EINDHOVEN UNIVERSITY OF TECHNOLOGY

A DFA attack on White-box implementations of AES with external encoding

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Alessandro Amadori, Wil Michiels and Peter Roelse

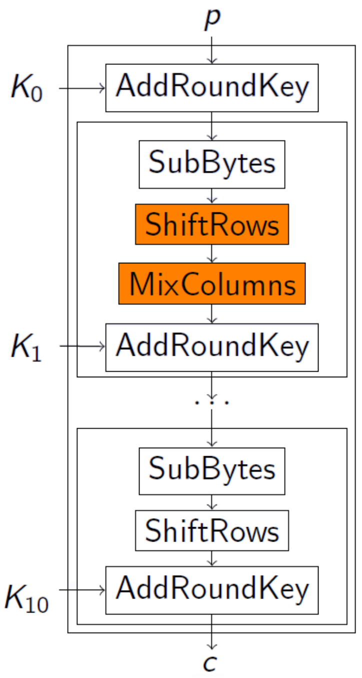
Department of Mathematics and Computer Science

White-box Cryptography and Side Channel Attacks

A very quick introduction

Advanced Encryption Standard

- AES-128 is a block cipher
  - 128-bit plaintext
  - 128-bit key
  - Rearranged bits
  - 10 rounds



Attacks in a White-box Scenario

- In a White-box Attack scenario an attacker:
  - has full access to implementation;
  - can modify part of the implementation;
  - can observe the execution of the algorithm;

- Algebraic attacks on source-code generally require:
  - Reverse engineering;
  - De-obfuscation;
  - Attack-strategies based on the implementation;

Side Channel Attacks (DCA/DFA)

- Advantages:
  - Can be automated;
  - Require little-to-no reverse engineering.
- Differential Computational Analysis (DCA) is the software counterpart of Differential Power Analysis (DPA).
- Differential Fault Analysis (DFA) introduces faults during execution.
  - Inject faults at Round 9 (4 faulty output bytes);
  - Set up system:

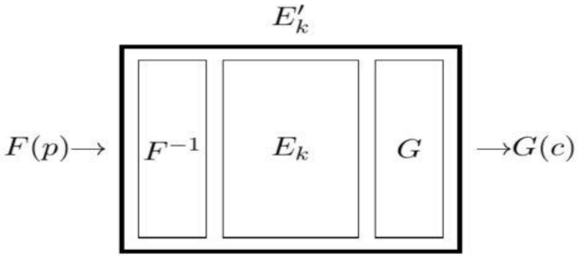
$$
\begin{aligned}
S^{-1}(x_0 \oplus k_0) \oplus S^{-1}(X_0 \oplus k_0) &= 2\,(\, S^{-1}(x_1 \oplus k_1) \oplus S^{-1}(X_1 \oplus k_1)\,) \\
S^{-1}(x_2 \oplus k_2) \oplus S^{-1}(X_2 \oplus k_2) &= S^{-1}(x_1 \oplus k_1) \oplus S^{-1}(X_1 \oplus k_1) \\
S^{-1}(x_3 \oplus k_3) \oplus S^{-1}(X_3 \oplus k_3) &= 3\,(\, S^{-1}(x_1 \oplus k_1) \oplus S^{-1}(X_1 \oplus k_1)\,)
\end{aligned}
$$

- Solve the system to obtain the round key.

External Encodings

- Input or output of the executable may be encoded
  - Composition of random non-linear and linear functions
  - Input is encoded/output is decoded by another party

  - Prevent from code-lifting
  - Prevent from some algebraic attacks



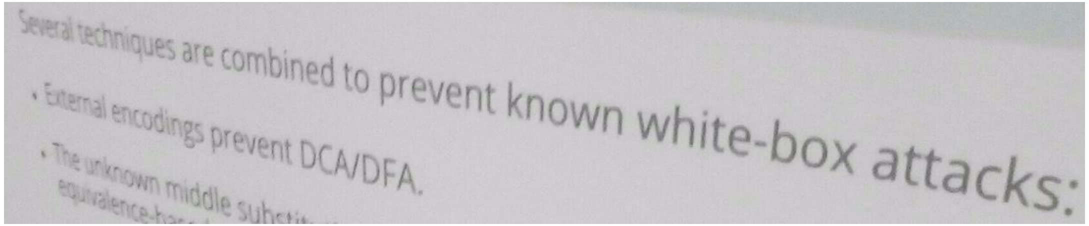
External encodings as countermeasures to SCA

- “Therefore, DFA attacks on encoded outputs are not feasible either.”

Unboxing the White-box, Sanfelix, Mune, de Haas, BlackHat 2016.

- “Another potential countermeasure against DCA is the use of external encodings. This was the primary reason why we were not able to extract the secret key […]”

Differential Computation Analysis: Hiding your White-Box Designs is Not Enough, Bos, Hubain, Michiels, Teuwen, CHES 2016.



Polynomial-based White-Box AES, Ranea, Preneel, Poster at CHES, 2018*.

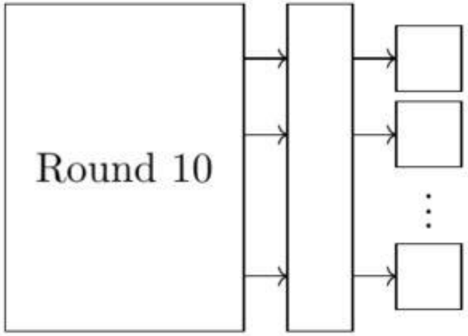
*Photo Courtesy by Lorenz Panny

Attack WB implementations with simple output External Encodings with DFA

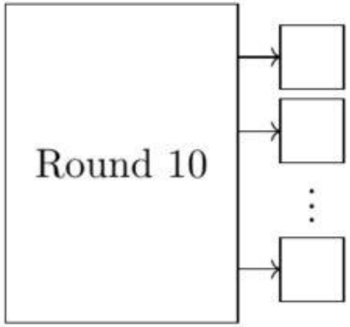
TU/e

Our Model

- External encodings proposed by Chow et al.: 128-bit matrix multiplication and non-linear byte encodings.
- Main objective: Use first-order fault injection attack to extract key
  - External encoding given by non-linear byte encodings.



Chow et al.



Our model

Our Assumptions

- No reverse engineering;
- Operations may not be aligned;
- For any S-box in/out *x* there exists at least 1 location in a *single* execution where we can change *x* to any of its possible 256 values
  - Masking, internal encodings and embedding
- Adversary can guess with good probability the location of an S-box
  - E.g. Checking if 4 output bytes have been altered
    - Different values for different faults

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Before we start off:

a quick thing

- $E_i()$ $\rightarrow$ i^{th} output byte encoding
- $\oplus$ $\rightarrow$ bitwise XOR
- x_i $\rightarrow$ i^{th} correct output byte
- X_i $\rightarrow$ i^{th} faulty output byte
- S() $\rightarrow$ AES S-box
- MC() $\rightarrow$ AES MixColumns
- Ignore Round 10 ShiftRows

Outline of the Attack

- Step 1: Pre-computation
- Step 2: Reconstruction of the 9^{th} round output up to affine bit-functions
- Step 3: Reconstruction of the 9^{th} round output up to affine byte-functions
  - Step 3/4: Reduction of number of variables
- Step 4: Complete reconstruction of the 9^{th} round SubBytes output
- Step 5: Recovery of the 8^{th} round key

Step 1: Pre-computation

- Construct bins of plaintexts $\boldsymbol{M_0}, \boldsymbol{M_1}, \ldots, \boldsymbol{M_{15}}$
    - Necessary to perform Step 2
  - One for every output byte
  - Every p in $\boldsymbol{M_i}$ satisfies the following properties:
    - For all p in $\boldsymbol{M_i}$, i^{th} ciphertext output bytes are unique
    - The output values of two other indexes in the same column are fixed

    - Example: $\boldsymbol{M_0} = \{p_0, p_1, \ldots, p_{255}\}$

$$
\begin{aligned}
p_0 &\rightarrow c_0 = (0x02, 0x34, 0x56, \ldots) \\
p_1 &\rightarrow c_1 = (0xf4, 0x34, 0x56, \ldots) \\
&\ldots \\
p_{255} &\rightarrow c_{255} = (0xc6, 0x34, 0x56, \ldots)
\end{aligned}
$$

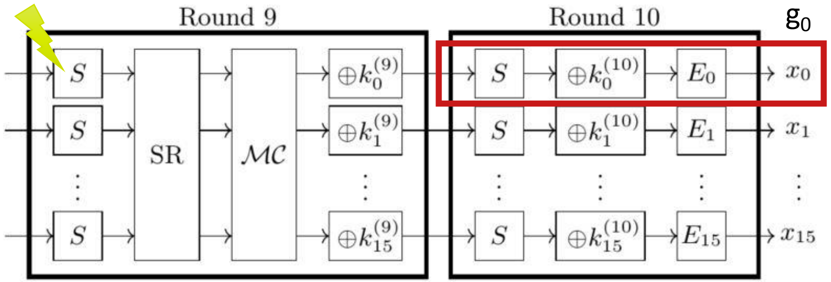
Step 2

- Inject faults at round 9;
- As for DFA, set up the system:

$$g_0^{-1}(x_0) \oplus g_0^{-1}(X_0) = 2(\, g_1^{-1}(x_1) \oplus g_1^{-1}(X_1)\,)$$

$$g_2^{-1}(x_2) \oplus g_2^{-1}(X_2) = \quad g_1^{-1}(x_1) \oplus g_1^{-1}(X_1)$$

$$g_3^{-1}(x_3) \oplus g_3^{-1}(X_3) = 3(\, g_1^{-1}(x_1) \oplus g_1^{-1}(X_1)\,)$$



- $g_i^{-1}(x_i) = S^{-1}(E_i^{-1}(x_i) \oplus k_i)$

- The output of g_i^{-1} is the input of Round 10.

Step 2 (cont.)

- Using a theorem from the BGE attack, if we have functions $g_i(\oplus_\alpha(g^{-1}_i(.)))$, we can derive a non-linear function g_i
  - $g_i = g_i \circ g_i^{-1}$
  - g_i is an affine unknown function
- $g_0^{-1}(x_0) \oplus g^{-1}_0(X_0) = 2(g_1^{-1}(x_1) \oplus g^{-1}_1(X_1))$ $\Rightarrow$ $X_0 = g_0(g_0^{-1}(x_0) \oplus \underbrace{2(g_1^{-1}(x_1) \oplus g_1^{-1}(X_1))}_{\alpha})$
- To provide a correct construction:
  - one byte must assume all possible values
  - an output byte must stay fixed
- We use the bin ***M****ᵢ*
  - We inject all byte values for every plaintext in ***M****ᵢ*
- Why a second fixed byte?

Step 2 (cont.)

- Faults must be introduced for every plaintext.
  - The same S-box must be affected
  - Possible execution misalignments for different plaintexts
- This is where the second fixed byte comes in action:
  - Comparing faulty outputs on fixed bytes:
    - It is possible to check if two injections affected the same S-Box
  - No information about which S-box
    - Not necessary

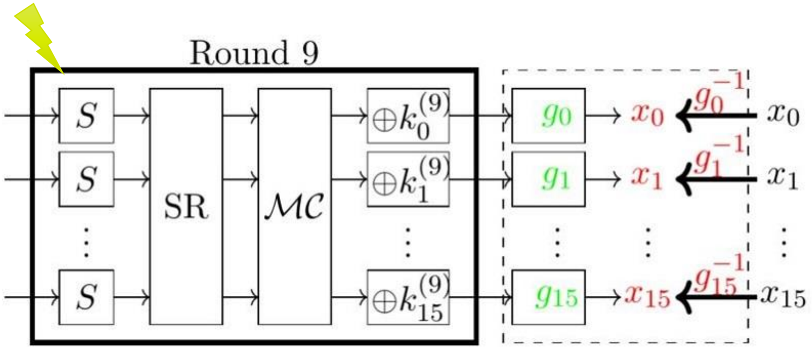
Step 3

- Inject faults at Round 9
  - Consider the set of equations

$$g_0^{-1}(x_0) \oplus g_0^{-1}(X_0) = 2(g_1^{-1}(x_1) \oplus g_1^{-1}(X_1))$$
$$g_2^{-1}(x_2) \oplus g_2^{-1}(X_2) = g_1^{-1}(x_1) \oplus g_1^{-1}(X_1)$$
$$g_3^{-1}(x_3) \oplus g_3^{-1}(X_3) = 3(g_1^{-1}(x_1) \oplus g_1^{-1}(X_1))$$

$$x_i = g_i^{-1}(x_i)$$

$$g_i^{-1}(x_i) = G_i^{-1}(x_i \oplus b_i)$$



Using another Theorem of BGE attack, if we have a function $G_i \circ \gamma \circ G_i^{-1}$ we derive a linear function g_i

- $G_i = g_i \circ \lambda_i^{-1}$
- λ_i^{-1} is an unknown non-zero factor

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Step 3 (cont.)

- We need to construct a function of the form $G_i \circ \gamma \circ G_i^{-1}$
    - γ is a particular known constant (derived from MC coefficients)

- We inject faults affecting 2 different S-boxes in different executions
  $G_0^{-1}(x_0 \oplus X_0) = 2(G_1^{-1}(x_1 \oplus X_1))$ $\quad$ $G_0^{-1}(x_0 \oplus X_0) = 2^{-1}3(G_1^{-1}(x_1 \oplus X_1))$
    - $G_0(2(G_1^{-1}(.))$ and $G_0(2^{-1}3(G_1^{-1}(.))$ $\Longrightarrow$ $G_0(\underbrace{\boxed{2^{-2}3}}_{\gamma}(G_0^{-1}(.)),$
    - γ is unknown but computable! (check the eigenvalues).
    - For some indexes, we can infer the targeted S-Boxes.
    - Any pair of positions and output bytes works!

- We construct an encoded output of Round 9 y_i such that
    - $y_i = g_i^{-1}(x_i)$
    - $y_i = \lambda_i y_i \oplus b_i$
    - y_i is the non-encoded output of Round 9

Step 3/4

Knowing that :

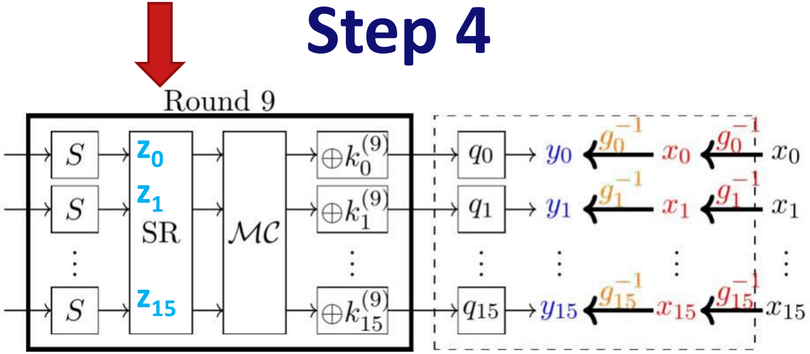
- $G_i = g_i \circ \lambda_i^{-1}$,
- $y_i = g_i^{-1}(x_i)$ and
- $$G_0^{-1}(x_0 \oplus X_0) = 2(G_1^{-1}(x_1 \oplus X_1))$$
  $$G_2^{-1}(x_2 \oplus X_2) = G_1^{-1}(x_1 \oplus X_1)$$
  $$G_3^{-1}(x_3 \oplus X_3) = 3(G_1^{-1}(x_1 \oplus X_1))$$

We construct a dependency among λ_i

$$\lambda_0^{-1}(y_0 \oplus Y_0) = 2(\lambda_1^{-1}(y_1 \oplus Y_1))$$
$$\lambda_2^{-1}(y_2 \oplus Y_2) = \lambda_1^{-1}(y_1 \oplus Y_1)$$
$$\lambda_3^{-1}(y_3 \oplus Y_3) = 3(\lambda_1^{-1}(y_1 \oplus Y_1))$$

- $\lambda_1^{-1} = c_1\lambda_0^{-1}$, $\lambda_2 = c_2\lambda_0^{-1}$, $\lambda_3 = c_3\lambda_0^{-1}$.
  - c_1, c_2, c_3 are computable.

Step 4



- We obtain an "encoded" S-Box output of round 9 ($z_0, z_1, \dots, z_{15}$) from ($y_0, y_1, \dots, y_{15}$) by reverting AES operations (without considering key addition).
- Inject faults at Round 8:

$$S^{-1}(\lambda_0^{-1} z_0 \oplus \beta_0) \oplus S^{-1}(\lambda_0^{-1} Z_0 \oplus \beta_0) = 2(S^{-1}(\lambda_4^{-1} z_1 \oplus \beta_1) \oplus S^{-1}(\lambda_4^{-1} Z_1 \oplus \beta_1))$$
$$S^{-1}(\lambda_8^{-1} z_2 \oplus \beta_2) \oplus S^{-1}(\lambda_8^{-1} Z_2 \oplus \beta_2) = S^{-1}(\lambda_4^{-1} z_1 \oplus \beta_1) \oplus S^{-1}(\lambda_4^{-1} Z_1 \oplus \beta_1)$$
$$S^{-1}(\lambda_{12}^{-1} z_3 \oplus \beta_3) \oplus S^{-1}(\lambda_{12}^{-1} Z_3 \oplus \beta_3) = 3(S^{-1}(\lambda_4^{-1} z_1 \oplus \beta_1) \oplus S^{-1}(\lambda_4^{-1} Z_1 \oplus \beta_1))$$

- The unknowns are λ_i^{-1} and β_i
  - They contain the remaining randomness

Step 4 (cont.)

- Exhaustive search is unfeasible,
  - 2^{64} operations
- We use a MITM approach with hash tables:
  - $S^{-1}(\lambda_4^{-1}z_1 \oplus \beta_1) \oplus S^{-1}(\lambda_4^{-1}Z_1 \oplus \beta_1)$ in every equation
- Consider

  $$2^{-1}(S^{-1}(\lambda_0^{-1}z_0 \oplus \beta_0) \oplus S^{-1}(\lambda_0^{-1}Z_0 \oplus \beta_0)) = S^{-1}(\lambda_4^{-1}z_1 \oplus \beta_1) \oplus S^{-1}(\lambda_4^{-1}Z_1 \oplus \beta_1)$$

- For all λ and β we compute $S^{-1}(\lambda z_1 \oplus \beta) \oplus S^{-1}(\lambda Z_1 \oplus \beta)$
  - Store them in an Hash Table
- For all λ and β we compute $2^{-1}(S^{-1}(\lambda z_0 \oplus \beta) \oplus S^{-1}(\lambda Z_0 \oplus \beta))$
  - Check if we have a match in the hash table
  - If yes: $(\lambda, \beta, \lambda, \beta)$ is a solution
  - $(\lambda_0^{-1}, \beta_0, \lambda_4^{-1}, \beta_1)$ ***must*** belong to the set of solutions
- We apply this process for ω faults

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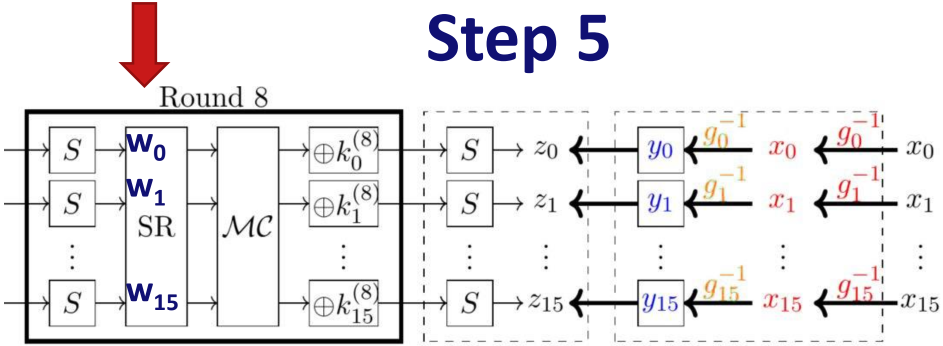
Step 4 (cont.)

- Higher $\varpi \rightarrow$ more accuracy
  - $\varpi = 8$ only one solution is found (in about 5 min)
- If injecting at the wrong spot: No solution for the system.
- After retrieving all the λ_i^{-1} and the β_i:
  - We are able to decode the output of the Round 9 S-box.
  - From encoded Round 9 S-Box output (z_0, z_1, … , z_{15}) compute $z_i = \lambda_i^{-1} z_i \oplus \beta_i$

Step 5



- From the decoded Round 9 S-box output ($z_0, z_1, \ldots, z_{15}$) compute the non-encoded Round 8 S-Box output ($w_0, w_1, \ldots, w_{15}$) as in Step 4.
- Inject faults at Round 7: set up and solve the standard equations

$$S^{-1}(w_0 \oplus k_0) \oplus S^{-1}(W_0 \oplus k_0) = 2(S^{-1}(w_{13} \oplus k_{13}) \oplus S^{-1}(W_{13} \oplus k_{13}))$$
$$S^{-1}(w_{10} \oplus k_{10}) \oplus S^{-1}(W_{10} \oplus k_{10}) = S^{-1}(w_{13} \oplus k_{13}) \oplus S^{-1}(W_{13} \oplus k_{13})$$
$$S^{-1}(w_7 \oplus k_7) \oplus S^{-1}(W_7 \oplus k_7) = 3(S^{-1}(w_{13} \oplus k_{13}) \oplus S^{-1}(W_{13} \oplus k_{13}))$$

- Obtain the values for k
  - MITM-approach is very efficient.
  - Round 8 key is MC(k)!
  - Revert the Key-Scheduling algorithm to obtain the encryption key.

Work load

- Step 1: $\rightarrow \sim 2^{31}$ WB encryptions, 0 operations
- Step 2: $\rightarrow \sim 2^{20}$ WB encryption, 2^{18} operations
- Step 3: $\rightarrow \sim 2^{10}$ WB encryptions, 2^{20} operations
  - Step 3/4: $\rightarrow$ 0 WB encryptions, 12 operations
- Step 4: $\rightarrow 4\omega$ WB encryptions, $\omega 2^{19}$ operations
- Step 5 $\rightarrow 4\omega'$ WB encryptions, $\omega' 2^{13}$ operations

$< 2^{32}$ WB encryptions

$< 2^{22}$ operations

Summary

- We perform the attack stepwise:
  - Construct last round up to some function
  - Remove the randomness and retrieve non-encoded state
  - Extract round-8 key

- Open Problems/Future work:
  - Work on assumptions
  - Consider stronger external encodings
    - Study what external encodings are safe
  - Reduce complexities

Thank you!

Any Questions?