounds is insufficient

Thank You !

ia.cr/2019/076