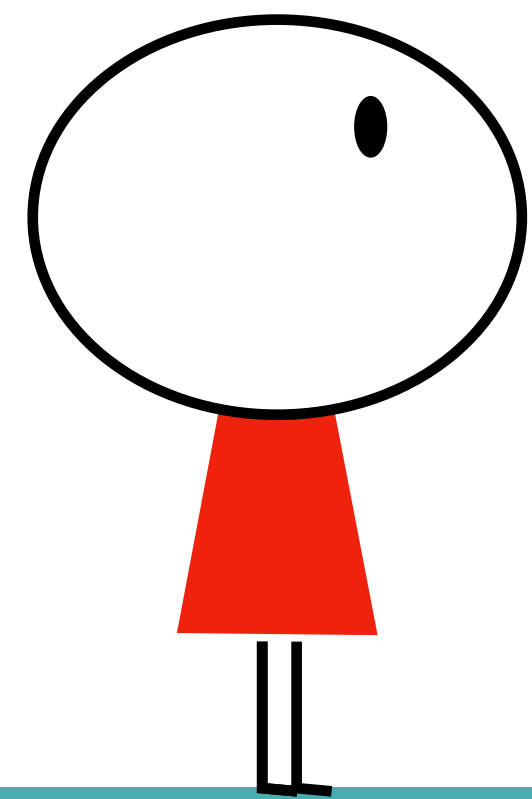


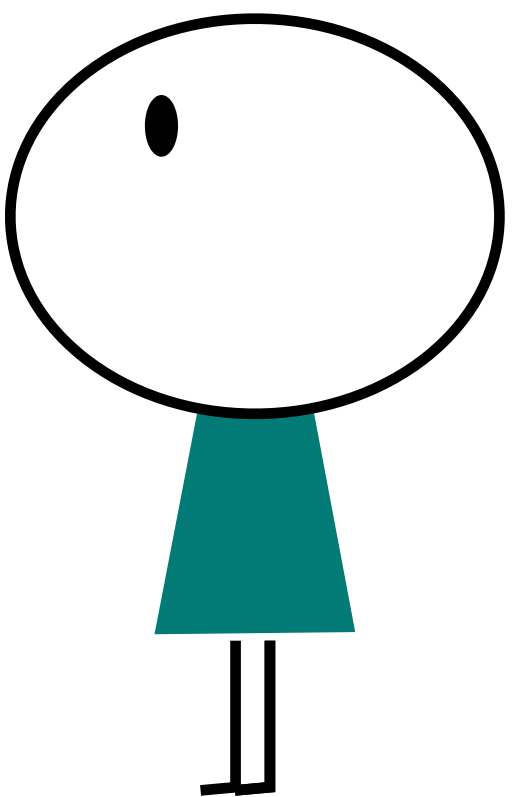
Side-channel countermeasures for lattice-based cryptography

- VeriSiCC Seminar -

Sept 22nd 2022



Mélissa Rossi



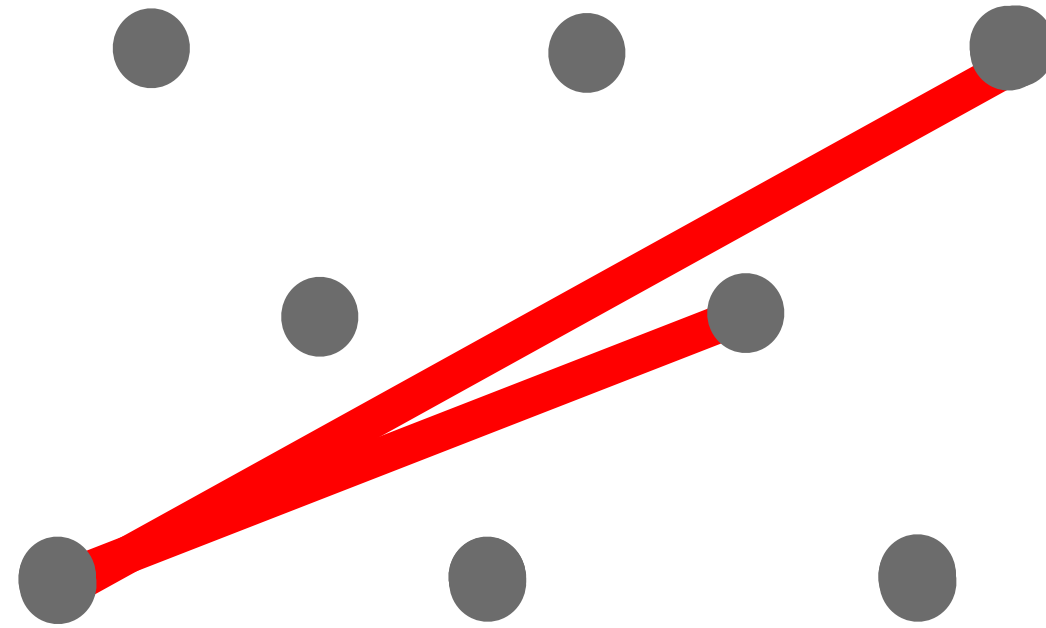
Lattices and hard problems

A lattice Λ is an additive subgroup generated by n linearly independent vectors of \mathbb{R}^n .



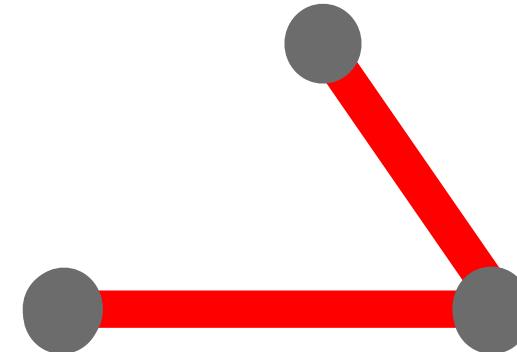
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Lattices and hard problems

A lattice Λ is an additive subgroup generated by n linearly independent vectors of \mathbb{R}^n .



Lattices and hard problems



Given a lattice Λ

Find the vector \mathbf{v} that has the smallest nonzero norm

Short Vector Problem (SVP)



Lattices and hard problems

« Linear system solving with noise »

Given the pair $(\mathbf{A} \in \mathbb{Z}_q^{m \times n}, \mathbf{b} = \mathbf{A}\mathbf{s} + \mathbf{e} \in \mathbb{Z}_q^m)$ where

\mathbf{A} is sampled uniformly at random

\mathbf{e} and \mathbf{s} are sampled following a small distribution χ

Find \mathbf{s}

Learning With Errors (LWE)

Lattice-based algorithms

● Signature schemes

● Public key encryption schemes



1 Strong hardness properties



Lattice-based algorithms

● Signature schemes

● Public key encryption schemes

1 Strong hardness properties

2 Simple designs (but complex analysis)



Lattice-based algorithms

Signature schemes

Public key encryption schemes

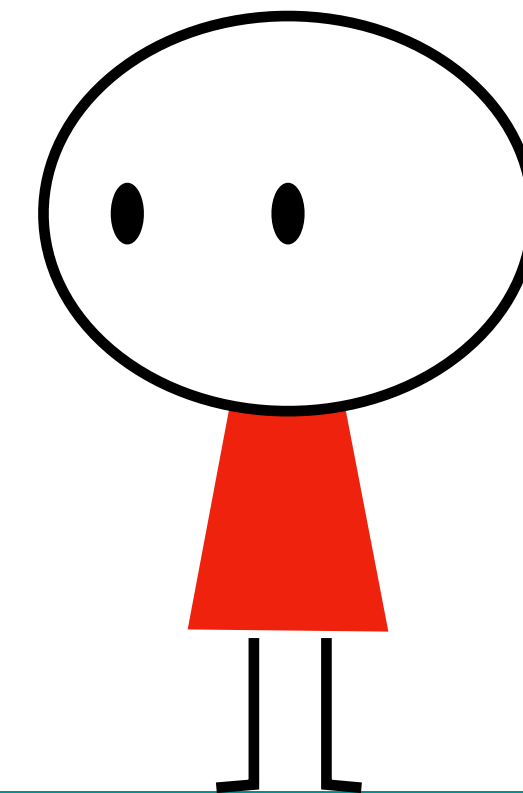
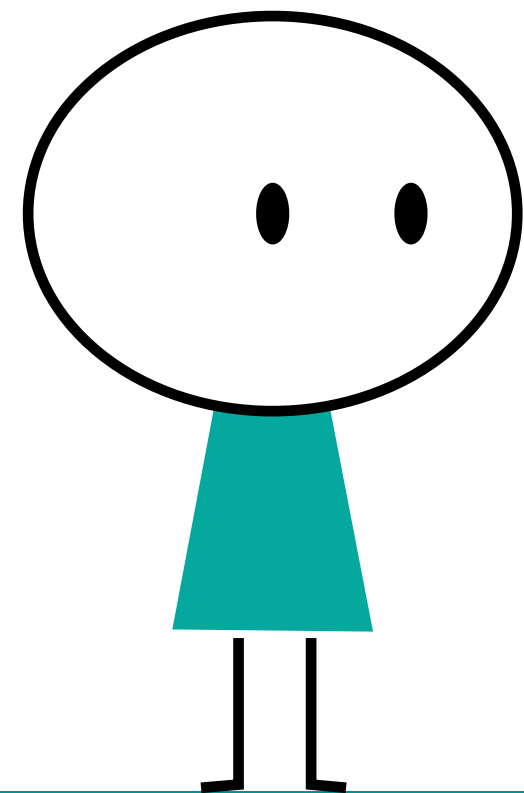
1 Strong hardness properties

2 Simple designs (but complex analysis)

3 Concrete candidates schemes
NIST round 2: 12 out of 26 candidates
NIST round 3: 5 out of 7 candidates
NIST first standards: 3
NIST round 4: ?

LWE-based public key encryption in a nutshell

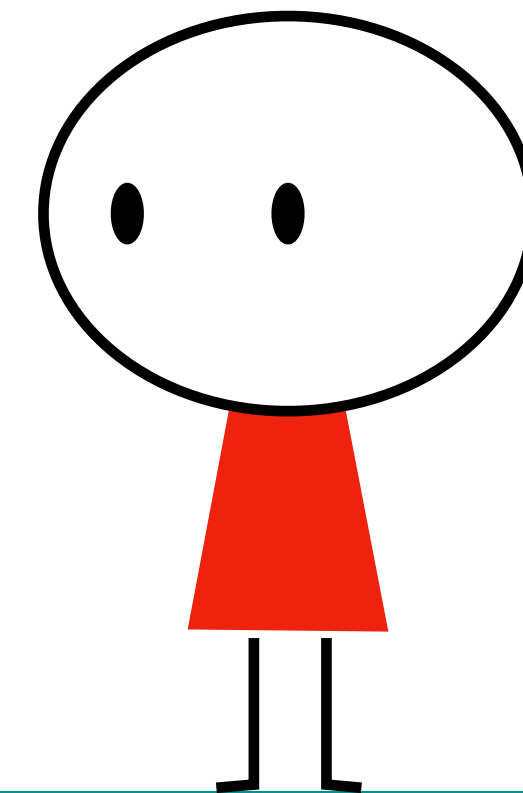
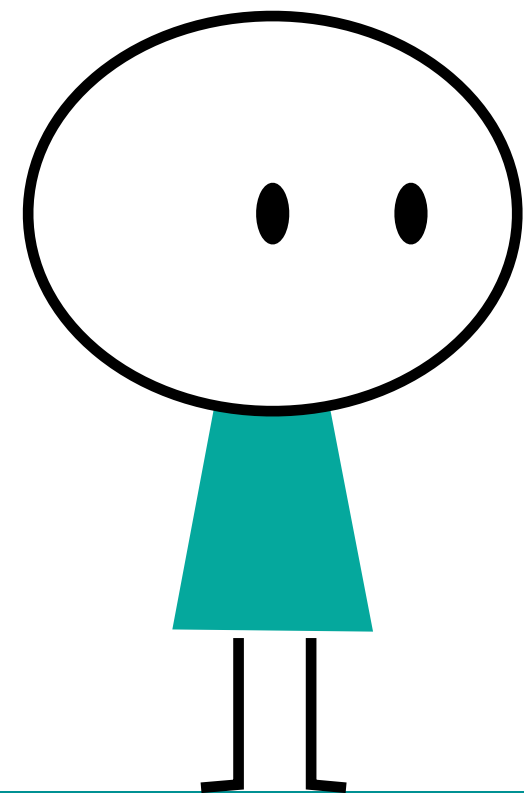
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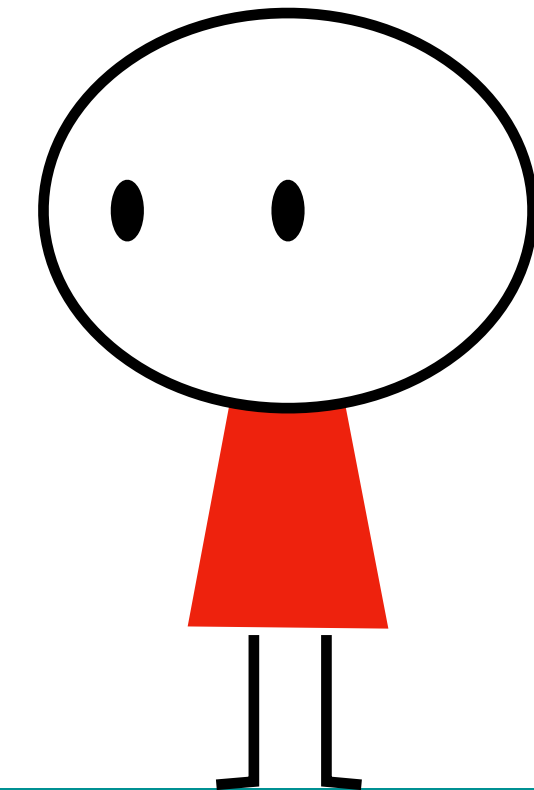
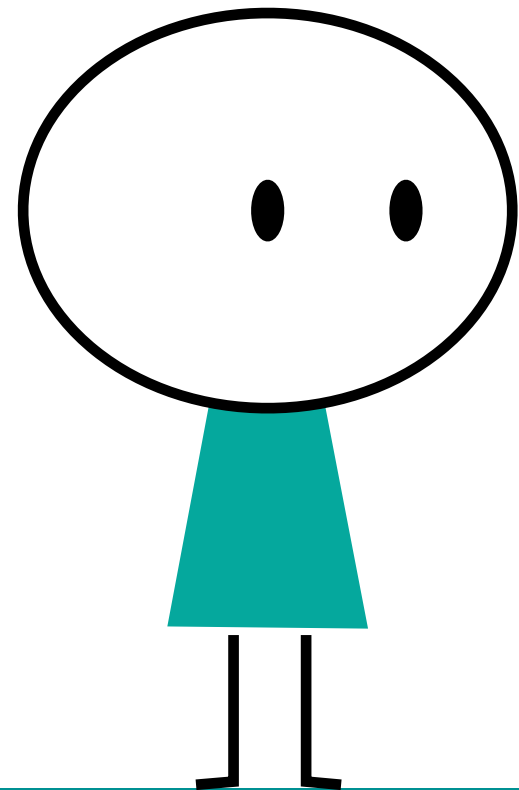
$$(\mathbf{A}, \mathbf{b} = \mathbf{Az} + \mathbf{e})$$



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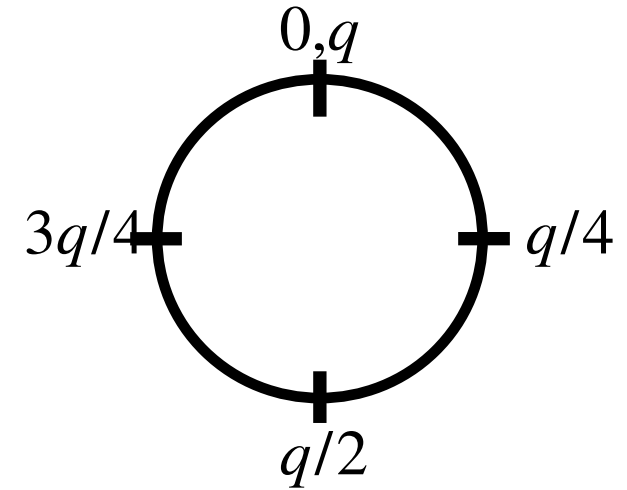
$$\mathbf{b}' = \mathbf{A}^T \mathbf{z}' + \mathbf{e}'$$

(\mathbf{A}, \mathbf{b})

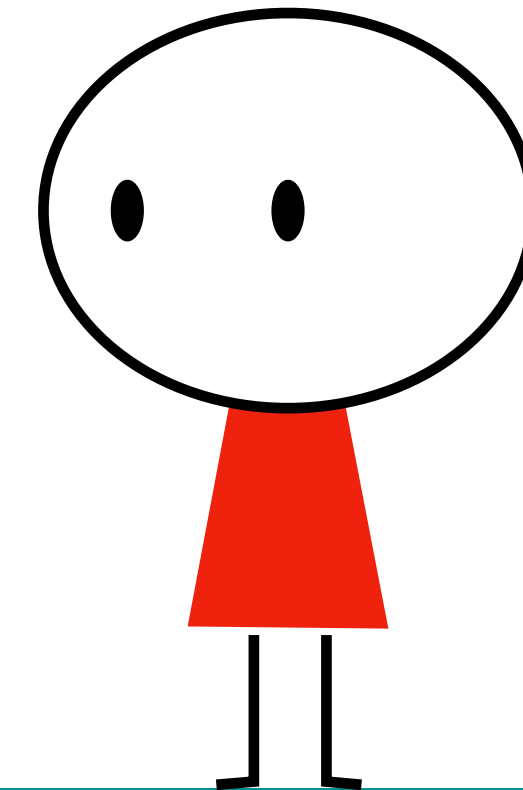
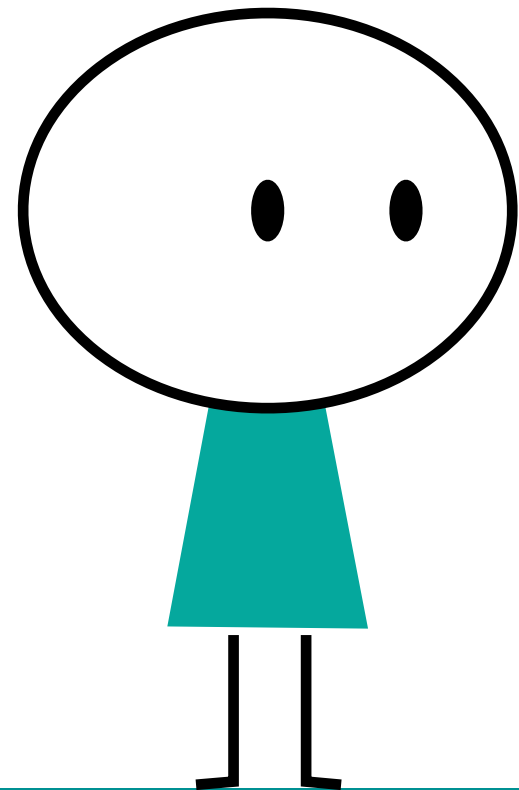
$$(\mathbf{A}, \mathbf{b} = \mathbf{A}\mathbf{z} + \mathbf{e})$$



$$v' = \mathbf{b}^T \mathbf{z}' + \mathbf{e}'' + \frac{q}{2}m$$



1-bit encryption



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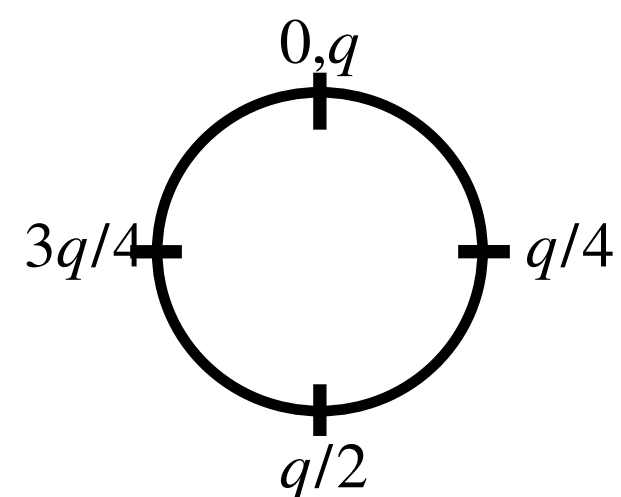
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$$\mathbf{b}' = \mathbf{A}^T \mathbf{z}' + \mathbf{e}'$$

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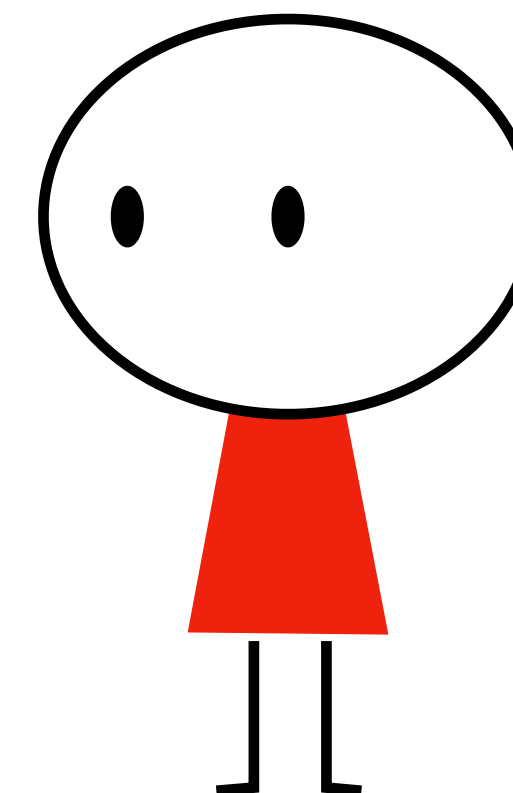
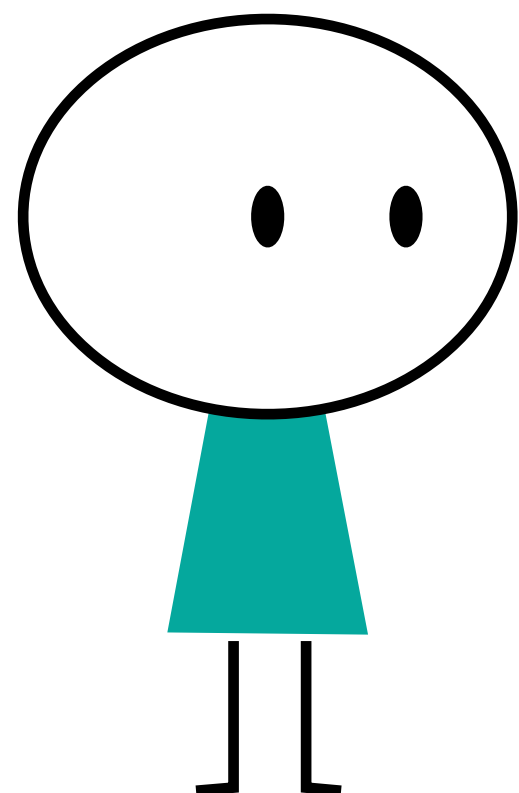
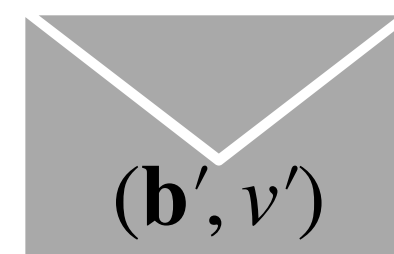
1-bit encryption



(\mathbf{A}, \mathbf{b})



$(\mathbf{A}, \mathbf{b} = \mathbf{A}\mathbf{z} + \mathbf{e})$



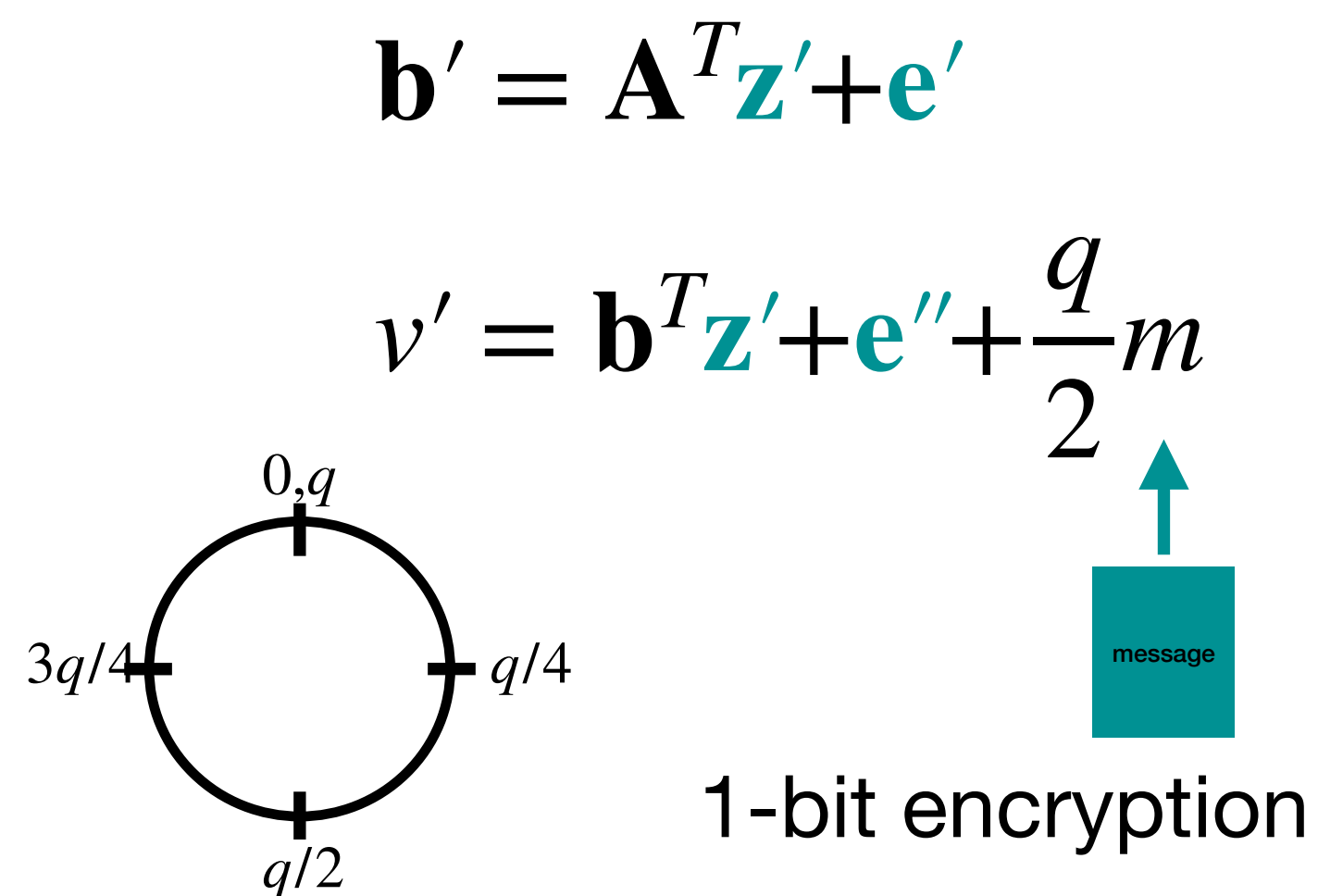
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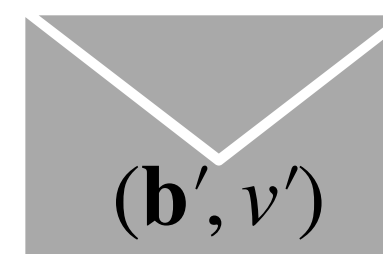
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(\mathbf{A}, \mathbf{b})



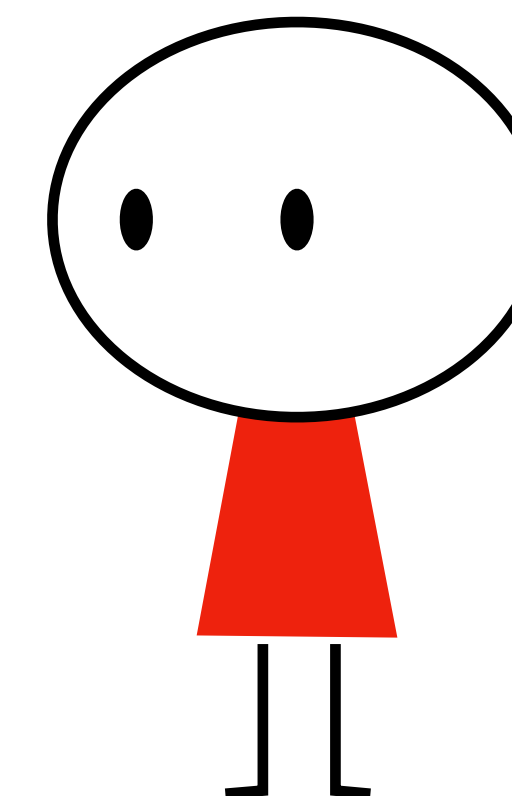
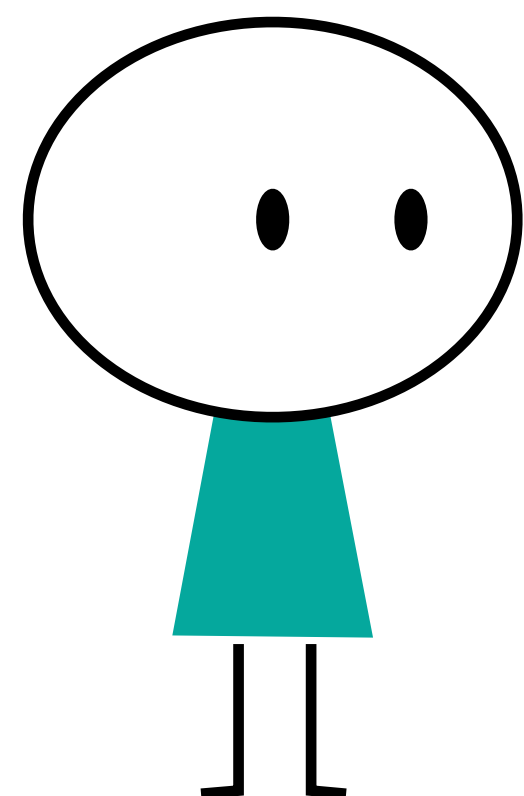
$(\mathbf{A}, \mathbf{b} = \mathbf{A}\mathbf{z} + \mathbf{e})$

$$m' = \left\lfloor \frac{2}{q} (v' - \mathbf{z}^T \mathbf{b}') \right\rfloor$$

$$m' \approx m + \left\lfloor \frac{2}{q} (\mathbf{e}^T \mathbf{z}' + \mathbf{e}'' - \mathbf{z}^T \mathbf{e}') \right\rfloor$$

1-bit decryption

Noise smaller than $1/2$



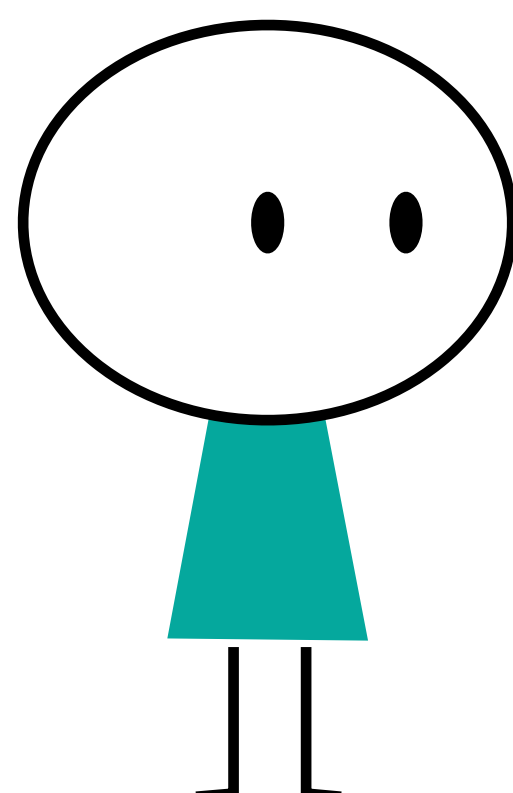
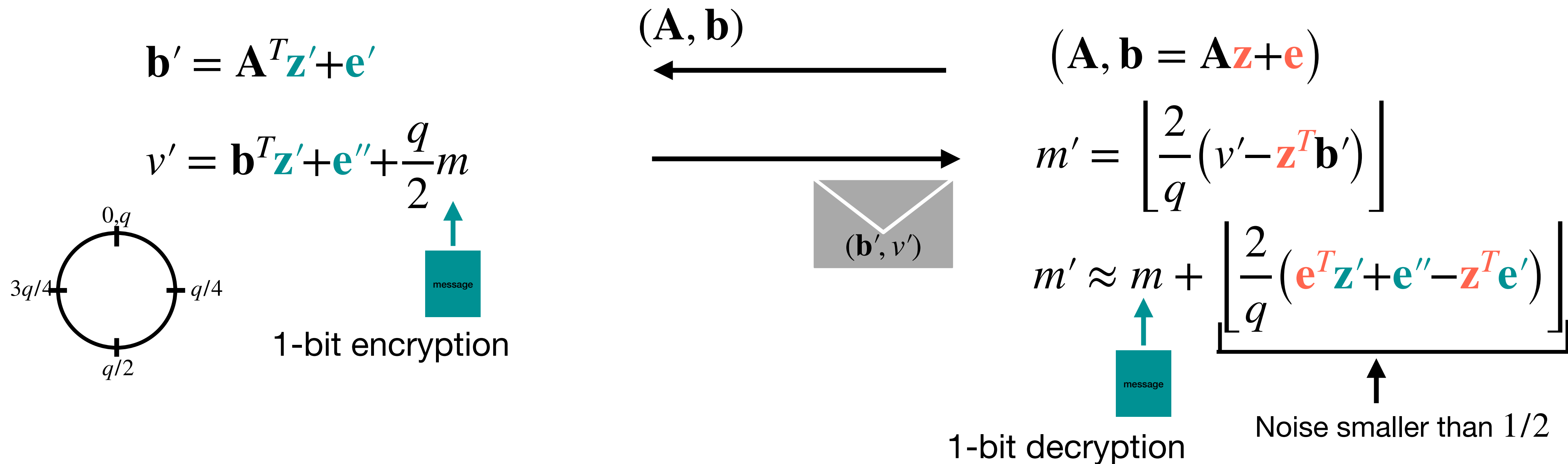
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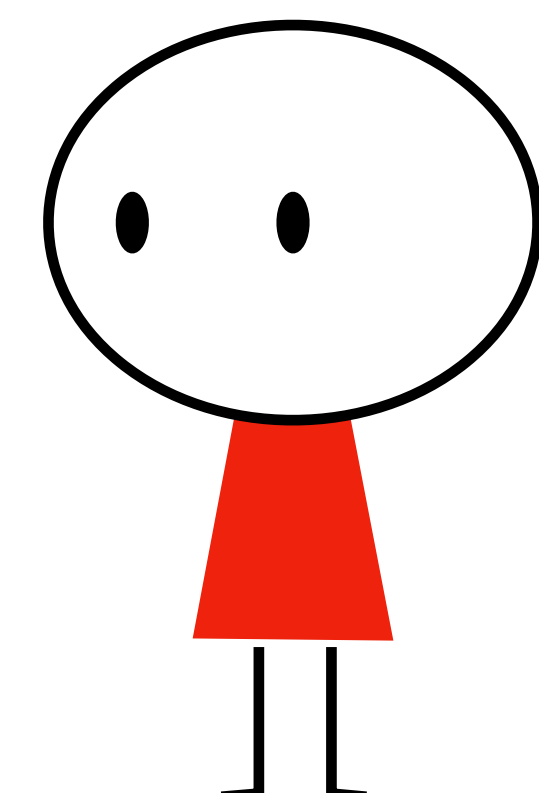
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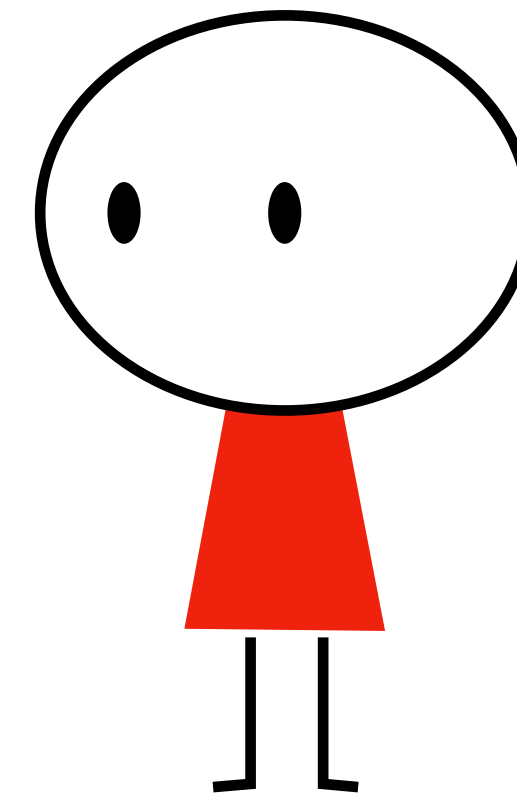
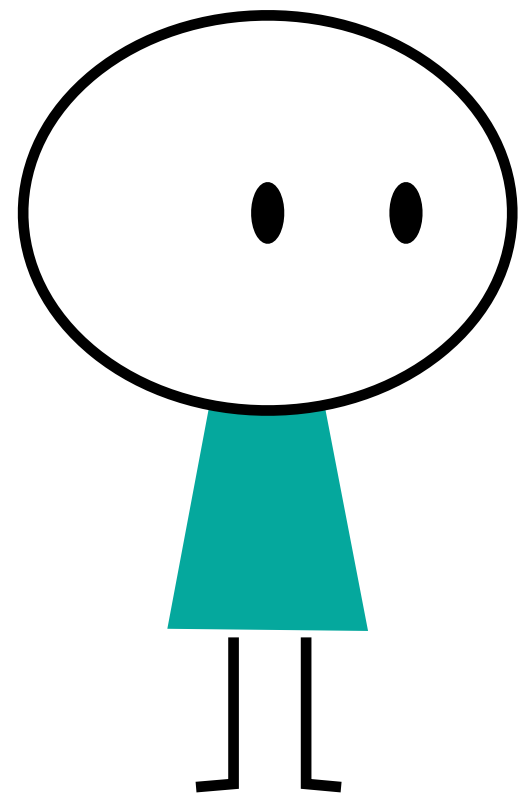
High level idea behind
 Crystals-Kyber (NIST standard)
 Frodo, Saber and NewHope



A Fiat-Shamir with aborts signature in a nutshell

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Short Integer Solution (SIS)

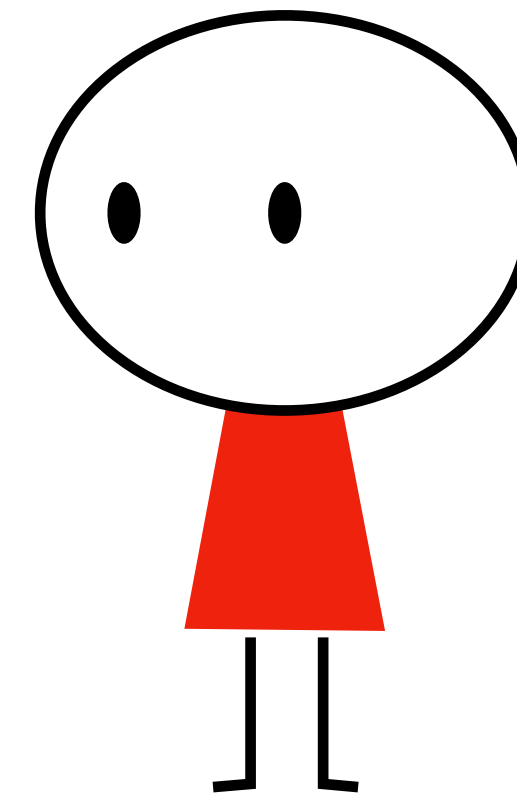
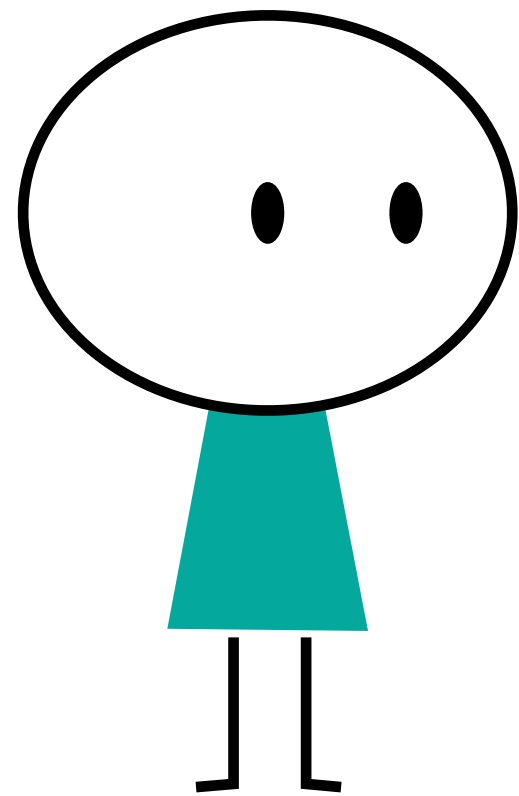


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Short Integer Solution (SIS)

$$(\mathbf{A}, \mathbf{t} = \mathbf{A}\mathbf{S} \bmod q)$$

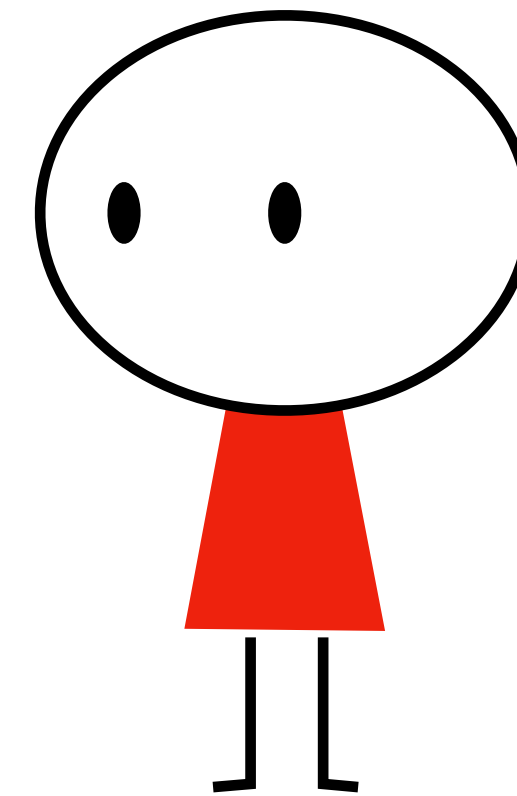
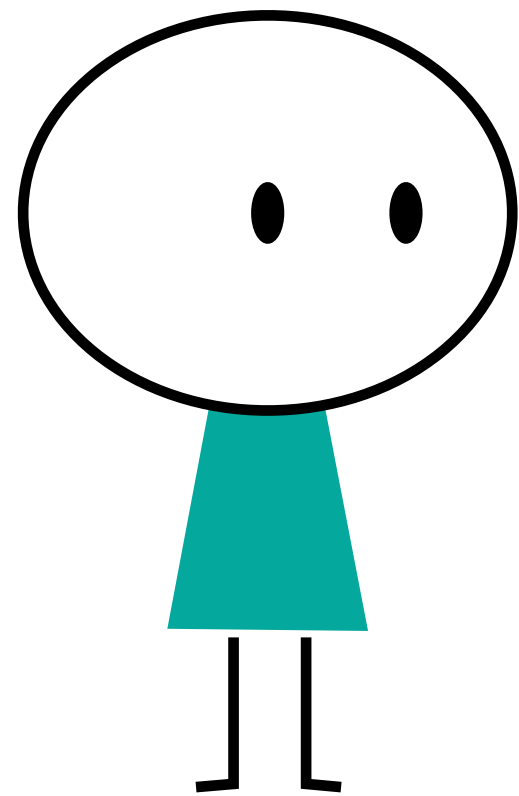


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$$(\mathbf{A}, \mathbf{t}) \longleftarrow (\mathbf{A}, \mathbf{t} = \mathbf{A}\mathbf{S} \bmod q)$$



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Short Integer Solution (SIS)

(\mathbf{A}, \mathbf{t})



$(\mathbf{A}, \mathbf{t} = \mathbf{A}\mathbf{S} \bmod q)$

Signature algorithm:

1: do

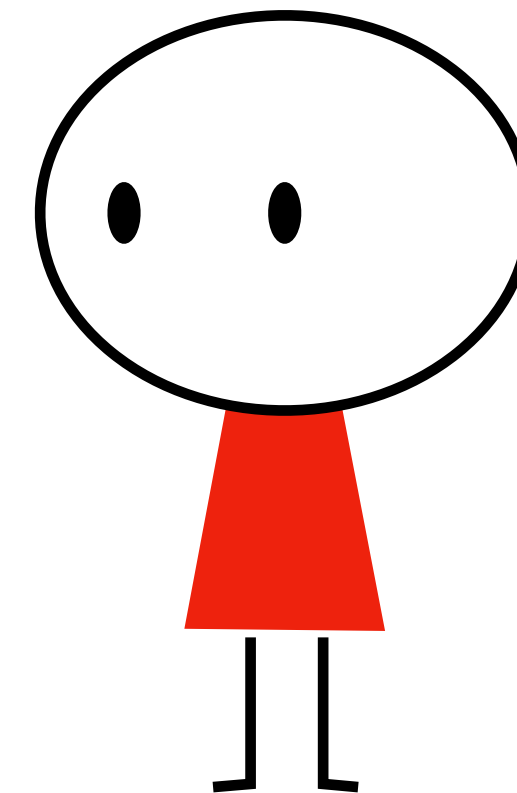
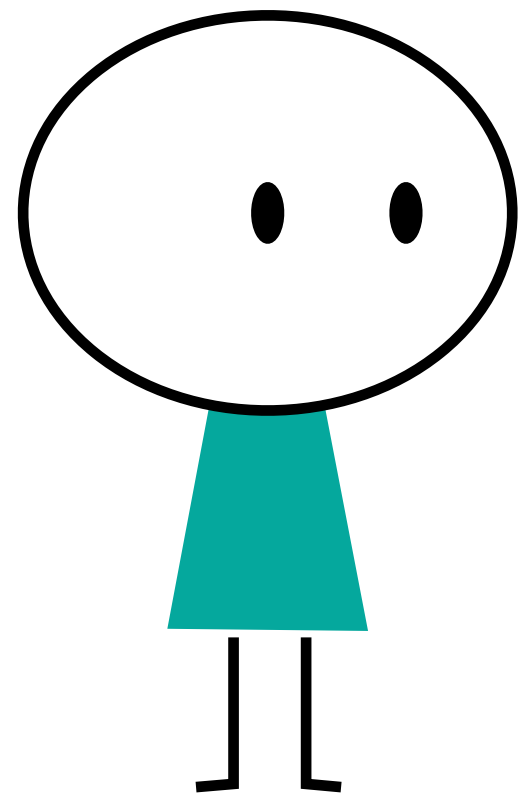
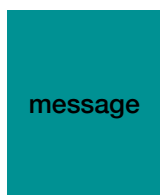
2: $\mathbf{y} \stackrel{\$}{\leftarrow} Y$

3: $\mathbf{c} \leftarrow H(\mathbf{A}\mathbf{y}, m)$

4: $\mathbf{z} \leftarrow \mathbf{c} \cdot \mathbf{S} + \mathbf{y}$

5: while Rejected($\mathbf{z}, \mathbf{c}, \mathbf{S}$)

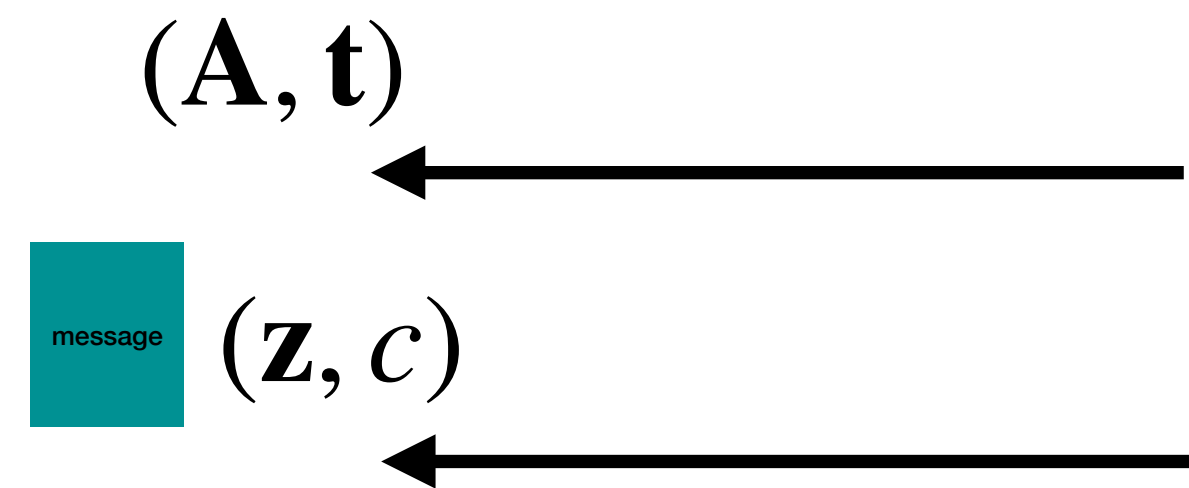
6: return (\mathbf{z}, \mathbf{c})



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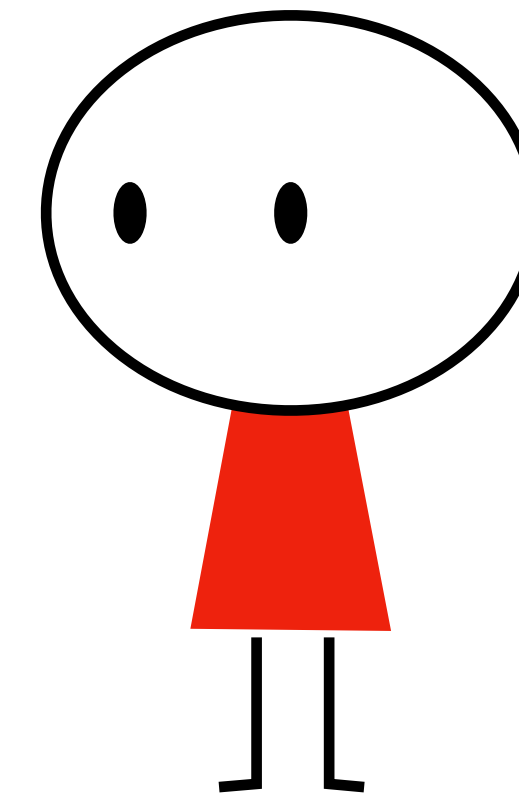
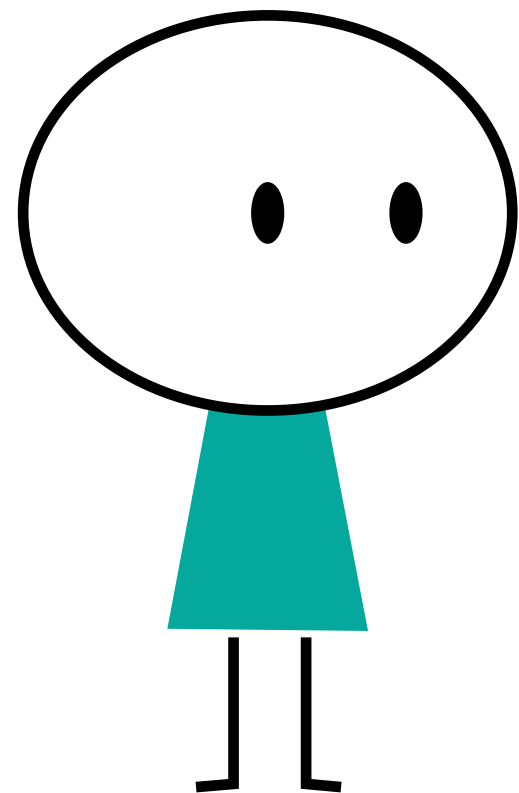
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Signature algorithm:

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Short Integer Solution (SIS)

Verification:

- 1: $\mathbf{r} \leftarrow \mathbf{A} \cdot \mathbf{z} - \mathbf{t} \cdot c$
- 2: $c' \leftarrow H(\mathbf{r}, m)$
- 3: if $c' = c$ and \mathbf{z} is small enough:
- 4: return Valid
- 5: else:
- 6: return Invalid

(\mathbf{A}, \mathbf{t})

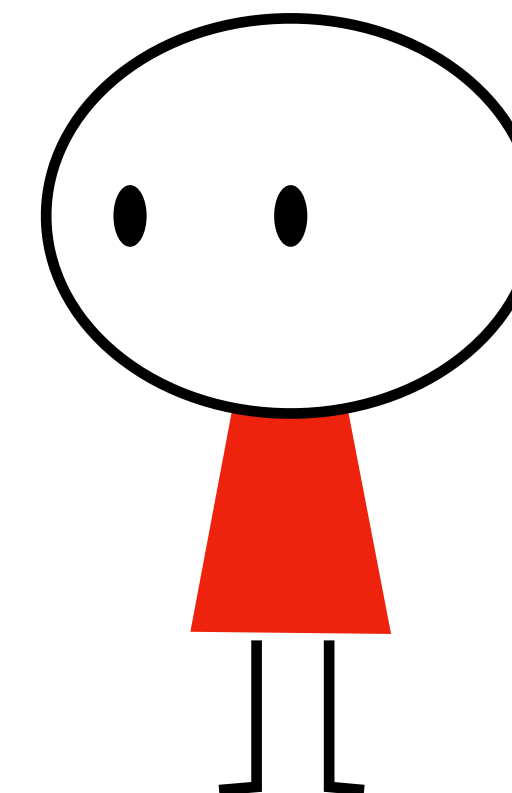
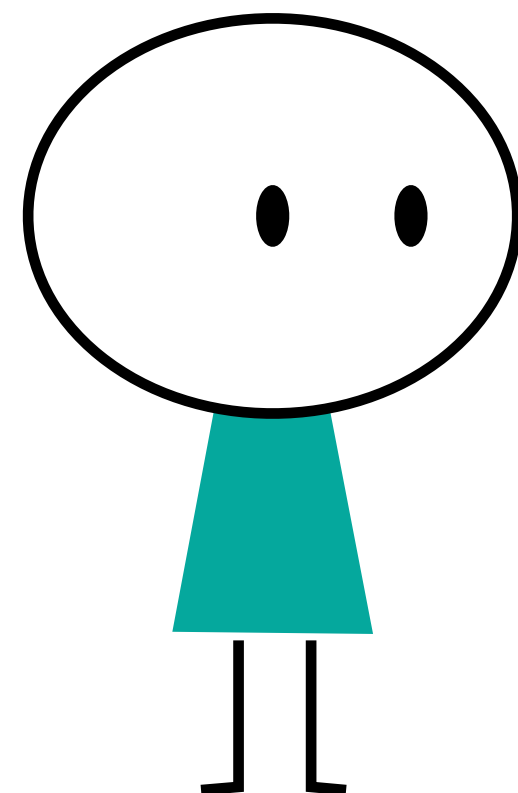


(\mathbf{z}, c)

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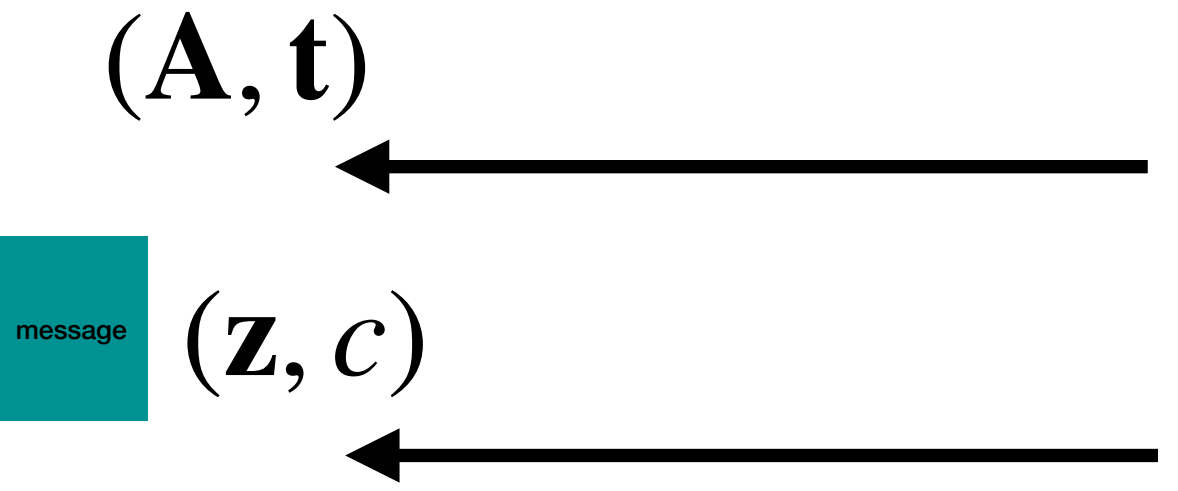
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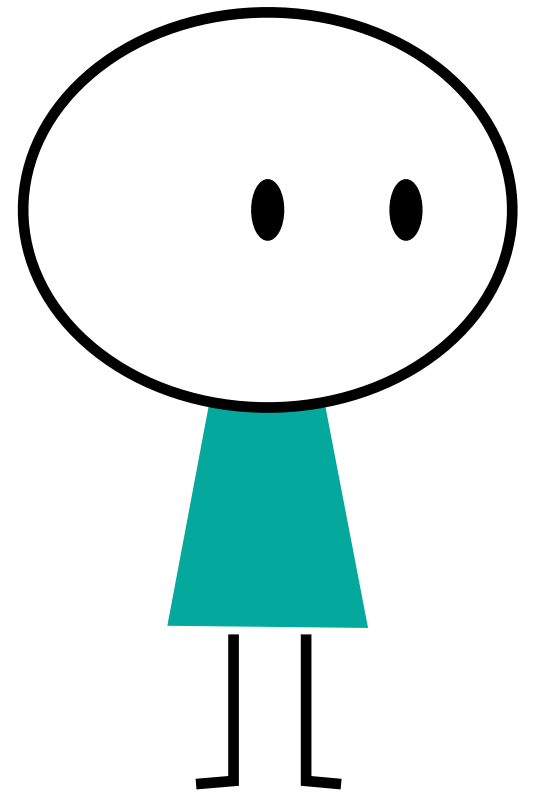
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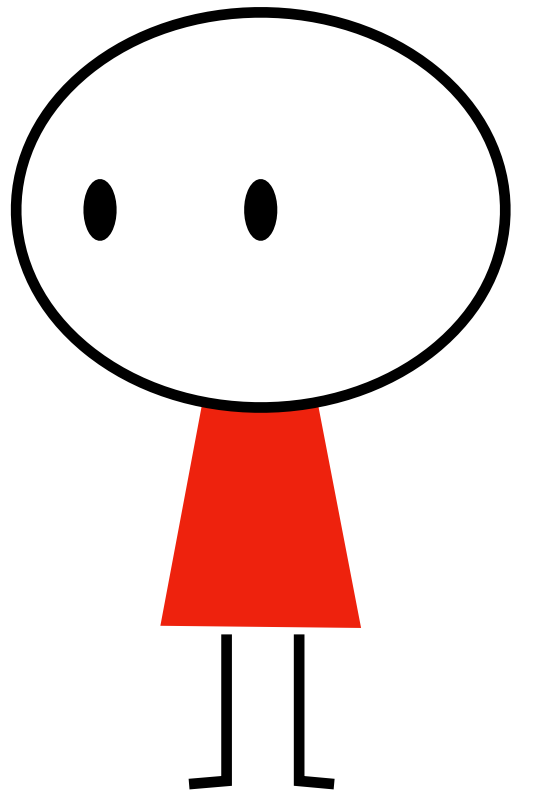
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Signature algorithm:

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High level idea behind
 Crystals-Dilithium (NIST standard)
 BLISS, GLP, BG

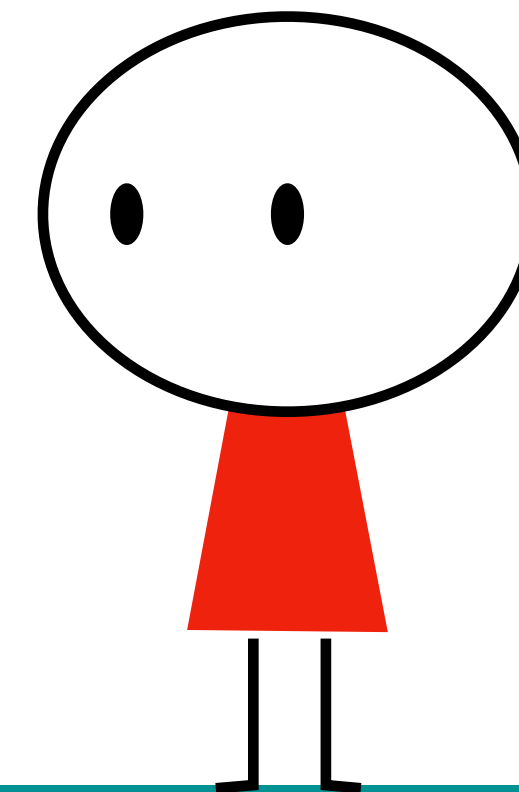
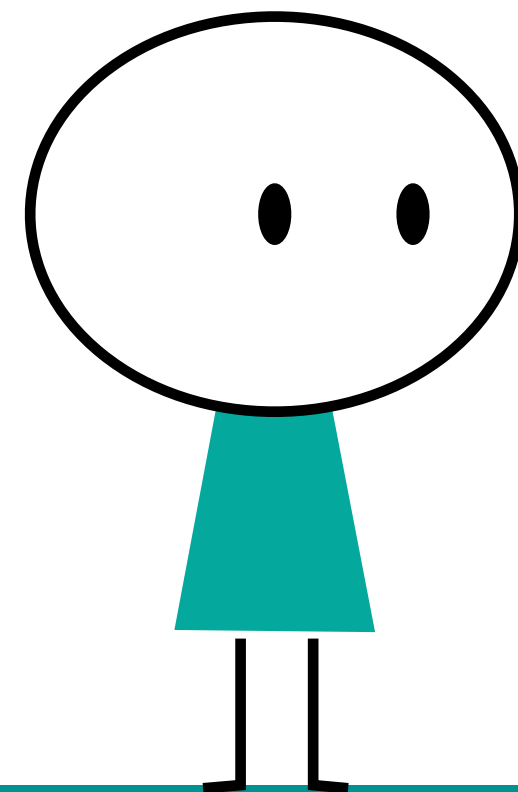


A Hash and sign in a nutshell

► C.Gentry, C. Peikert and V. Vaikuntanathan [STOC'08](#)

Generate matrices \mathbf{A} , \mathbf{B} such that

$$\begin{cases} \mathbf{B}\mathbf{A} = \mathbf{0} \\ \mathbf{B} \text{ has small coefficients} \end{cases}$$



A Hash and sign in a nutshell

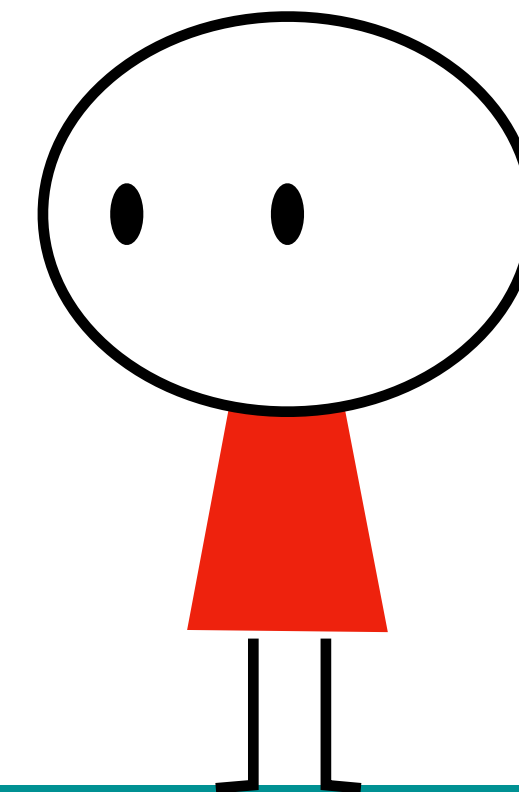
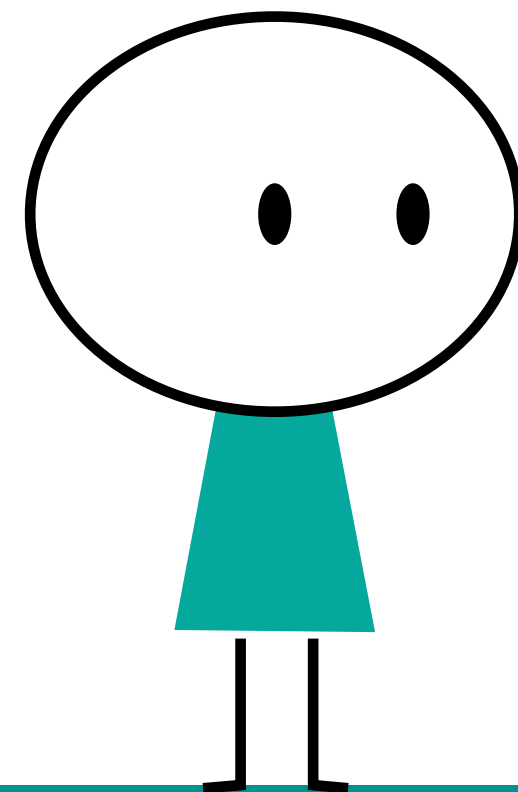
► C.Gentry, C. Peikert and V. Vaikuntanathan [STOC'08](#)

A



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A



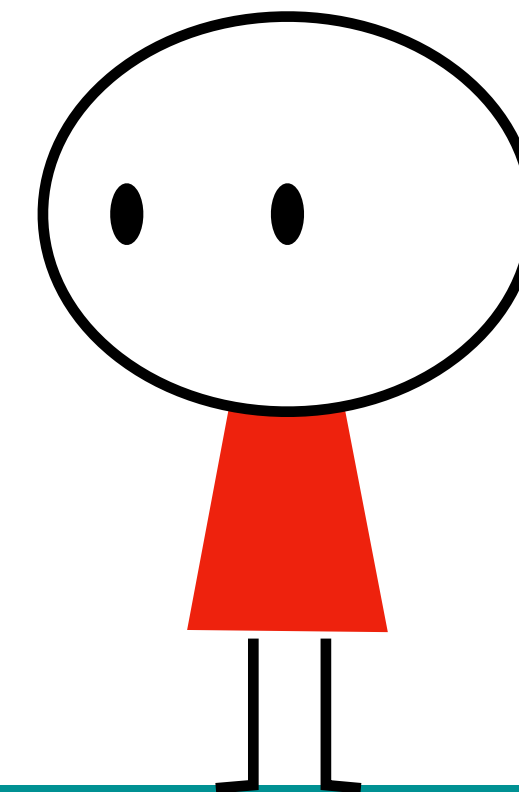
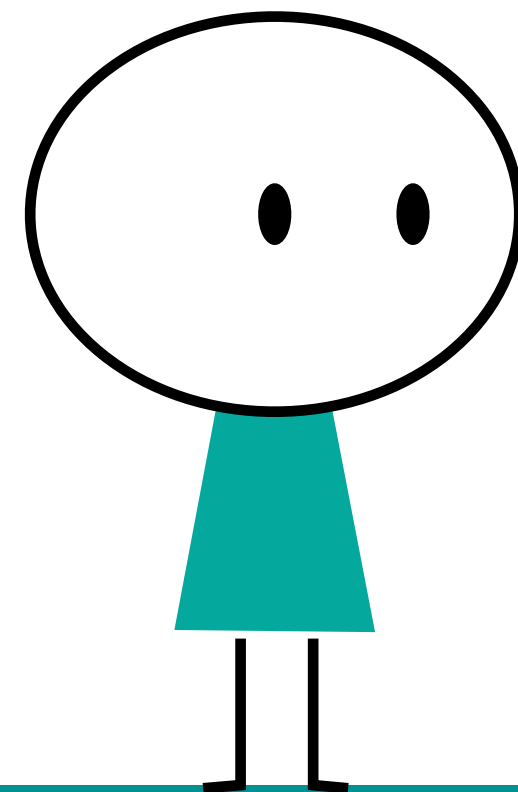
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Signature algorithm:

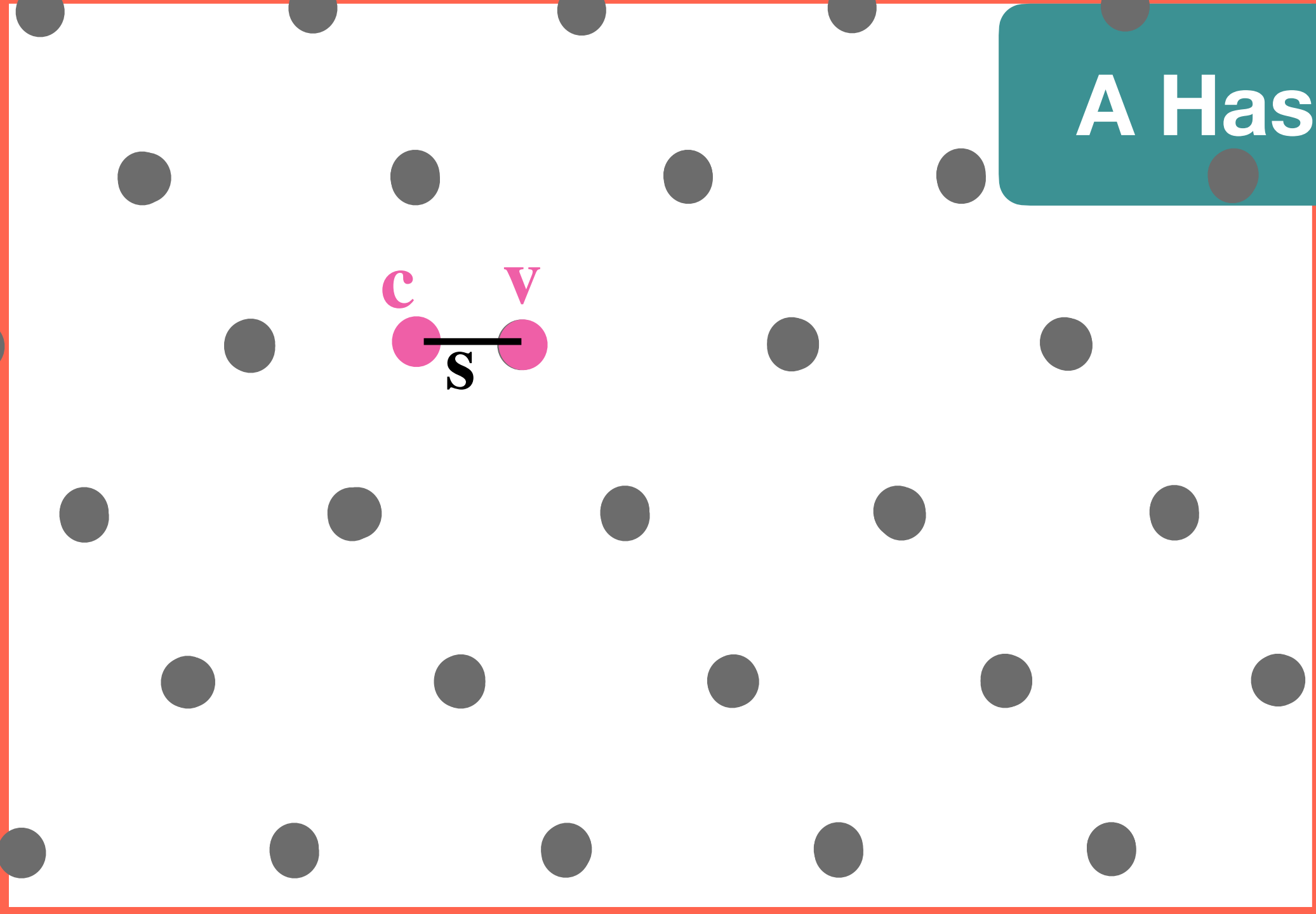
- 1: compute \mathbf{c} such that $\mathbf{c}\mathbf{A} = H(m)$
- 2: $\mathbf{v} \leftarrow$ a vector in $\Lambda(\mathbf{B})$ close to \mathbf{c}
- 3: return $\mathbf{s} \leftarrow \mathbf{c} - \mathbf{v}$

message



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A



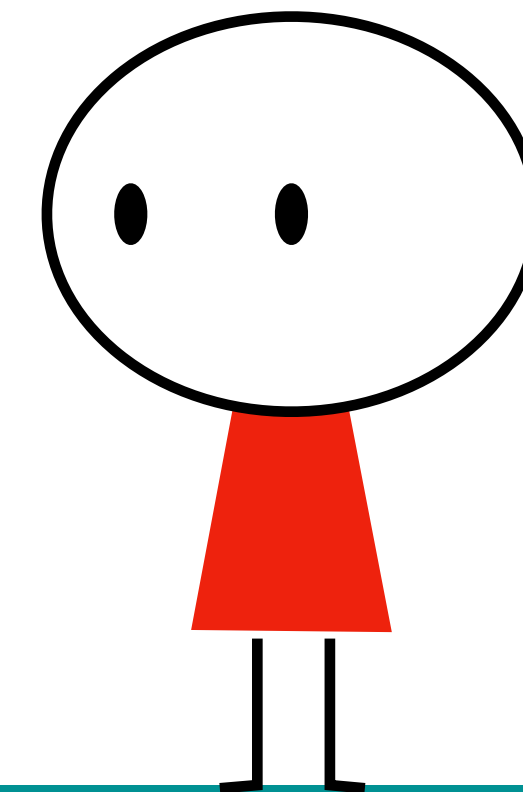
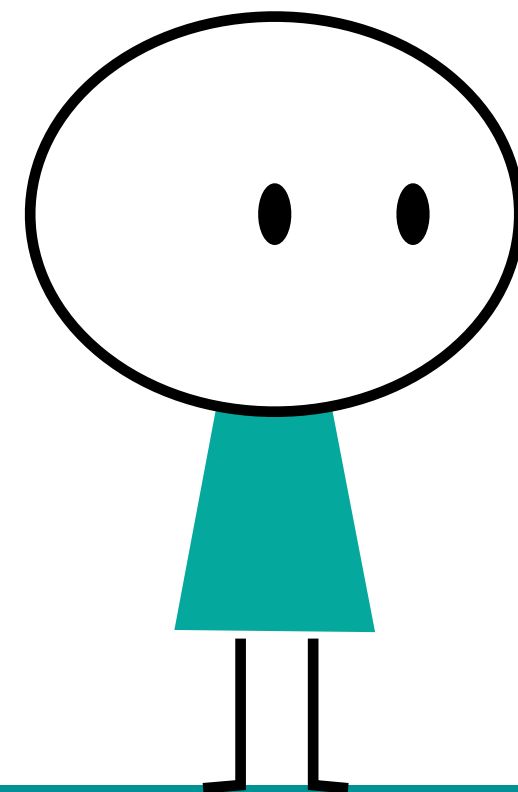
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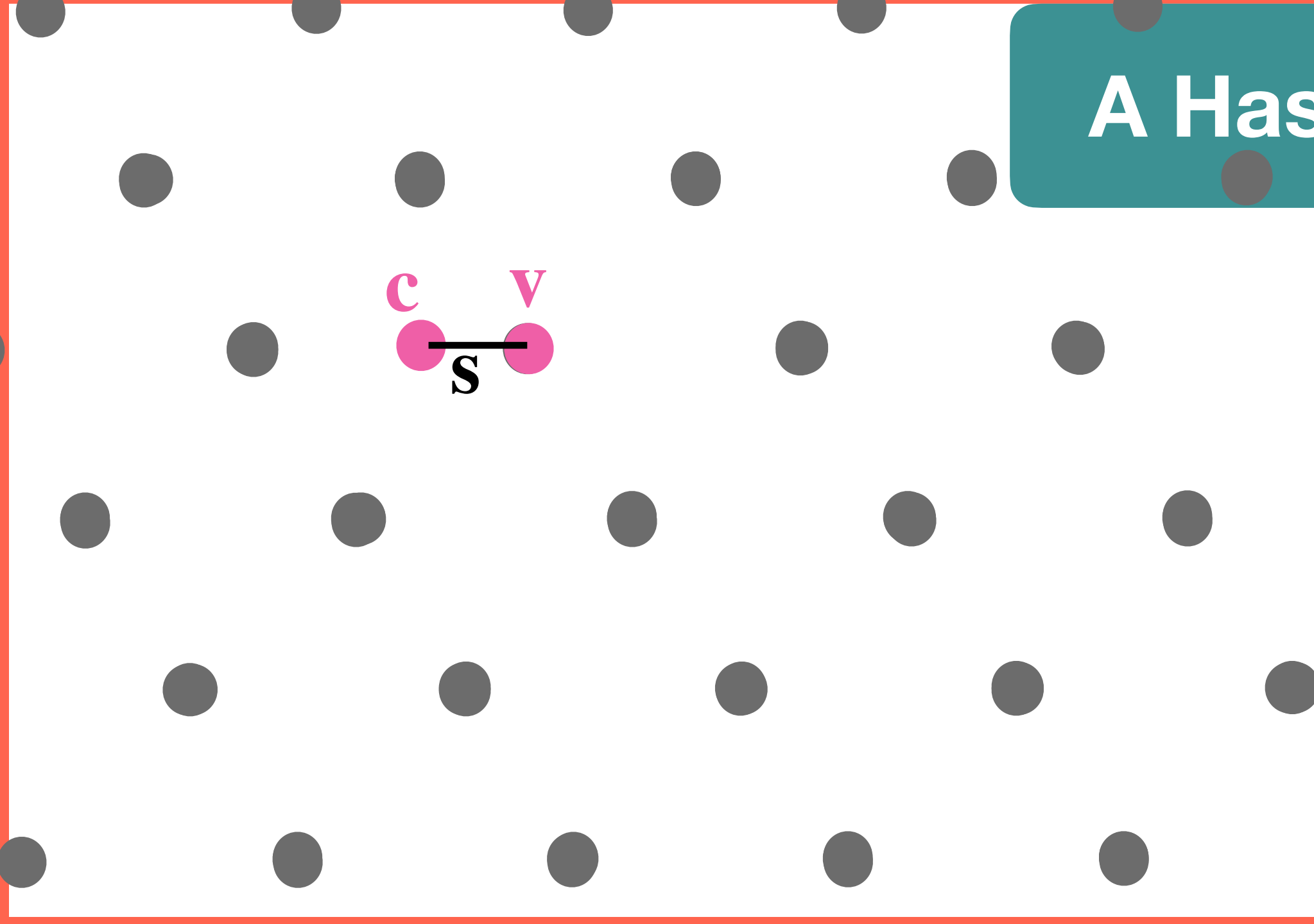
- 1: compute \mathbf{c} such that $\mathbf{c}\mathbf{A} = H(m)$
- 2: $\mathbf{v} \leftarrow$ a vector in $\Lambda(\mathbf{B})$ close to \mathbf{c}
- 3: return $\mathbf{s} \leftarrow \mathbf{c} - \mathbf{v}$

message



A Hash and sign in a nutshell

► C.Gentry, C. Peikert and V. Vaikuntanathan STOC'08



A



Generate matrices **A**, **B** such that

$$\begin{cases} \mathbf{BA} = \mathbf{0} \\ \mathbf{B} \text{ has small coefficients} \end{cases}$$

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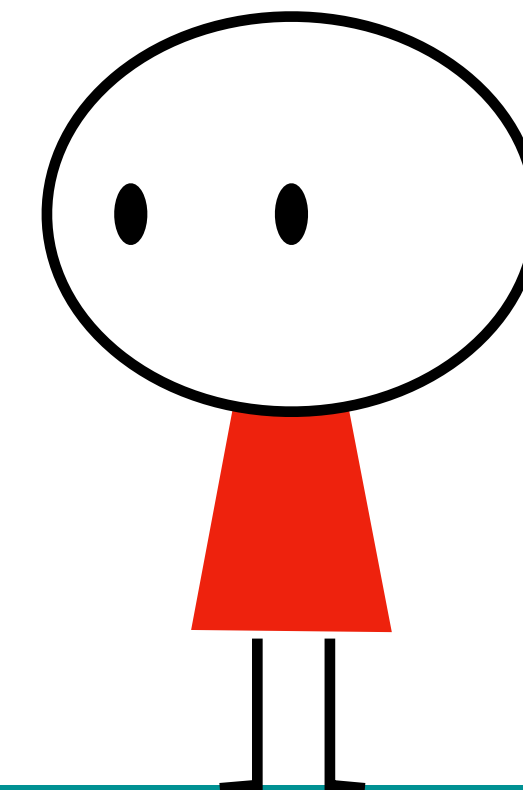
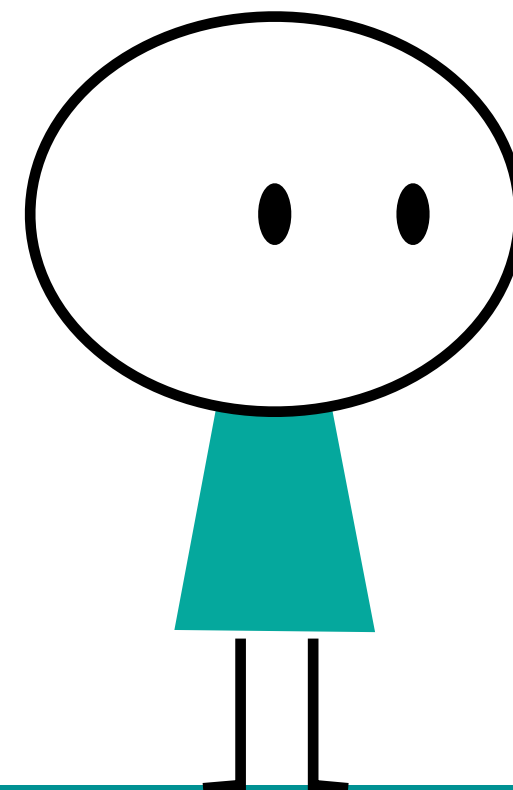
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s

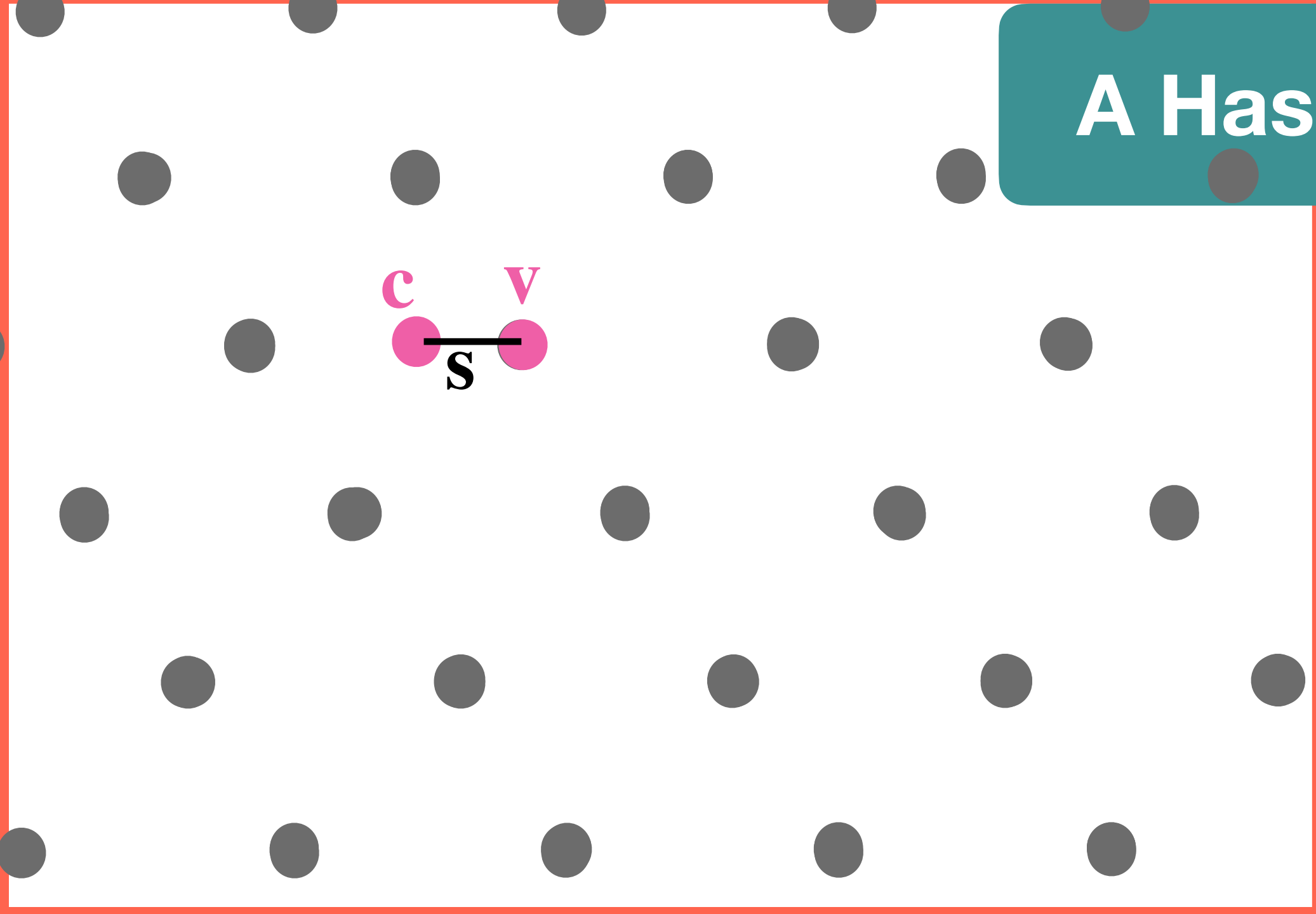


message



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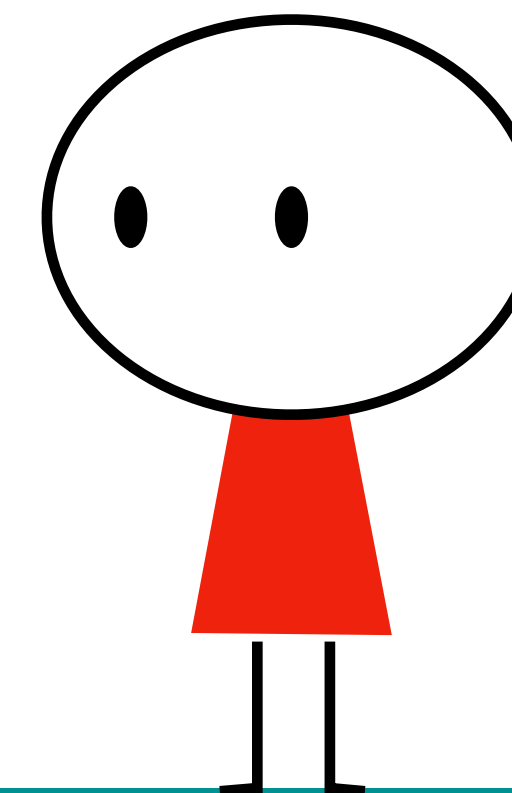
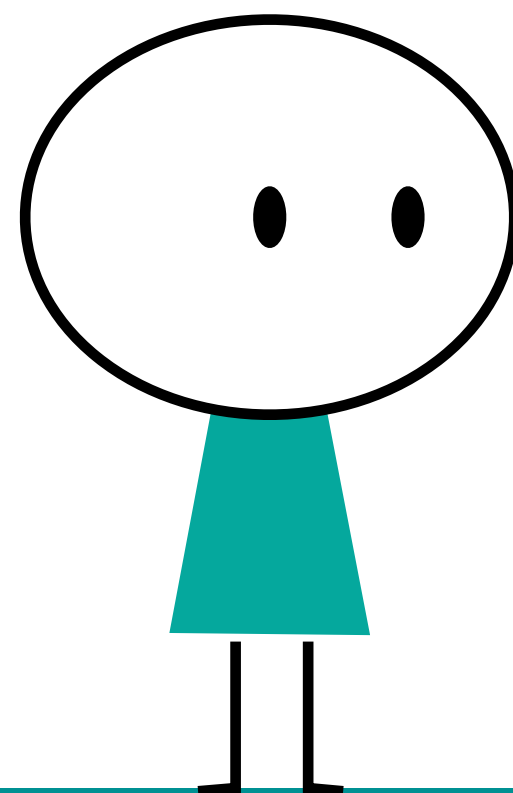


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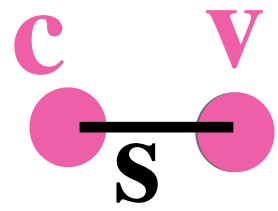
Verification:

- 1: if \mathbf{s} is short and $\mathbf{s}\mathbf{A} = H(m)$
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- 3: else:
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A Hash and sign in a nutshell

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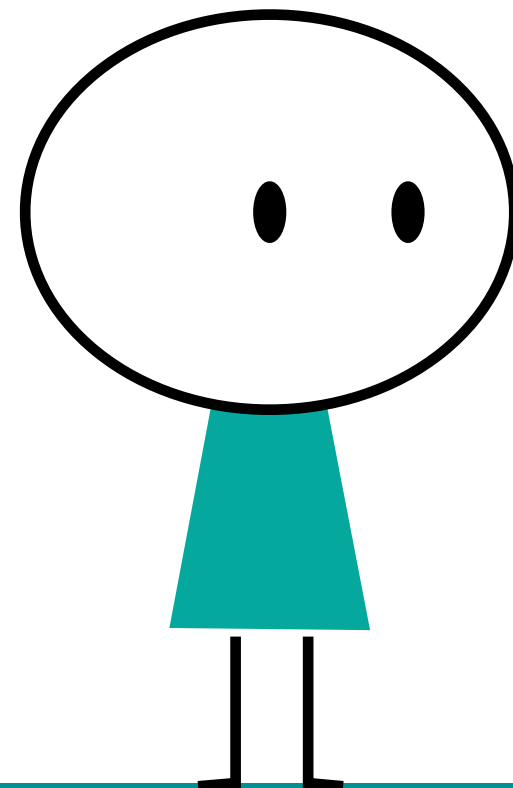


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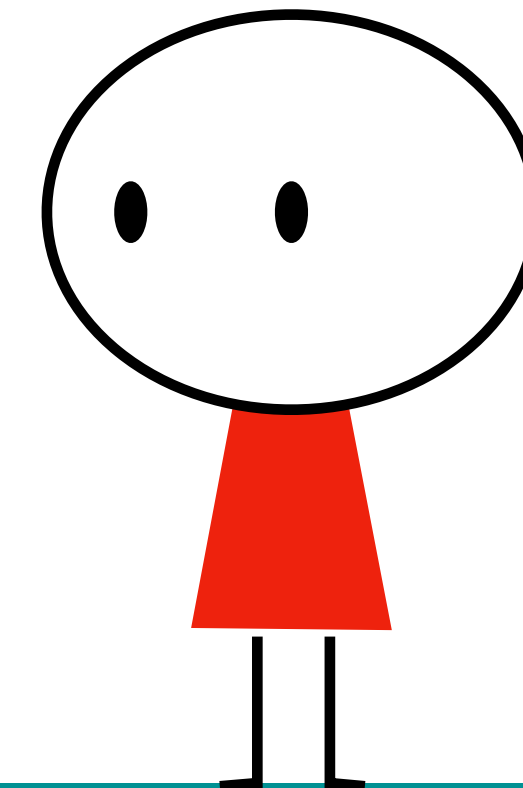
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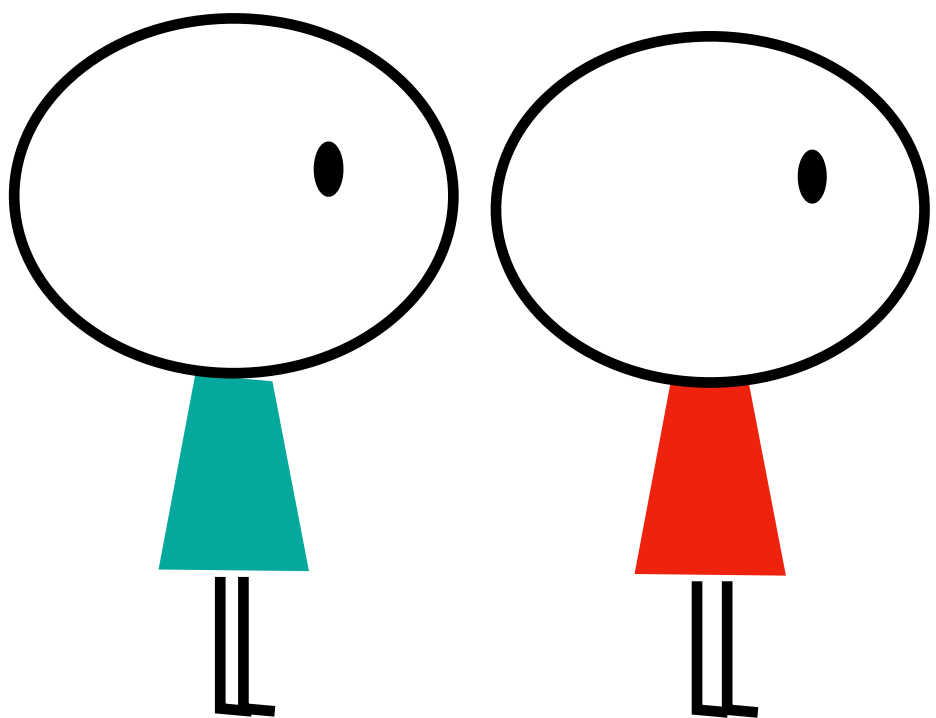
High level idea behind

Falcon (NIST standard)
GPV, Mitaka



Signature schemes

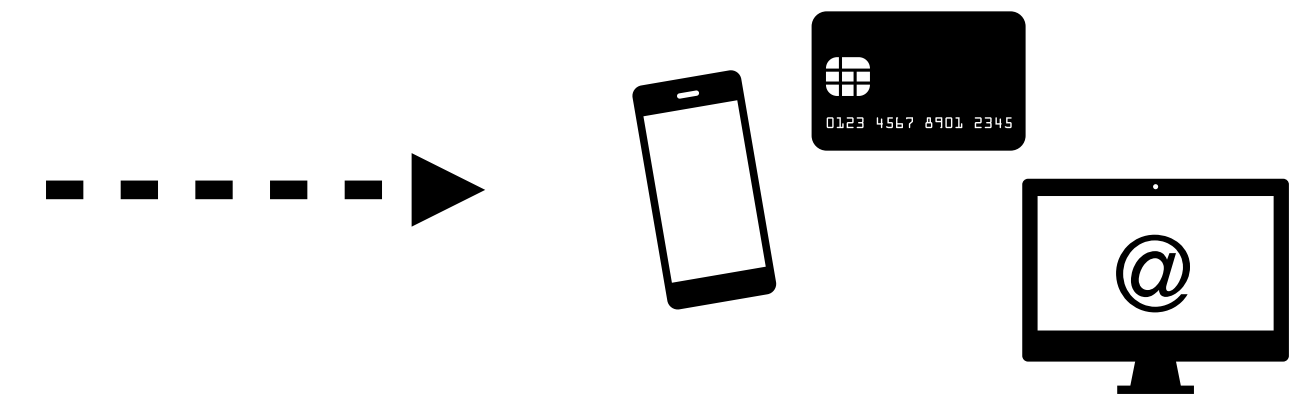
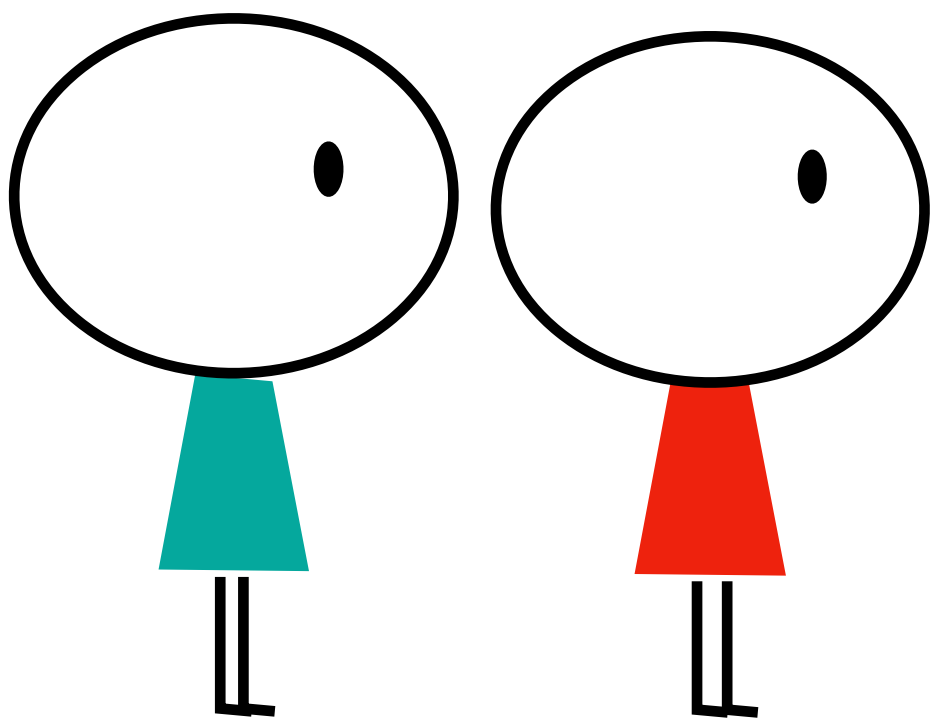
Public key encryption schemes



Lattice-based algorithms

Signature schemes

Public key encryption schemes



Lattice-based algorithms

Are you secure for real-world development ?

Are you **timing** resistant ?

Are you secure against **physical attacks**?

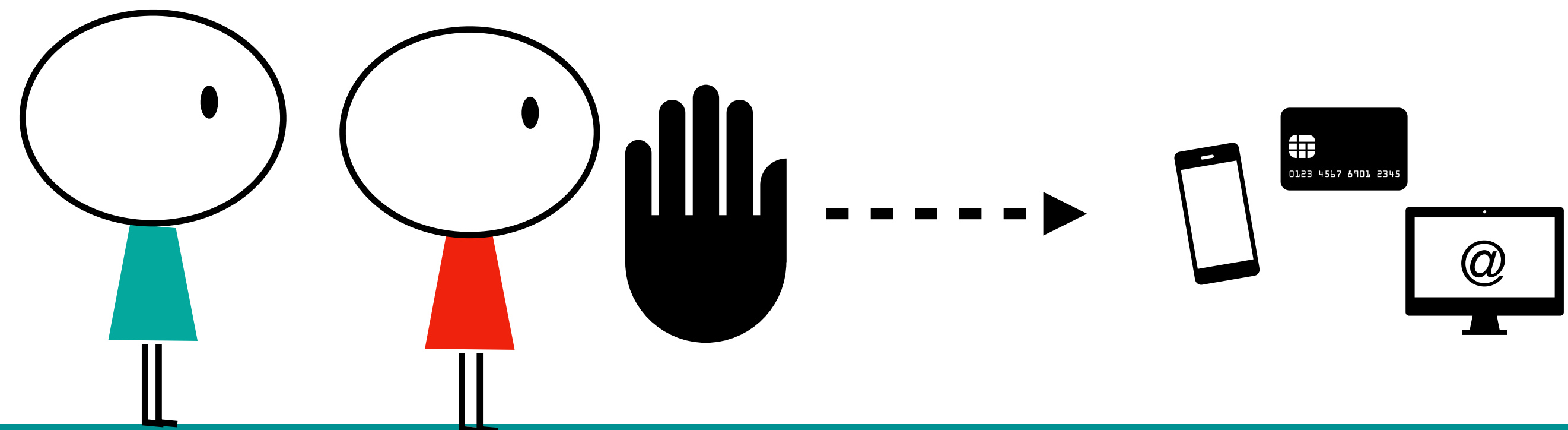
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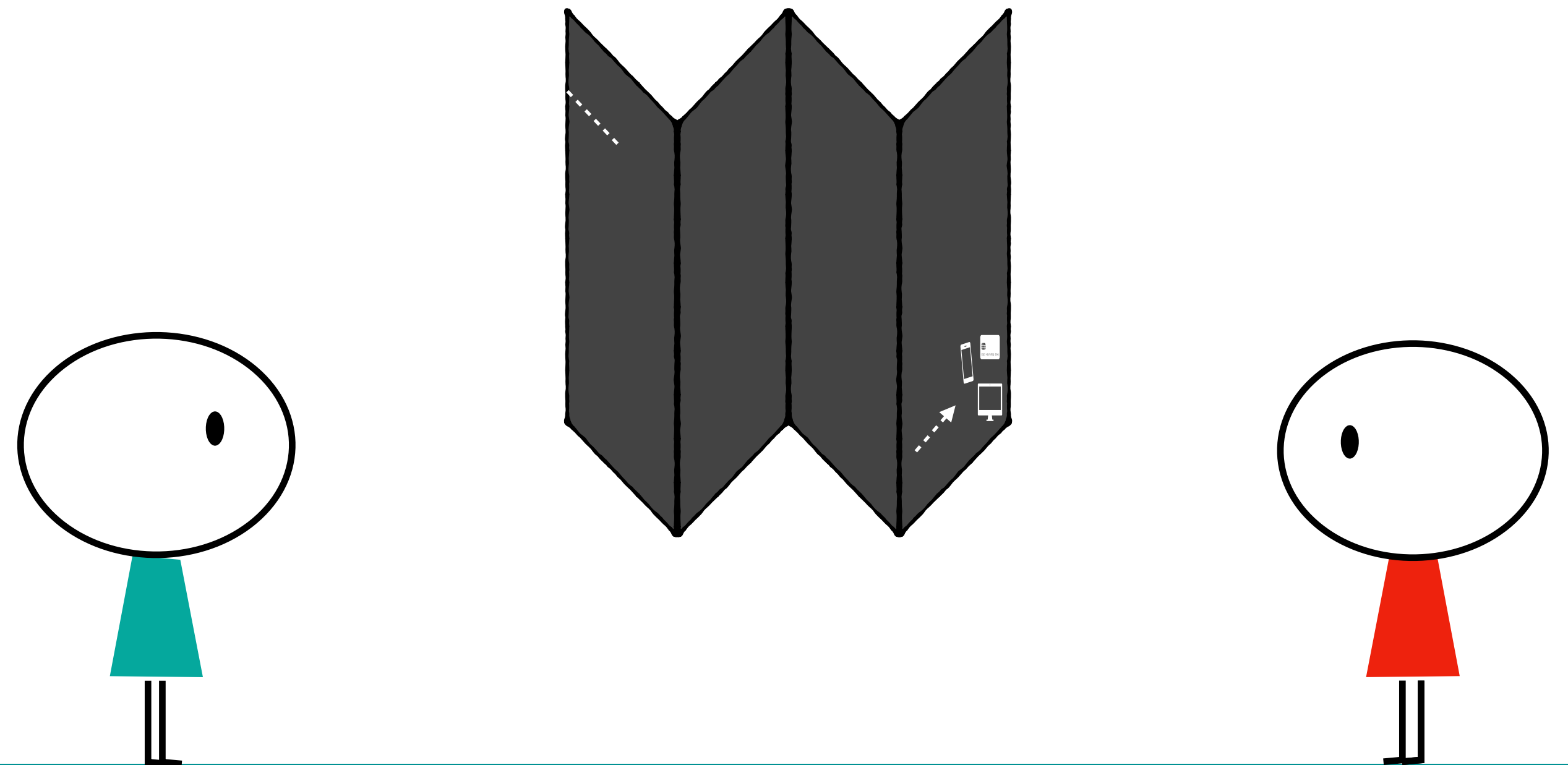
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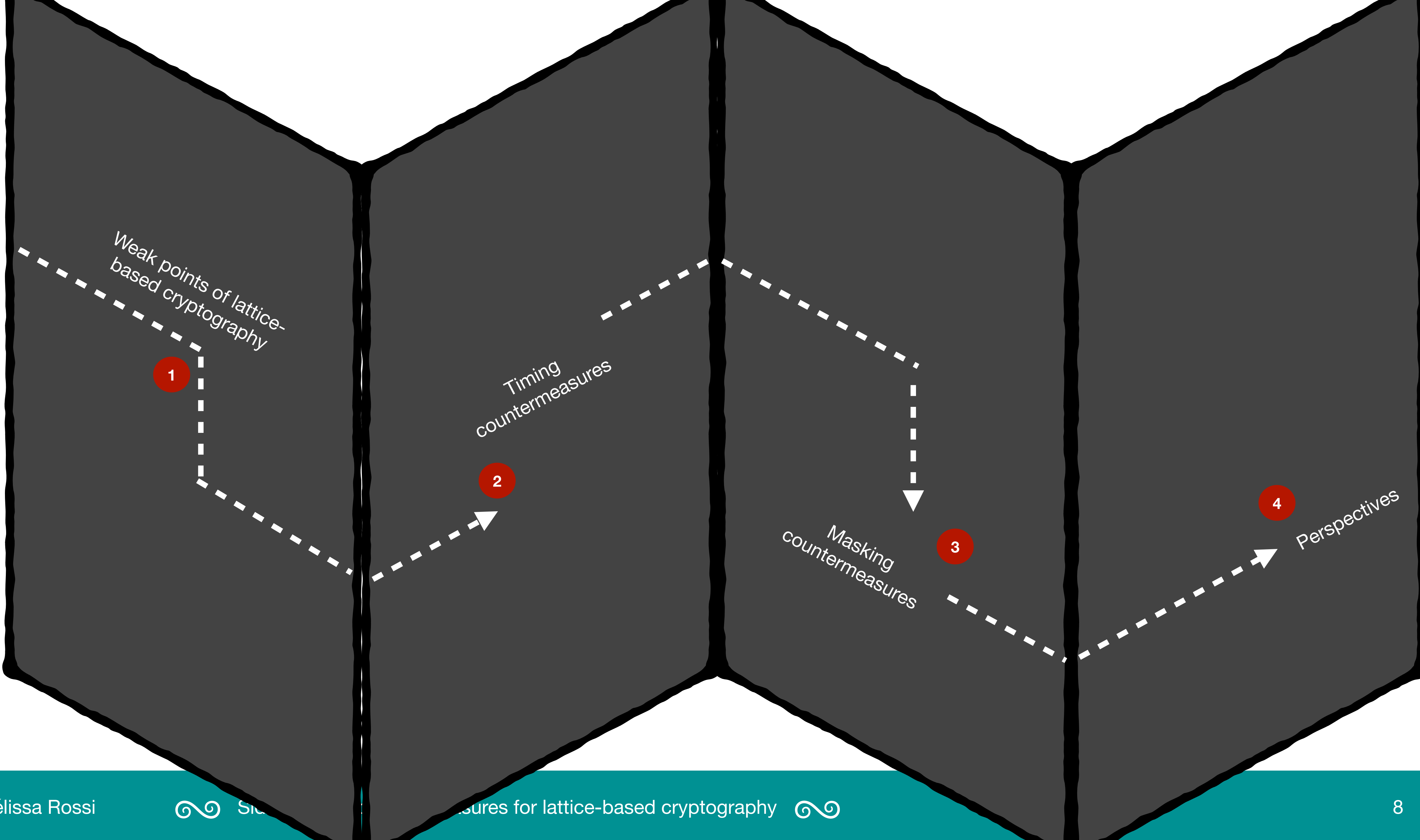
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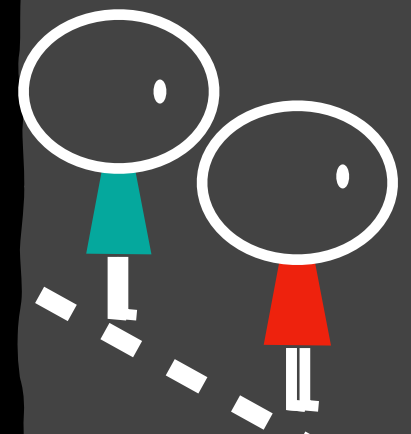
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Weak points of lattice-based cryptography

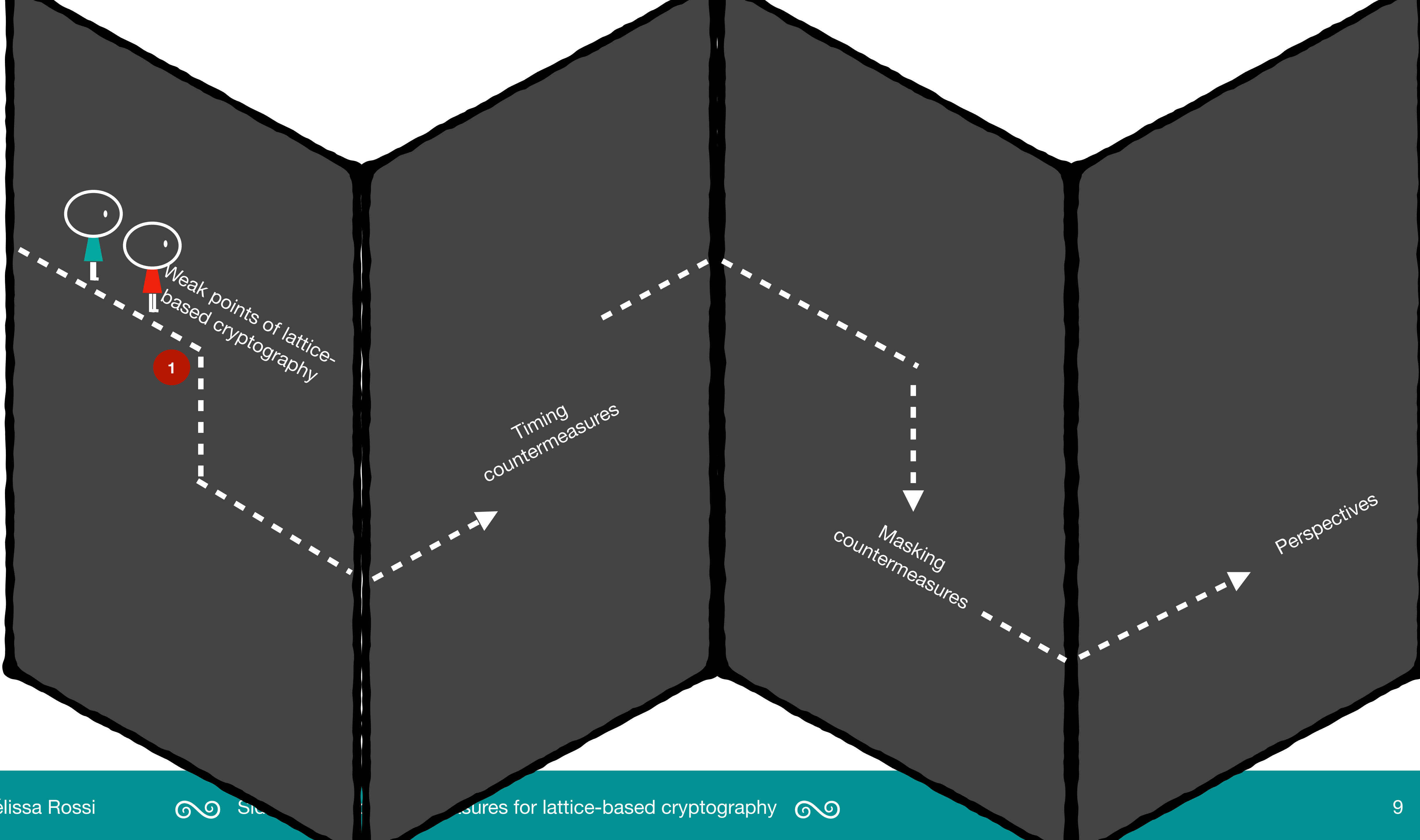
1

Timing countermeasures

Masking countermeasures

Perspectives





Lattice-based crypto has many side-channel weak points

$(\mathbf{A}, \mathbf{t} = \mathbf{A}\mathbf{S} \bmod q)$

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Usual suspects:

- A. Multiplication with the secret: known $\times \mathbf{s}$
- B. Complex internal sampling distributions (Cumulative Distribution Tables)
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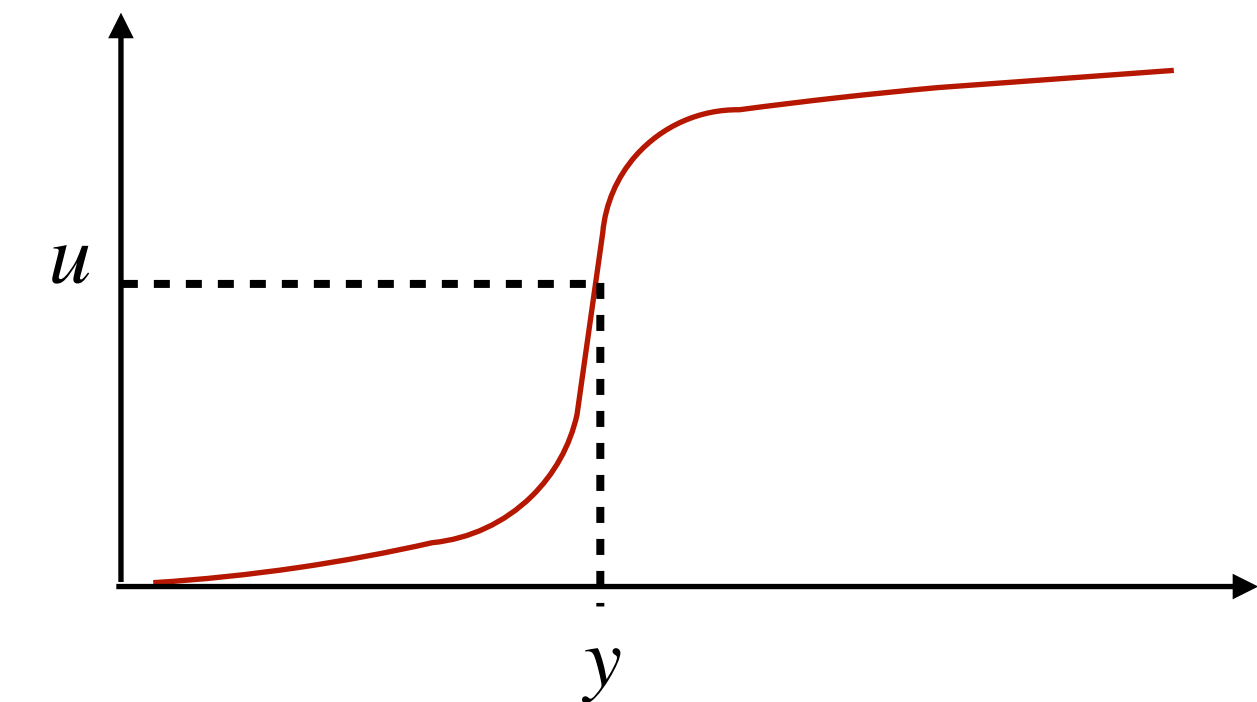
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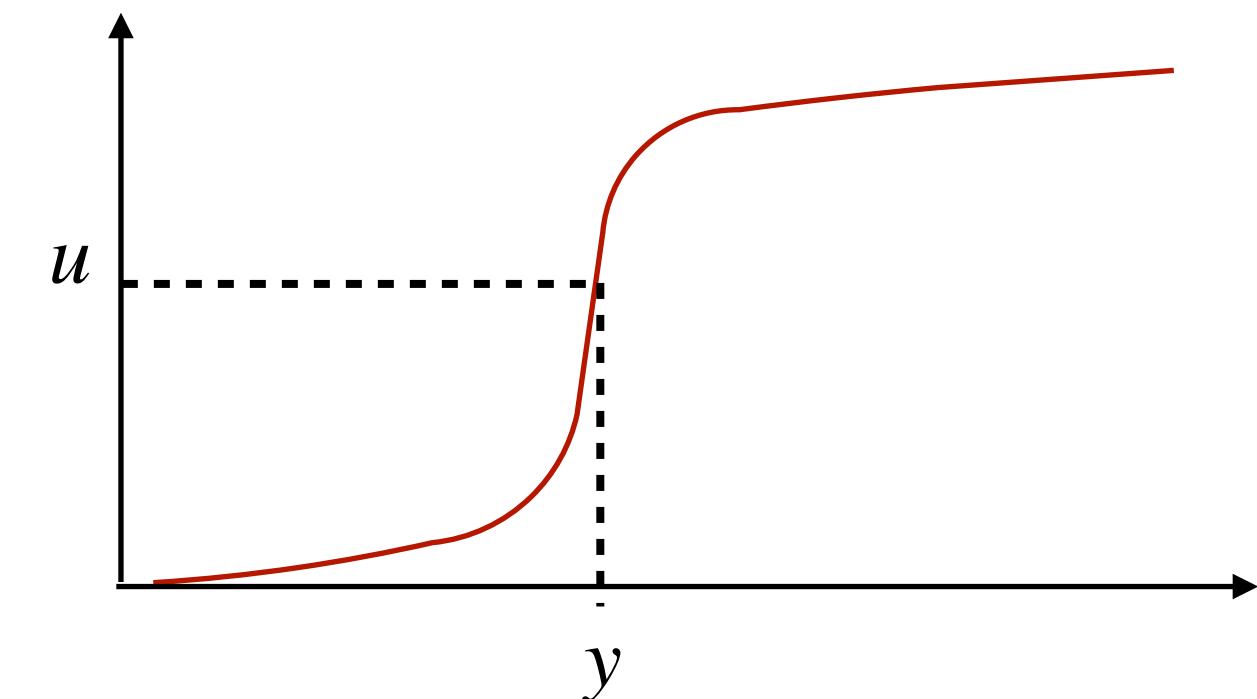
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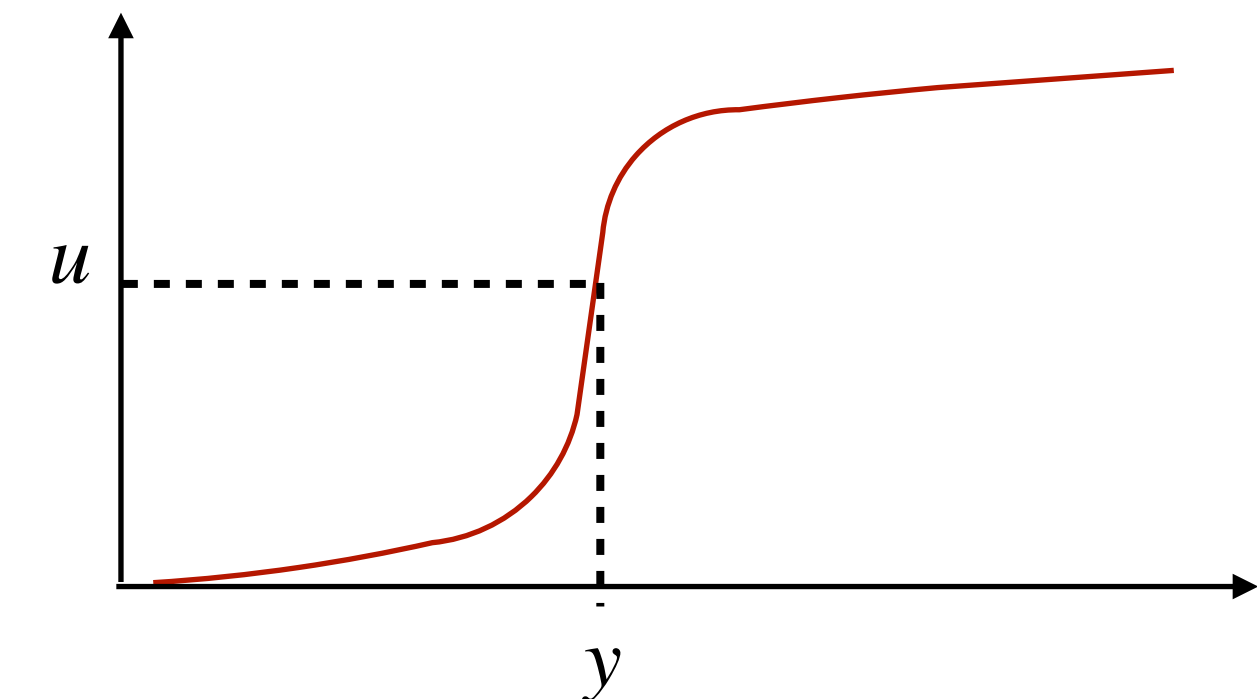
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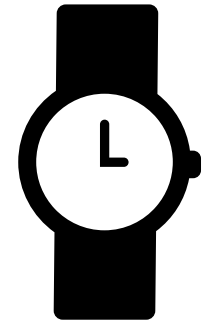
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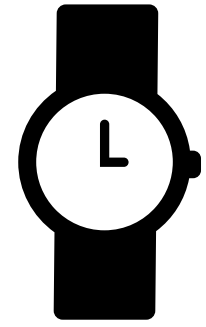
We will give three examples related to B and C

Timing attacks on internal distributions



Timing attack: the attacker knows the time that the algorithm takes e.g. the number of iterations.

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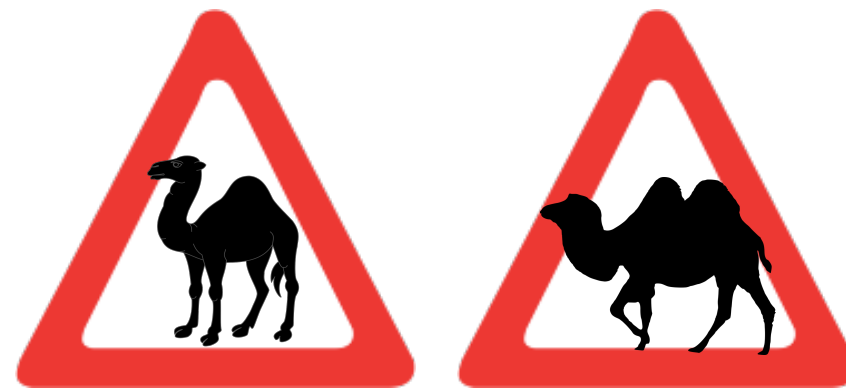
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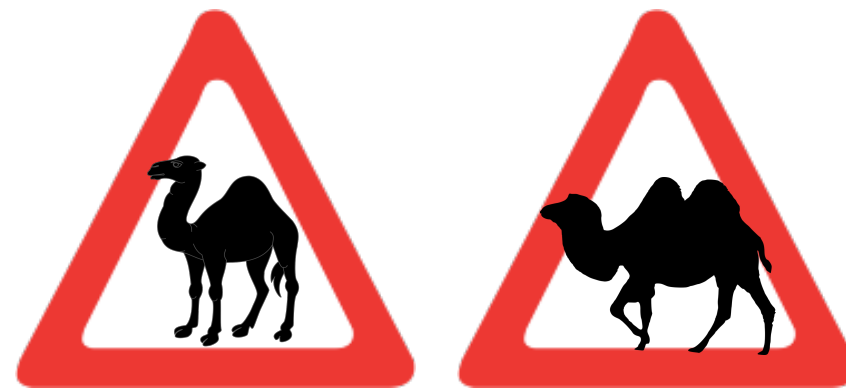
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An example presented in the next slide →



An example of timing attack on BLISS signature scheme

Signature algorithm:

1: do

2: $\mathbf{y} \xleftarrow{\$} Y$

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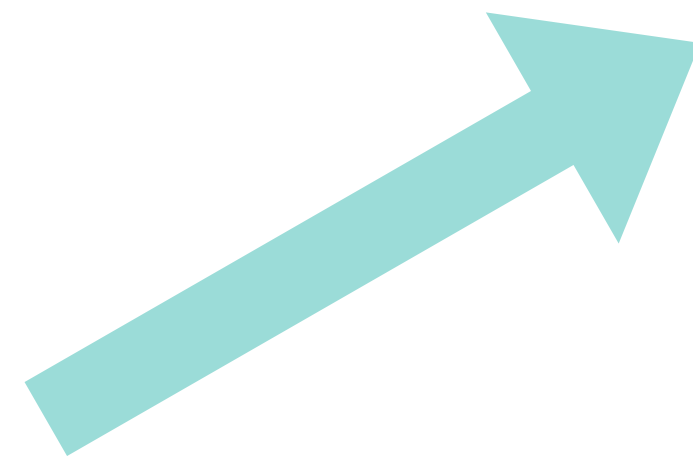
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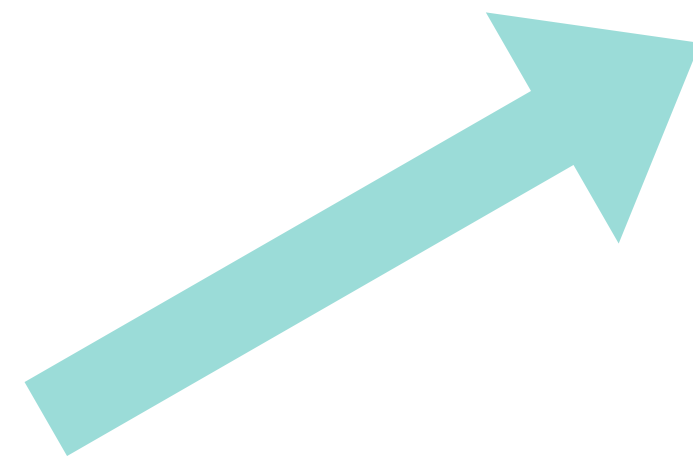
$$\text{Rejected}(\mathbf{z}, \mathbf{c}, \mathbf{S}) = \frac{1}{M \cdot \cosh\left(\frac{\langle \mathbf{z}, \mathbf{S}\mathbf{c} \rangle}{\sigma^2}\right) \cdot \exp\left(-\frac{\|\mathbf{S}\mathbf{c}\|^2}{2\sigma^2}\right)}$$



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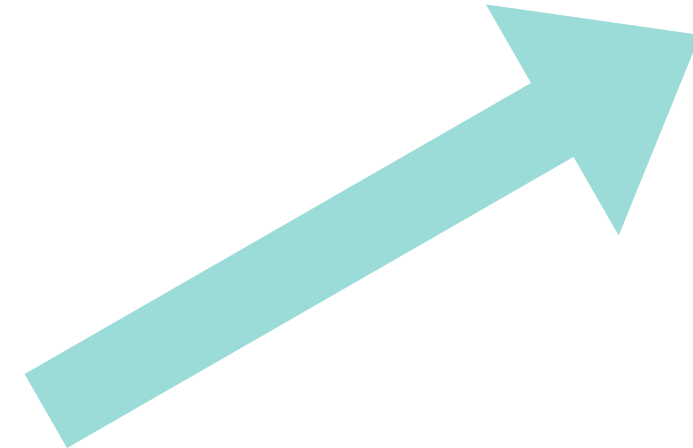
➔ computed by sampling two Bernoulli $\mathcal{B}_{1/\cosh(x)}$ and $\mathcal{B}_{\exp(-x)}$



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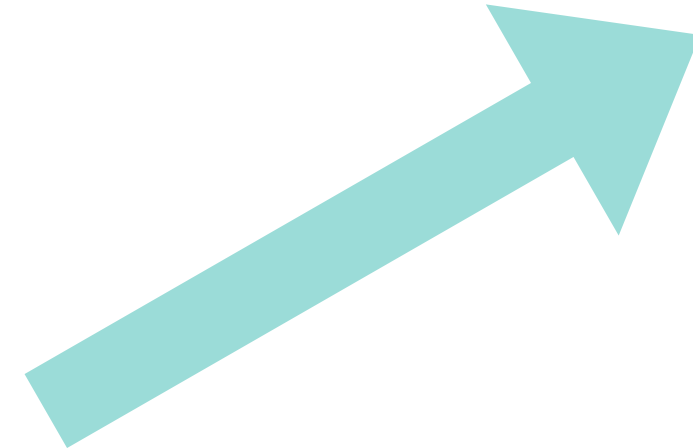
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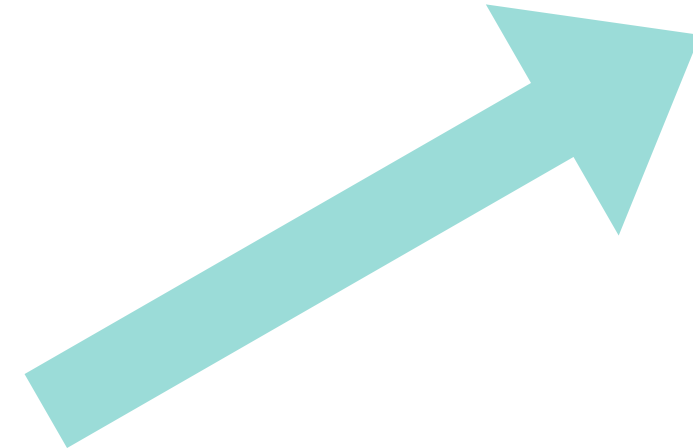
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Correctness

The distribution of a is indeed $\mathcal{B}_{1/\cosh(x)}$.

▸ L. Ducas, A. Durmus, T. Lepoint and V. Lyubashevsky CRYPTO'13

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→ Probability of going from step 5 to step 6:

$$\begin{aligned}\mathbb{P}(\overline{\bar{a} \wedge (b \vee c)}) &= 1 - \mathbb{P}(\bar{a}) \cdot \mathbb{P}(b \vee c) \\ &= 1 - (1 - \mathbb{P}(a)) \cdot (1 - \mathbb{P}(\bar{b} \wedge \bar{c})) \\ &= 1 - (1 - \exp(-x)) \left(1 - \frac{1 - \exp(-x)}{2} \right) \\ &= \frac{1 + \exp(-2x)}{2}\end{aligned}$$

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$$\begin{aligned}\mathbb{P}(\overline{\bar{a} \wedge (b \vee c)}) &= 1 - \mathbb{P}(\bar{a}) \cdot \mathbb{P}(b \vee c) \\ &= 1 - (1 - \mathbb{P}(a)) \cdot (1 - \mathbb{P}(\bar{b} \wedge \bar{c})) \\ &= 1 - (1 - \exp(-x)) \left(1 - \frac{1 - \exp(-x)}{2} \right) \\ &= \frac{1 + \exp(-2x)}{2}\end{aligned}$$

Depends on the input!

An example of timing attack on BLISS signature scheme

Sampling a Bernoulli with parameter $\cosh(x) : \mathcal{B}_{1/\cosh(x)}$

- 1: $x \leftarrow |x|$
- 2: $a \leftarrow \mathcal{B}_{\exp(-x)}$
- 3: $b \leftarrow \mathcal{B}_{1/2}$
- 4: $c \leftarrow \mathcal{B}_{\exp(-x)}$
- 5: if $\bar{a} \wedge (b \vee c)$ then restart
- 6: return a

Even if every Bernoulli sampling is constant time, there is still **timing attack!**

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Depends on the input!

Idea of the attack

Here $x = -|\langle z, \mathbf{S}c \rangle|$

We select the signatures (z, c) corresponding to **one iteration inside the Bernoulli sampling.**

It means that $\frac{1 + \exp(-2|\langle z, \mathbf{S}c \rangle|)}{2}$ is large.

Then, $|\langle z, \mathbf{S}c \rangle|$ is close to 0.

→ Can be solved with a phase retrieval algorithm (machine learning).

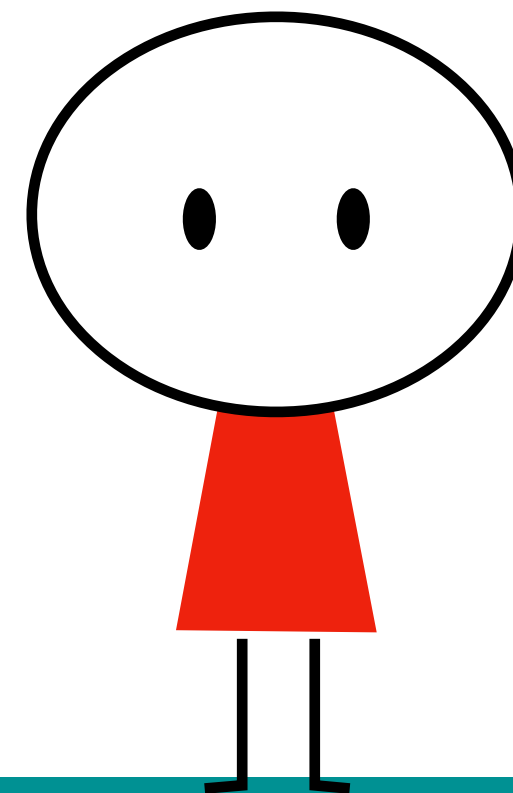
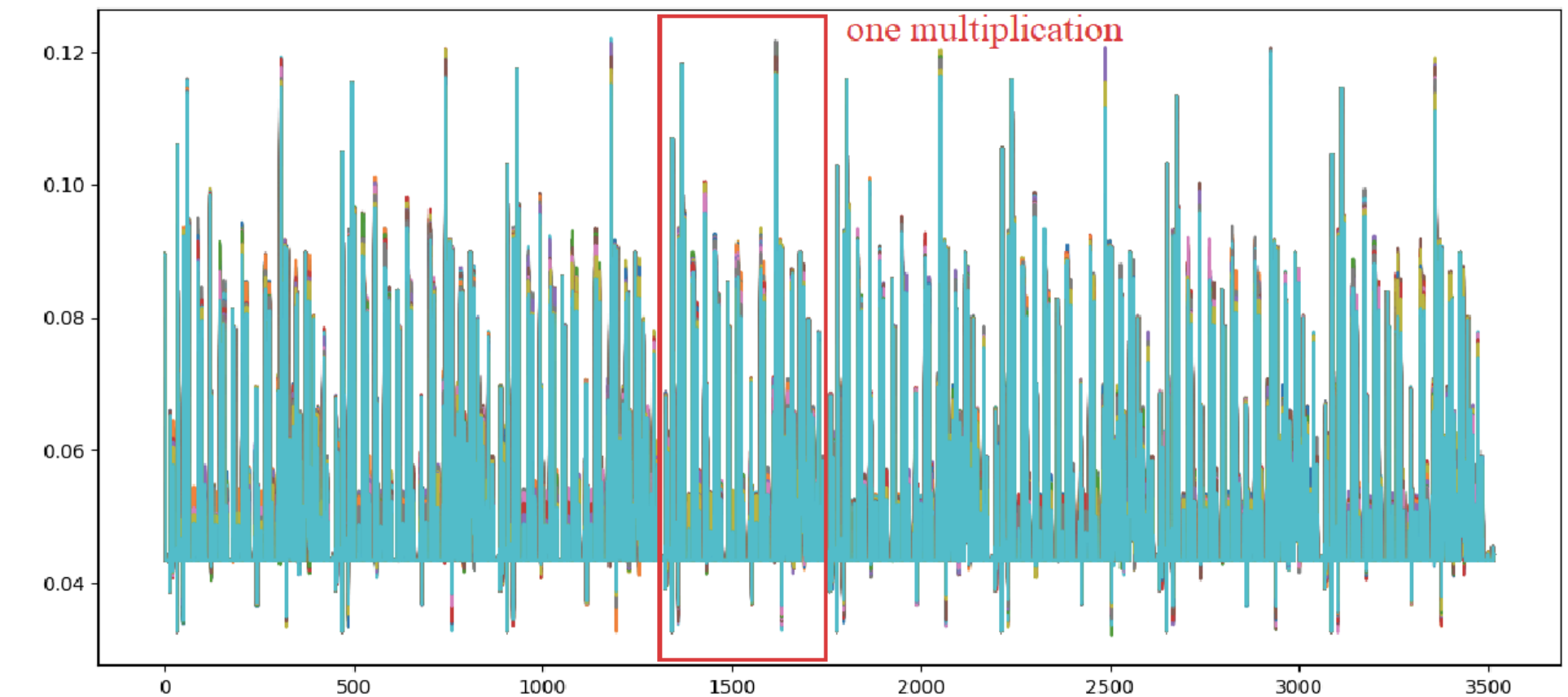
Full key recovery in an average of 40h on a powerful personal computer

Power consumption attacks

Power consumption attack: the attacker knows the power consumption of the device executing the algorithm. He has access to « traces ».

Many attacks as well

- ▶ R. Primas, P. Pessl, S. Magnard. [CHES'2017](#)
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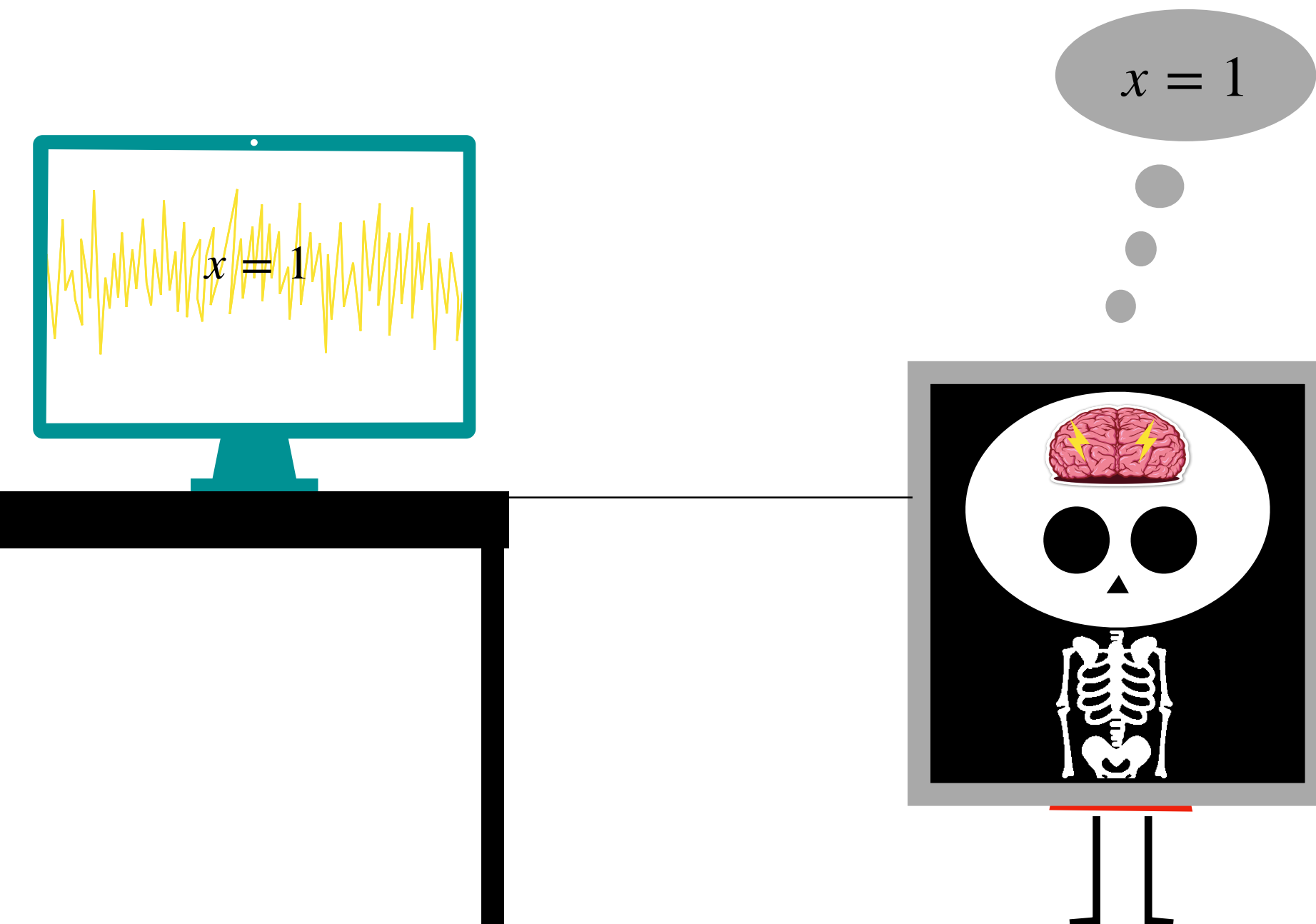
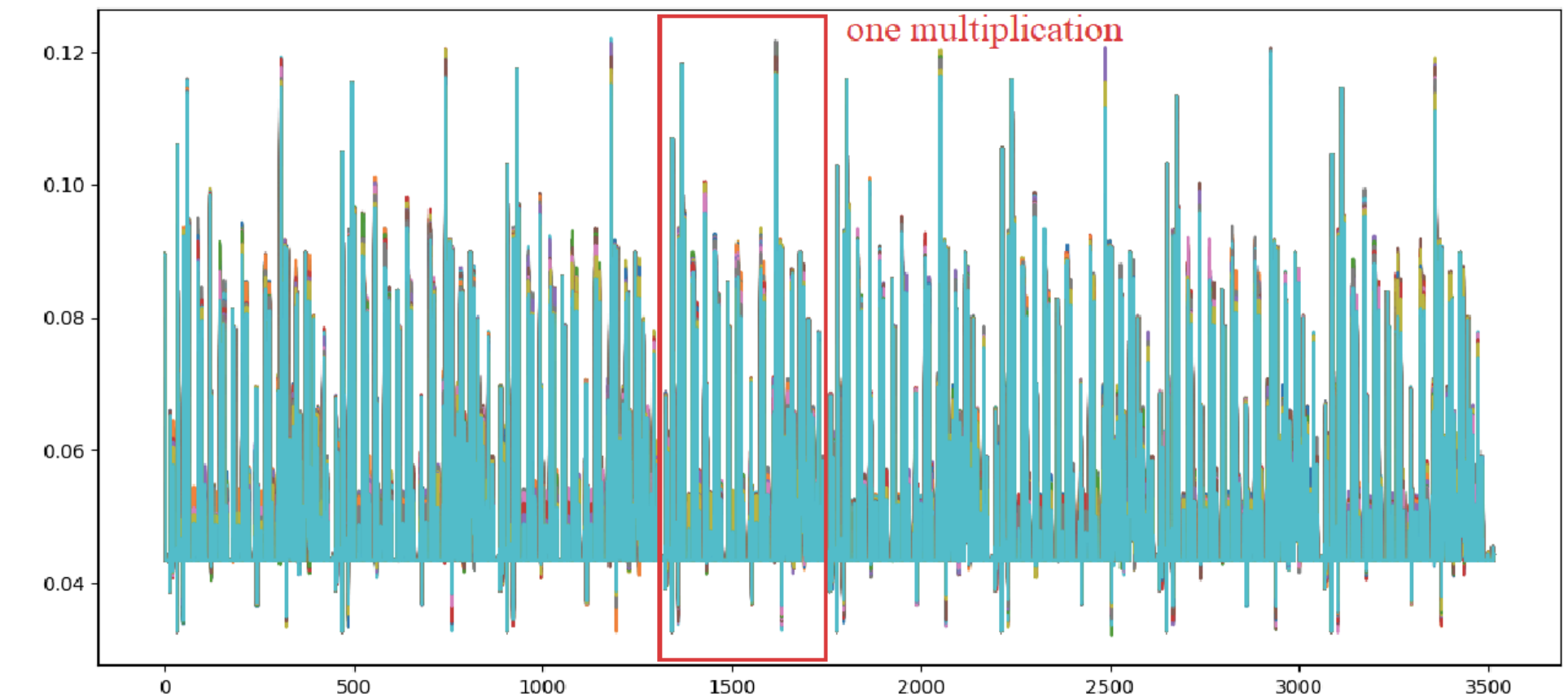


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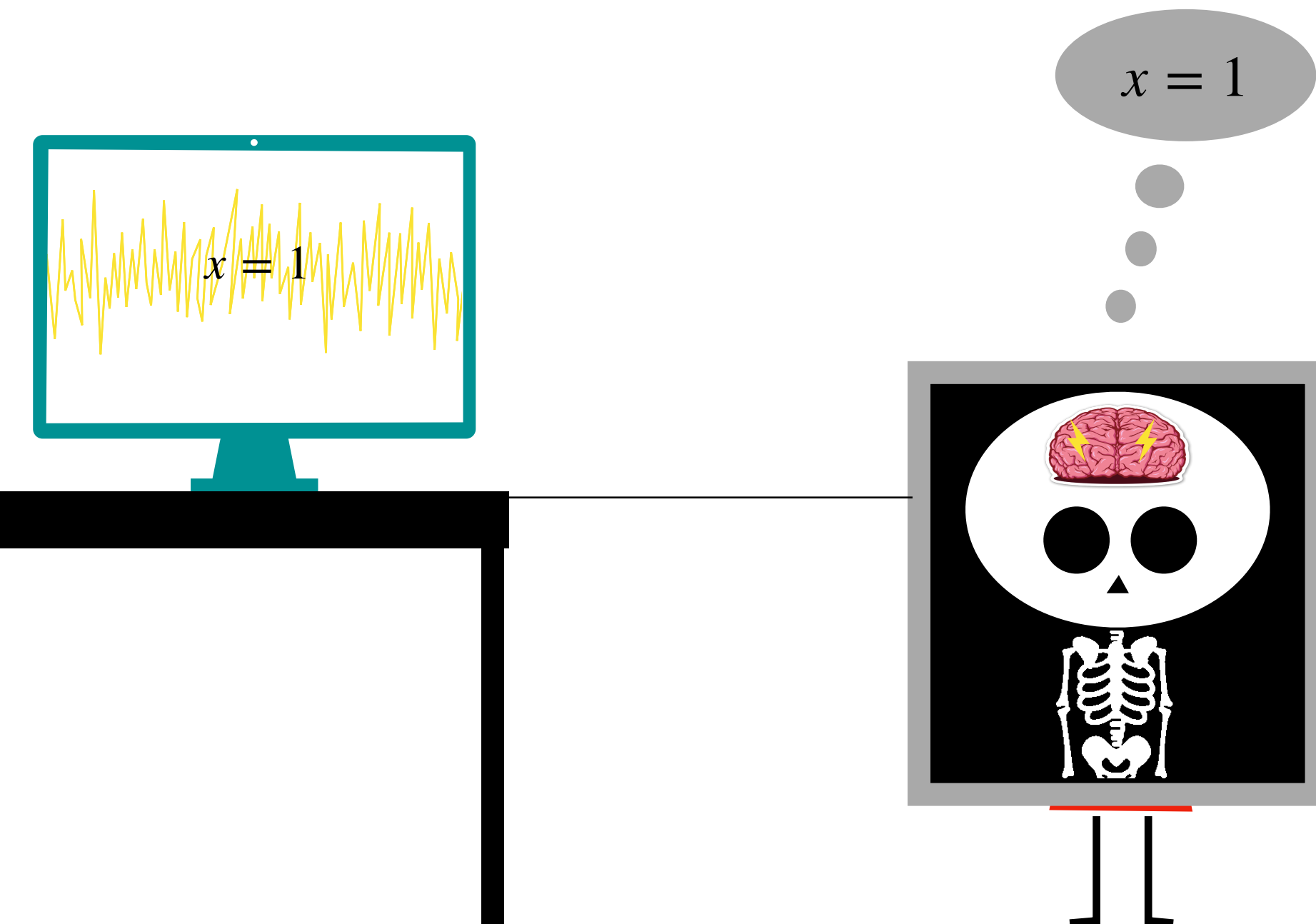
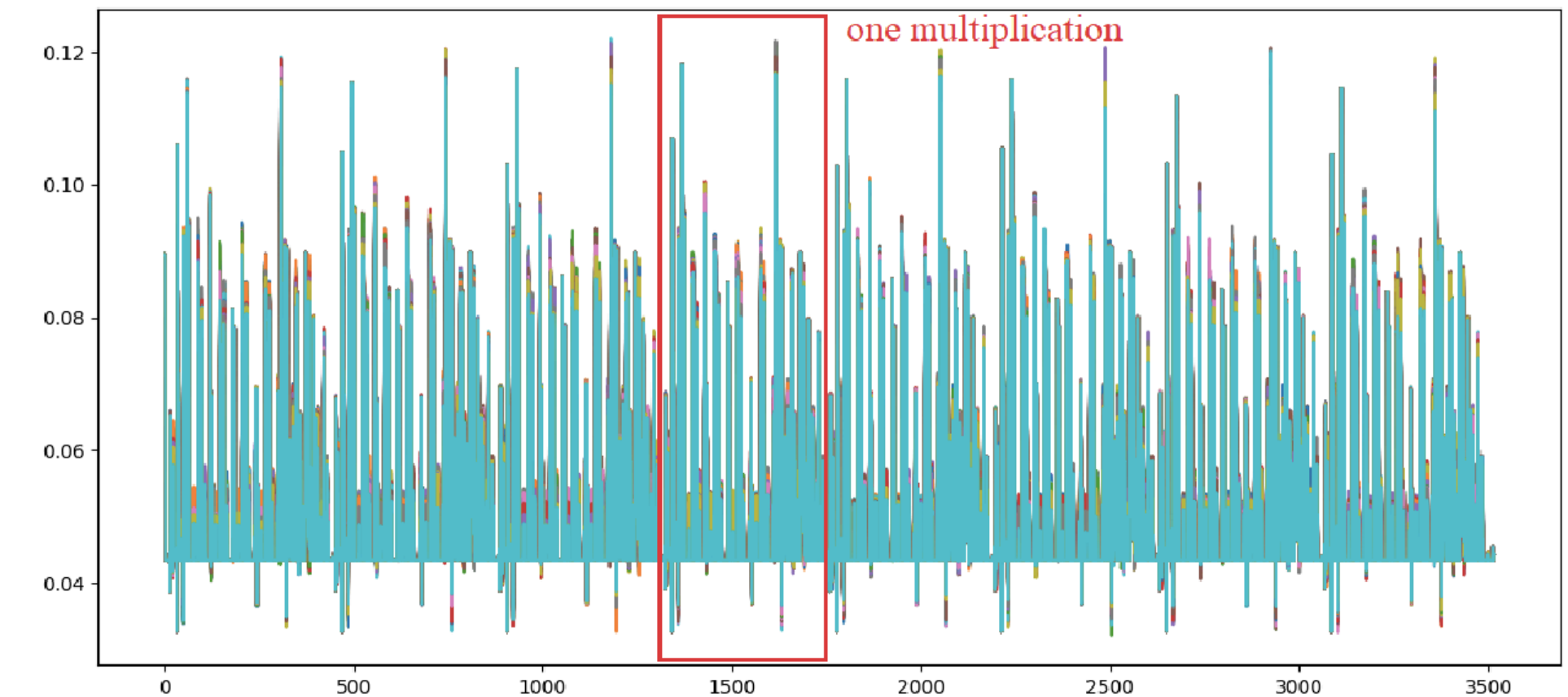
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An example presented in the next slides



Falcon signature scheme

Signature algorithm:

1: compute \mathbf{c} such that $\mathbf{c}\mathbf{A} = H(m)$

2: $\mathbf{v} \leftarrow$ a vector in $\Lambda(\mathbf{B})$ close to \mathbf{c}

3: return $\mathbf{s} \leftarrow \mathbf{c} - \mathbf{v}$



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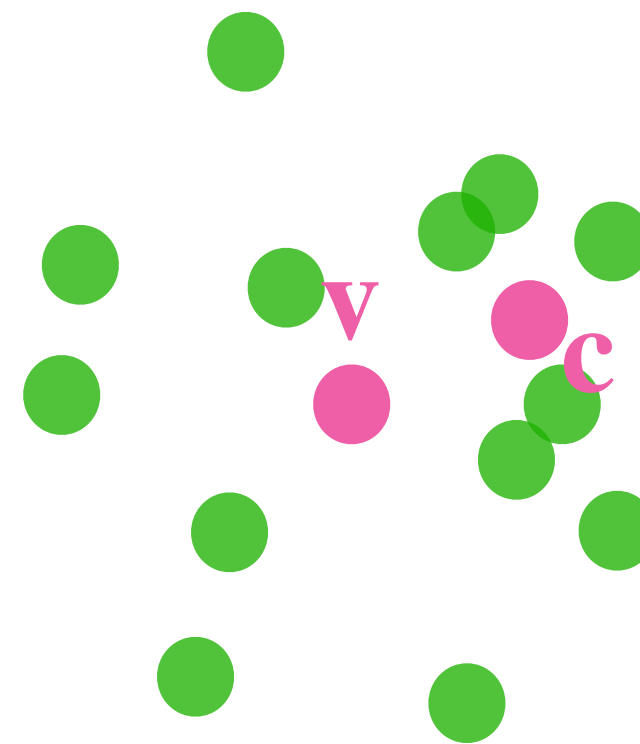
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Take the **closest vector**
Add a **Gaussian random shift** z_0



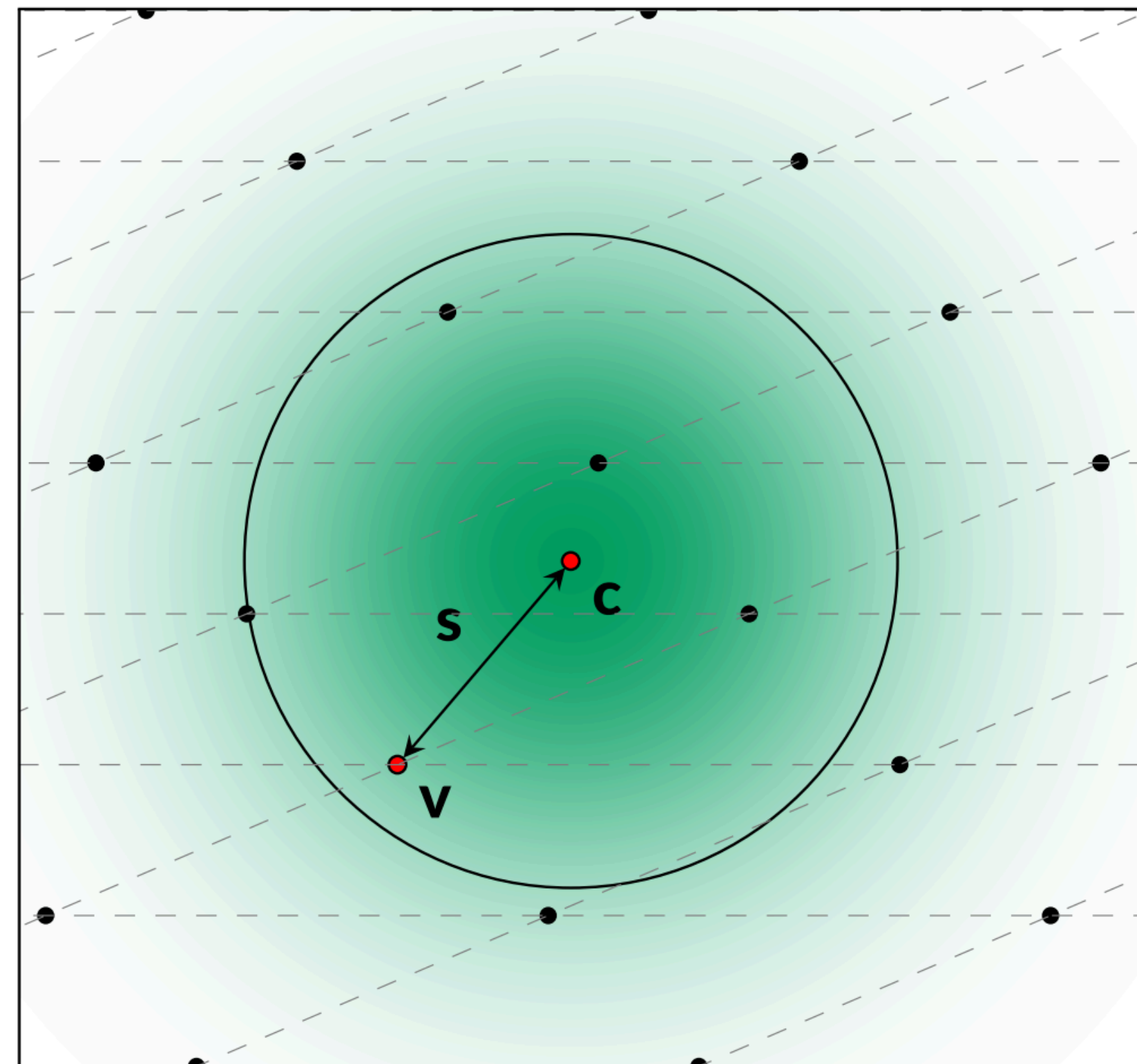
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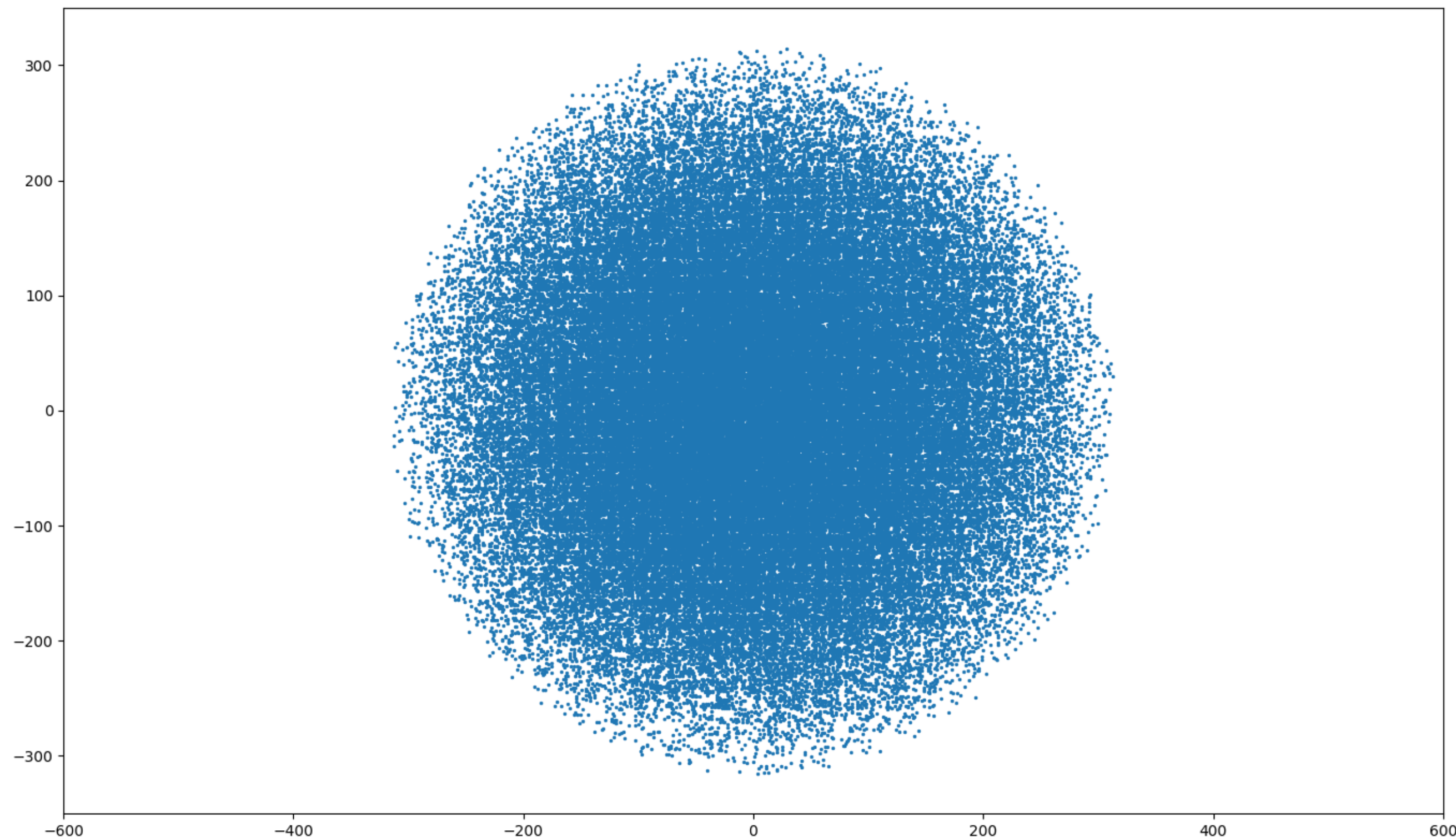
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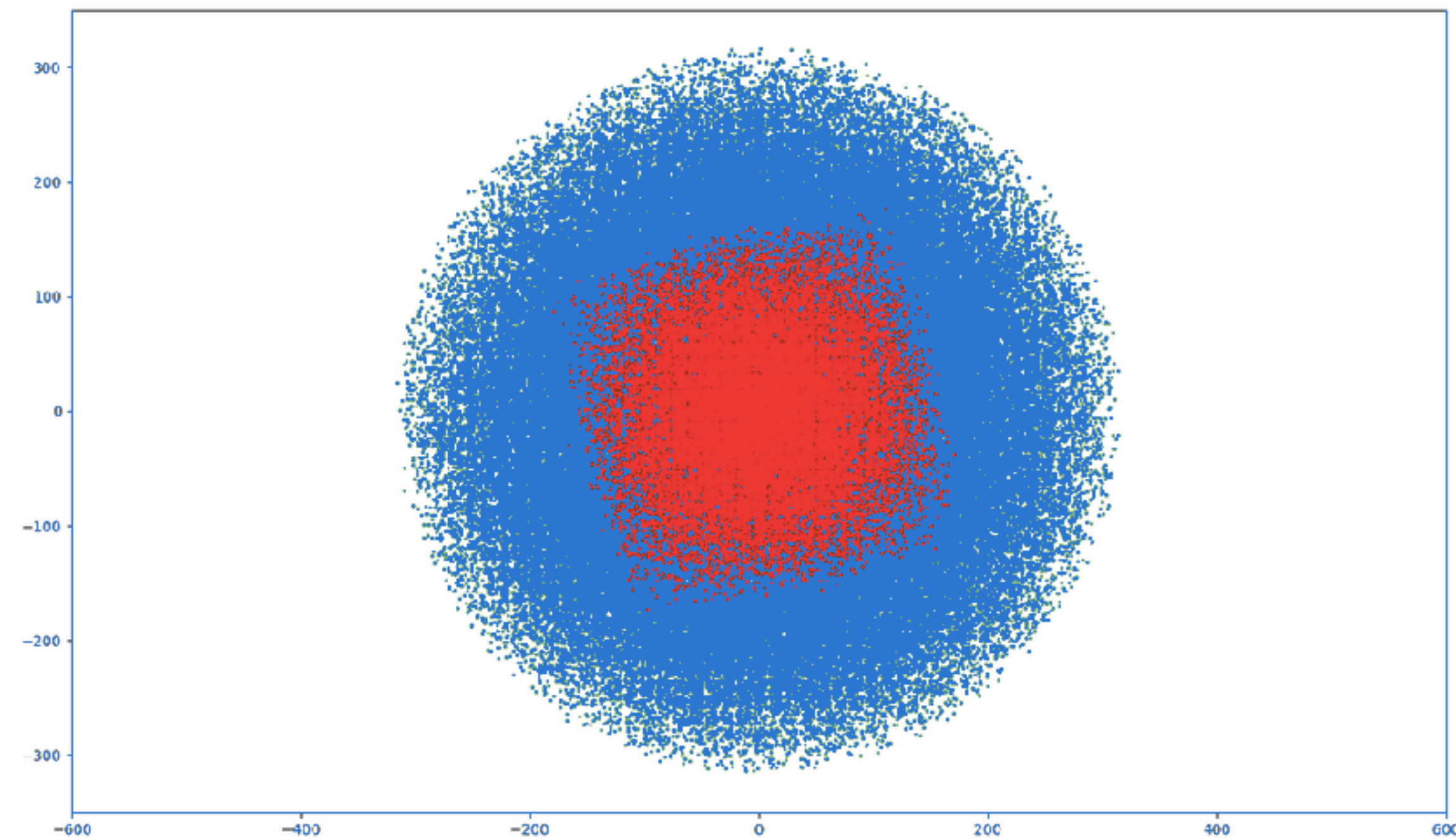
Distribution of signatures



Intuition of the power analysis attack of Falcon

Intuition of the attack

If we select the inputs such that the **Gaussian shift** is zero, we can “see” the hidden basis.



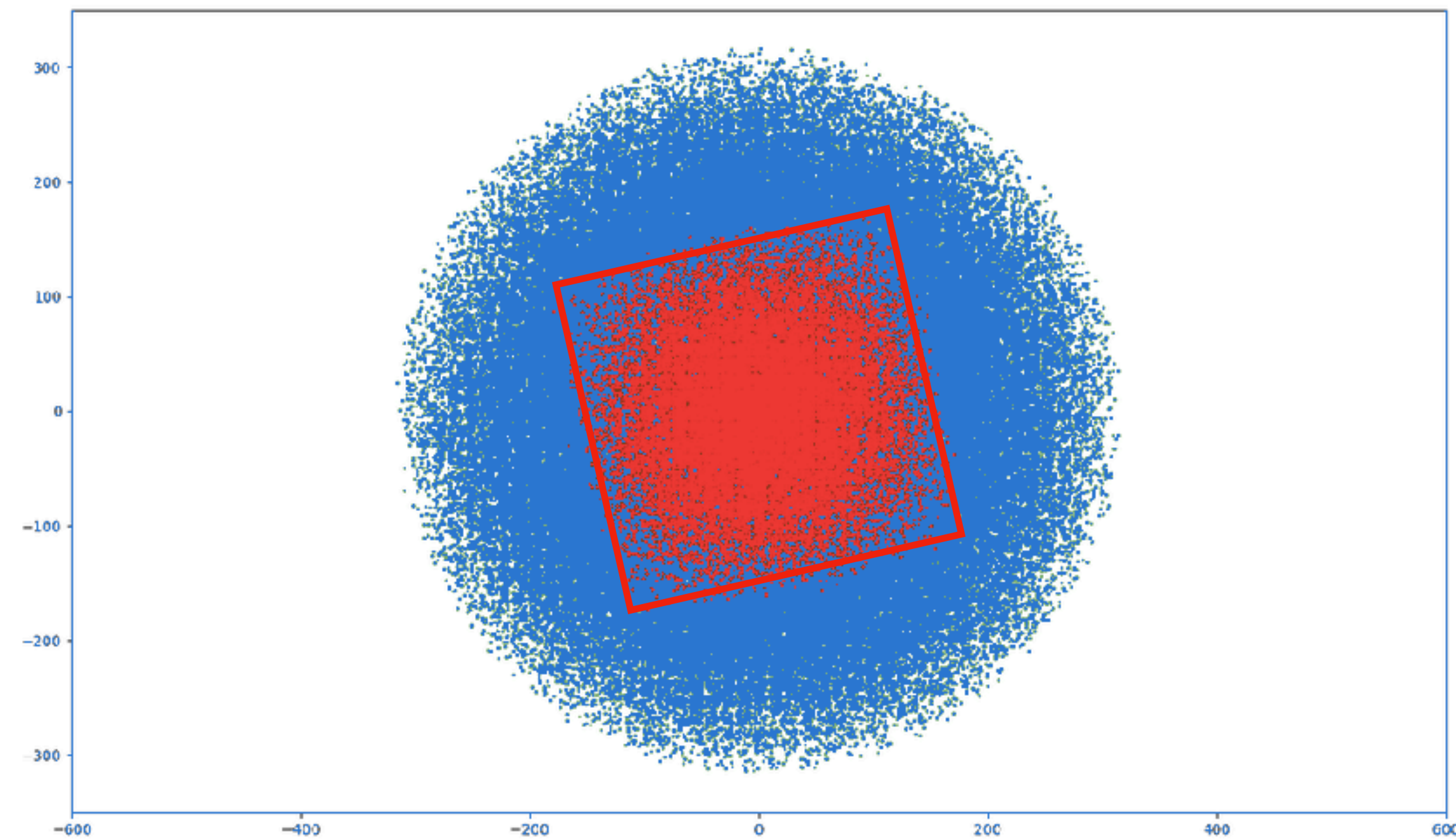
Non shifted signatures in red

What about high dimensions? There is a negligible amount of zero-shift in *all* 512 dimensions.

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Single trace power analysis of Falcon

We focus on one dimension.

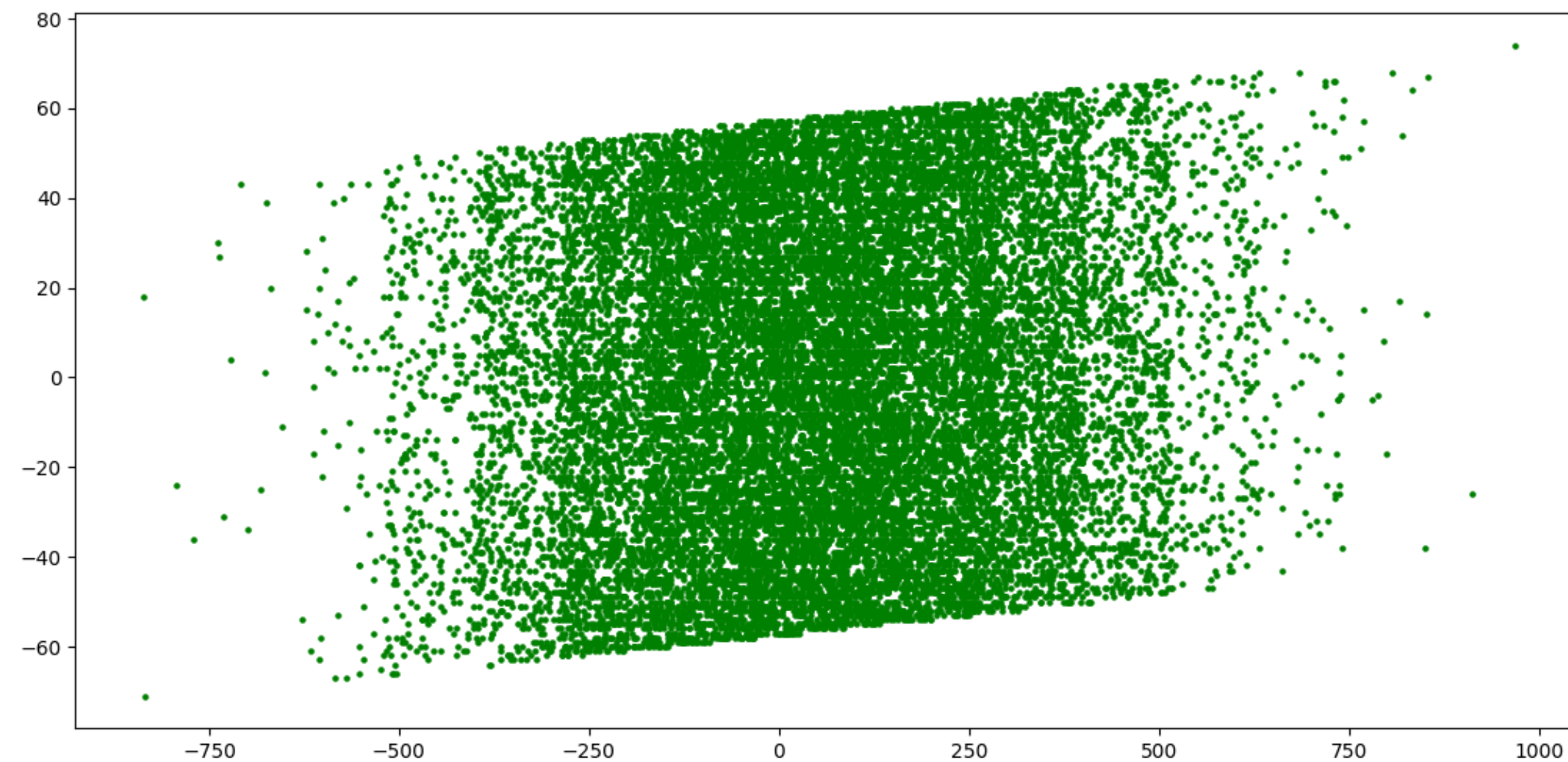
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Signatures for which $\text{shift} = 0$ in the first coordinate

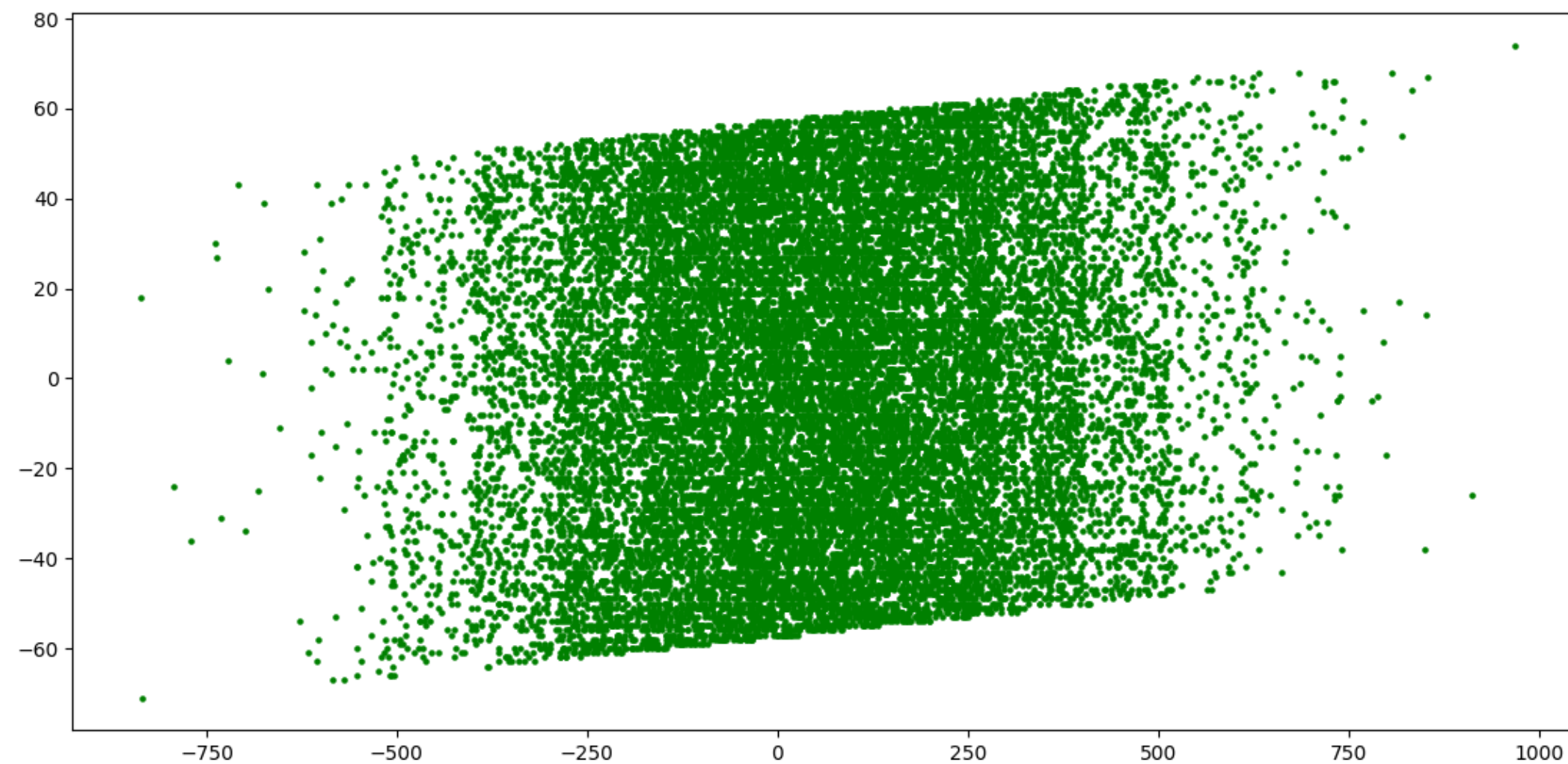


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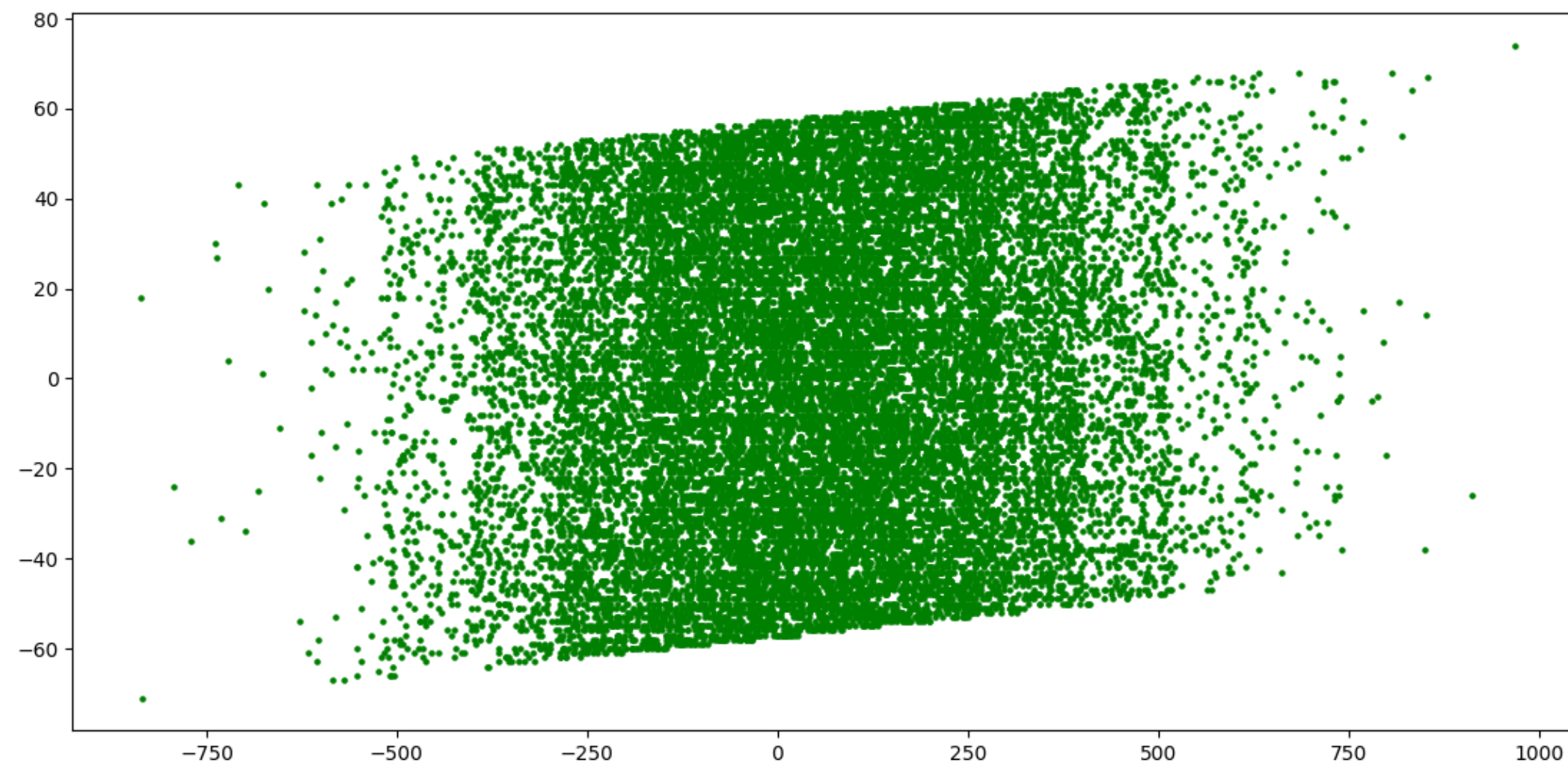
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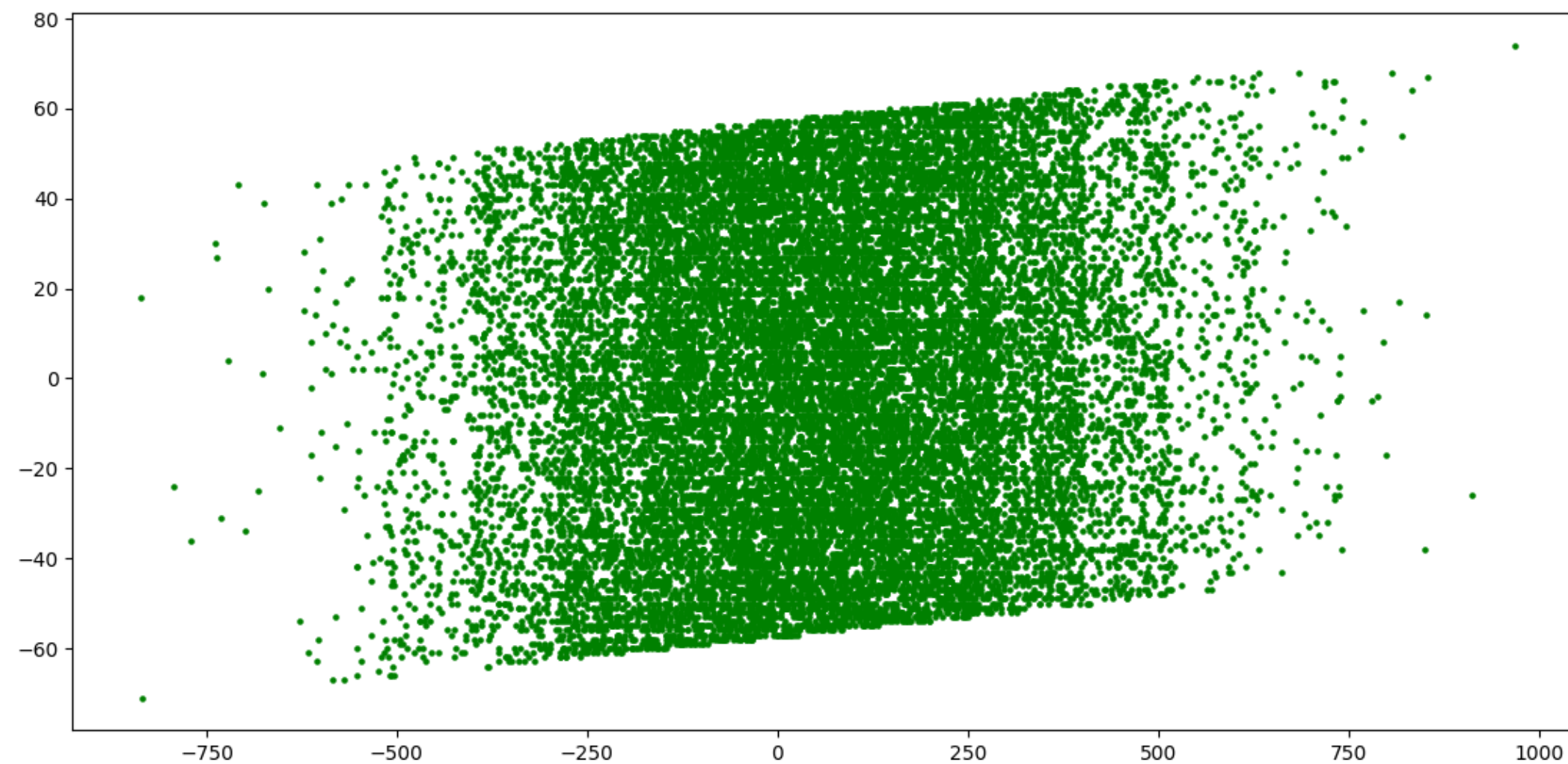
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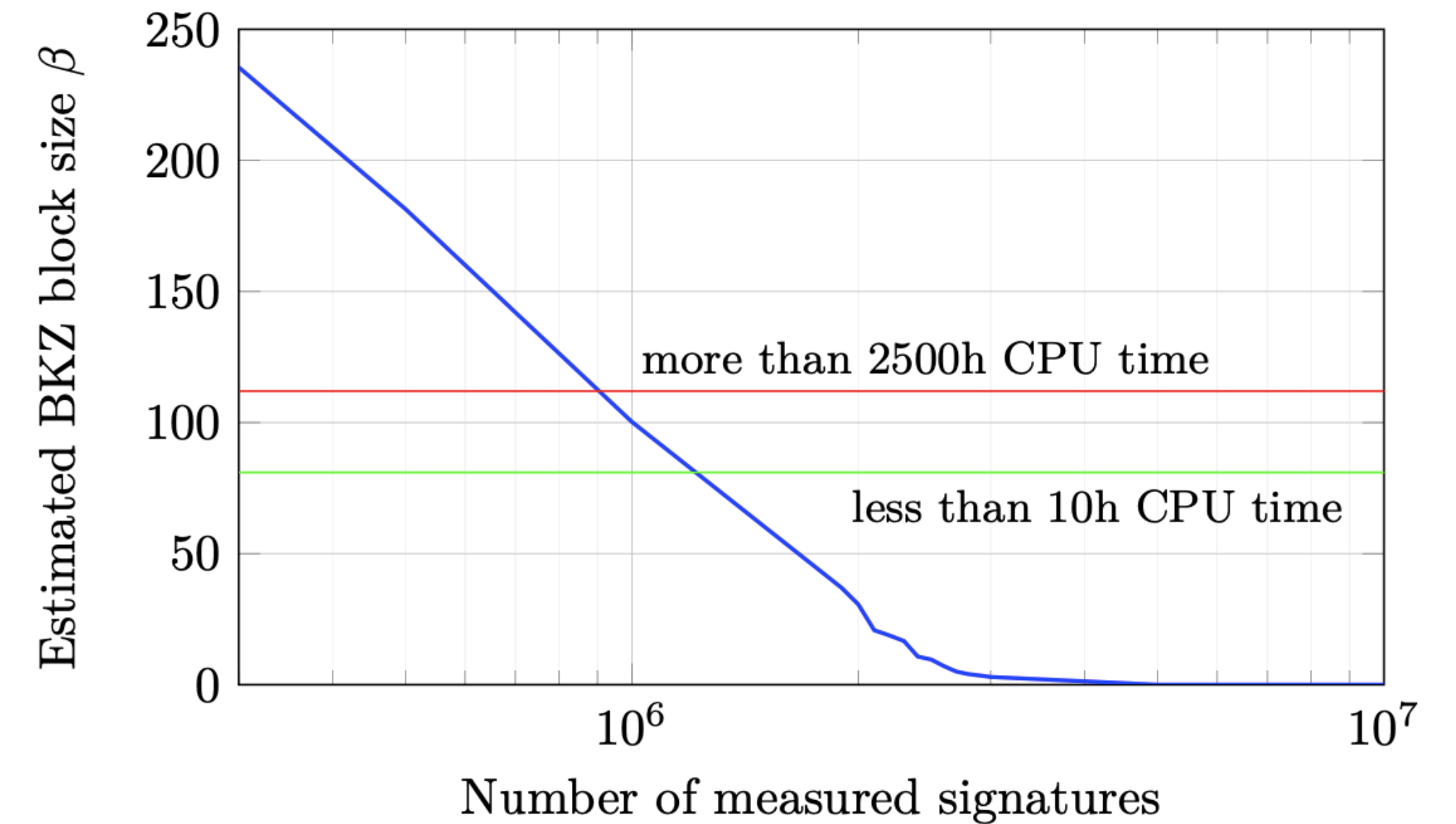
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Performance of the attack



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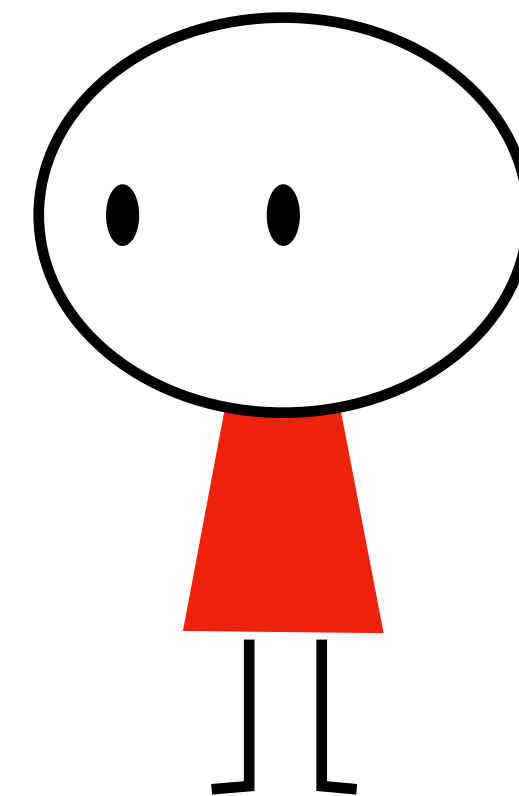
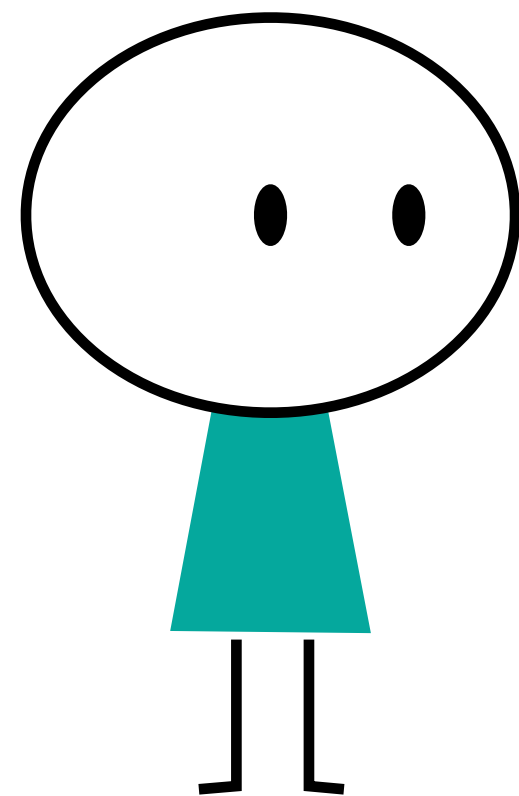
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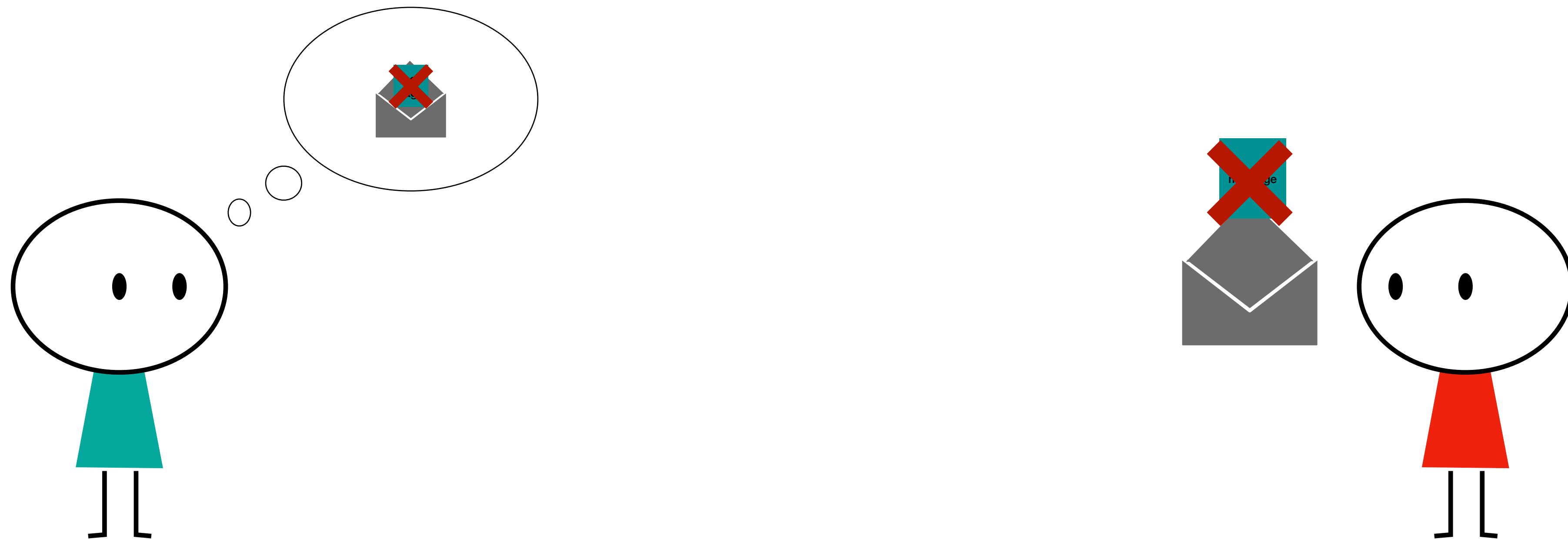
Decryption failure attacks

- ▶ J.-P. D'Anvers, Q. Guo, T. Johansson, A. Nilsson, F. Vercauteren, and I. Verbauwhede. [PKC'19](#)
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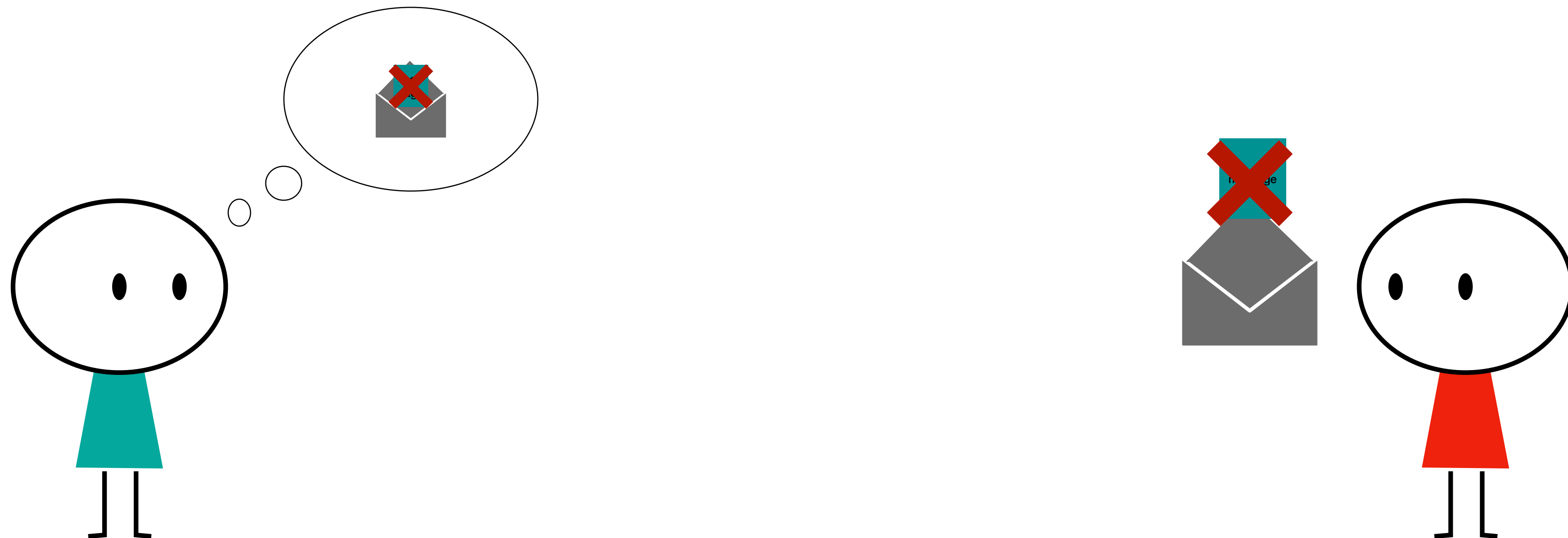
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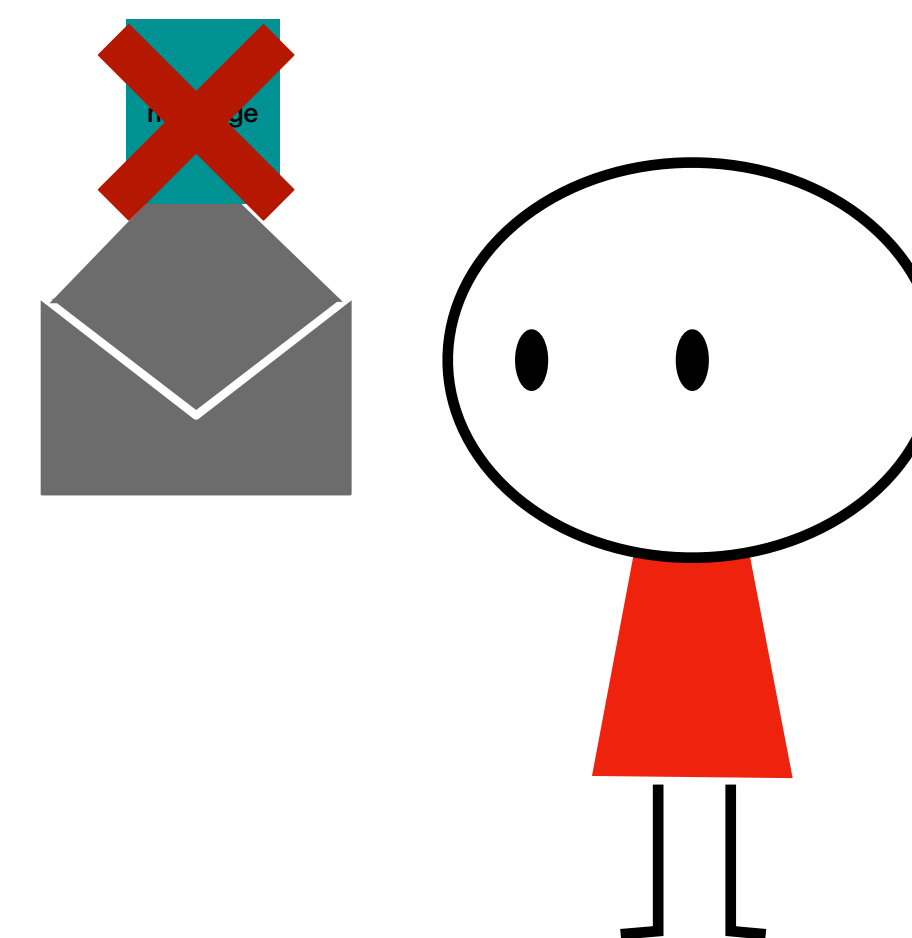
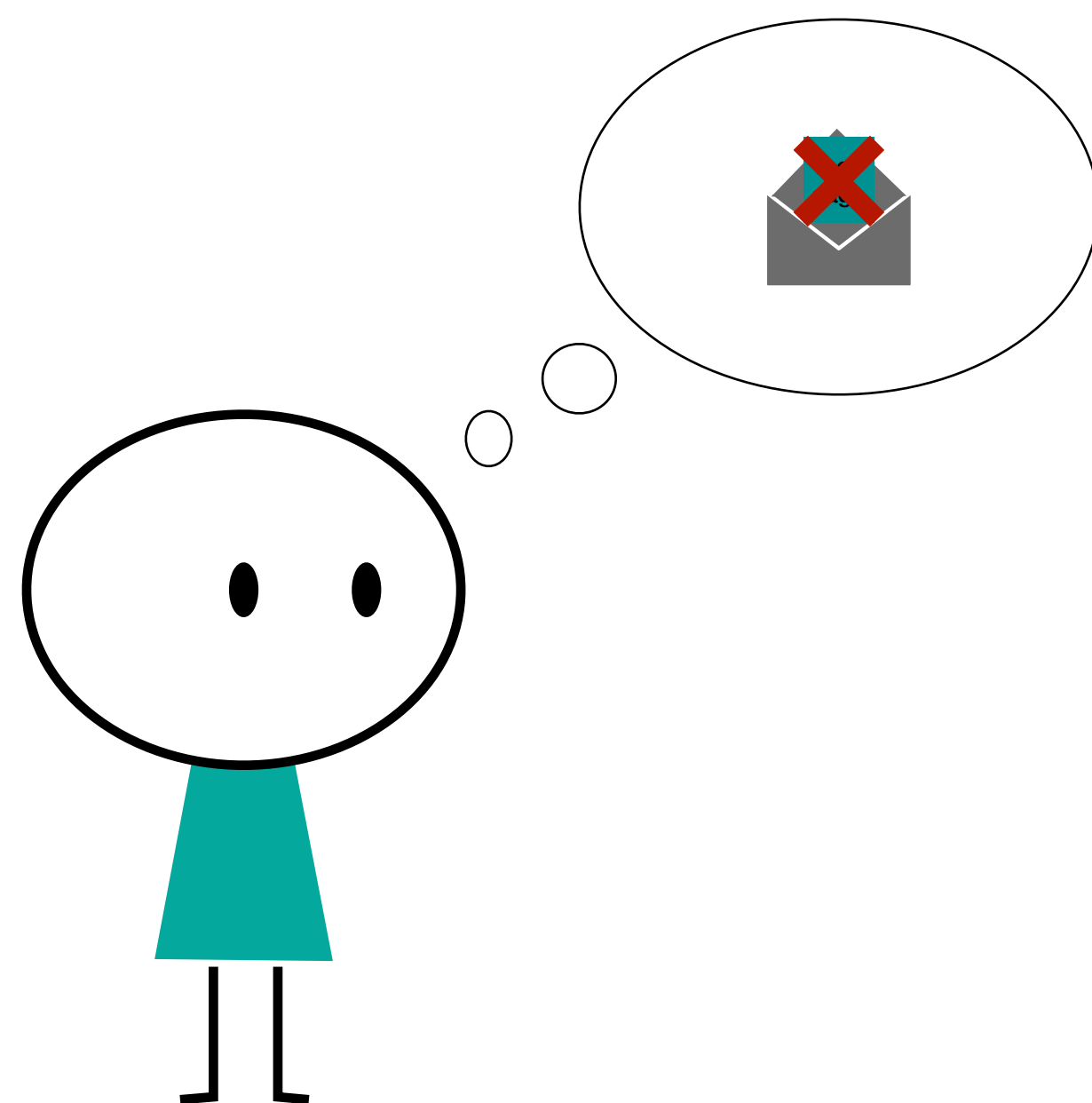
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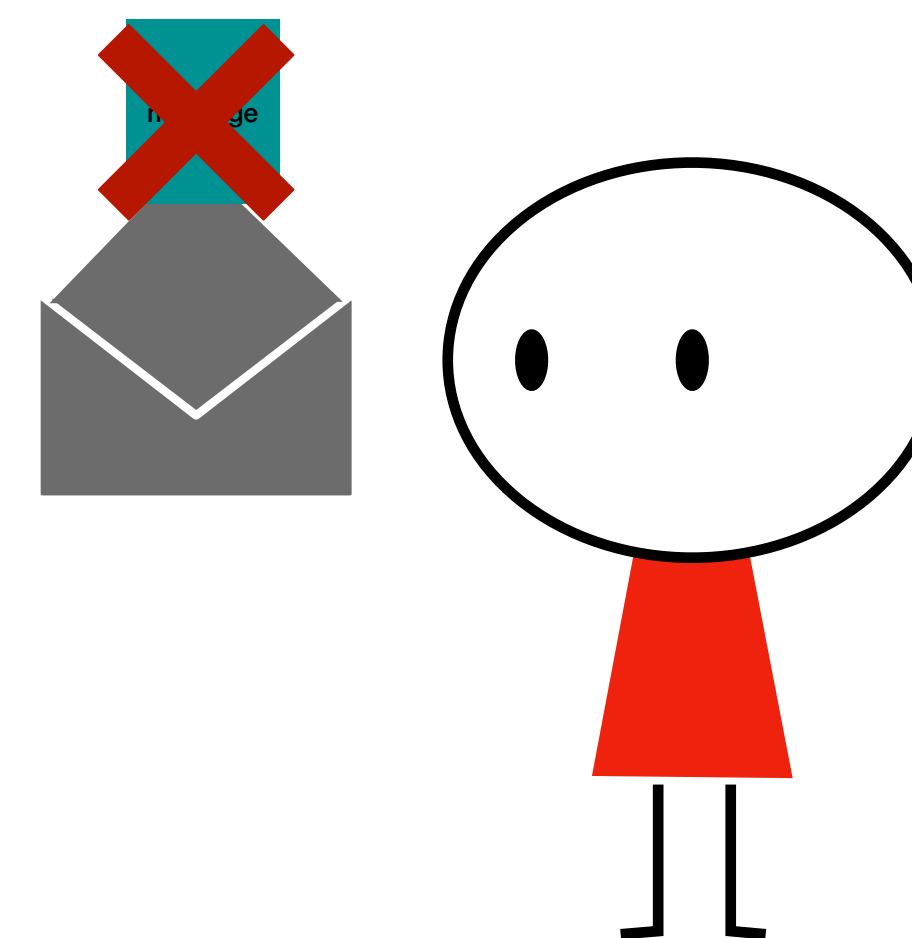
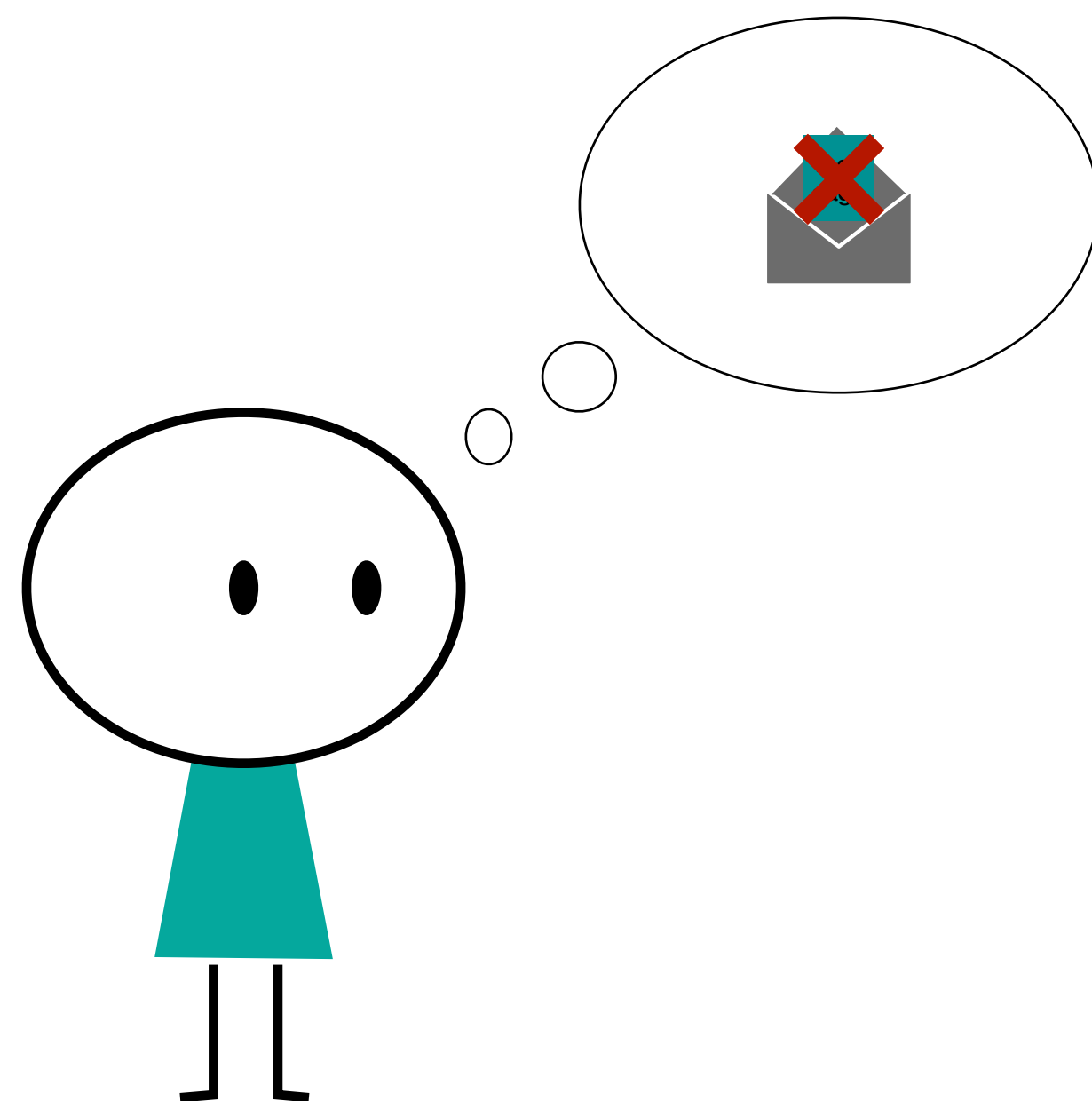
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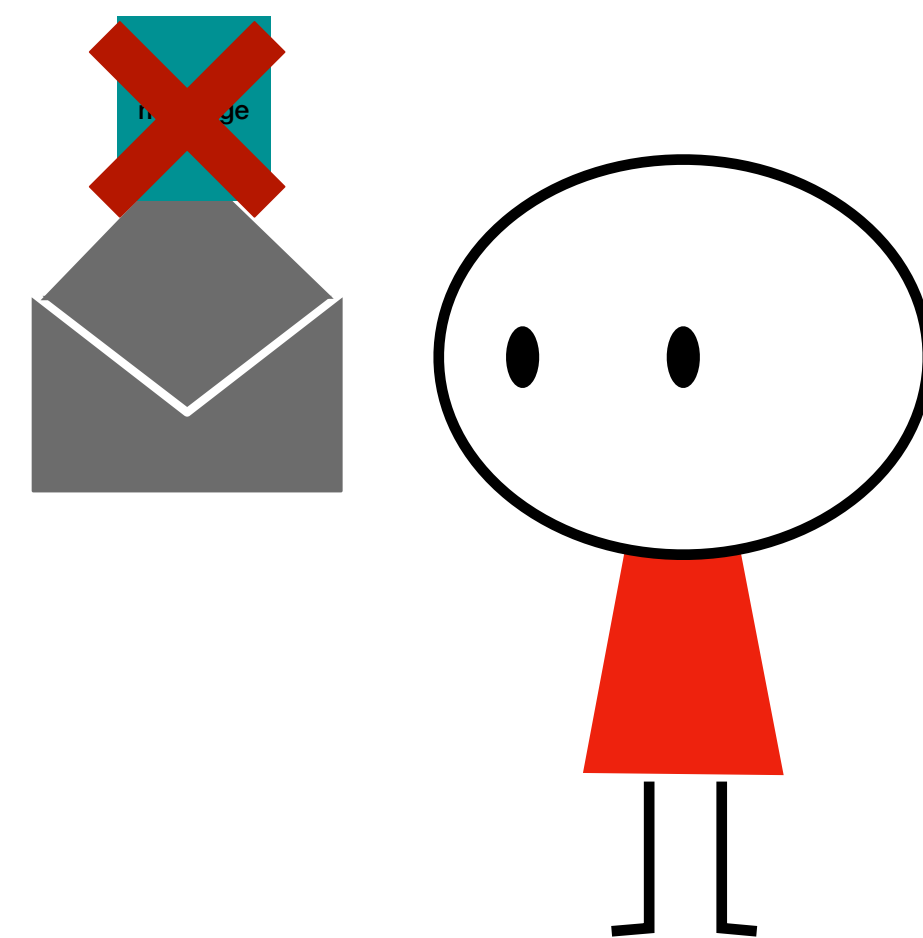
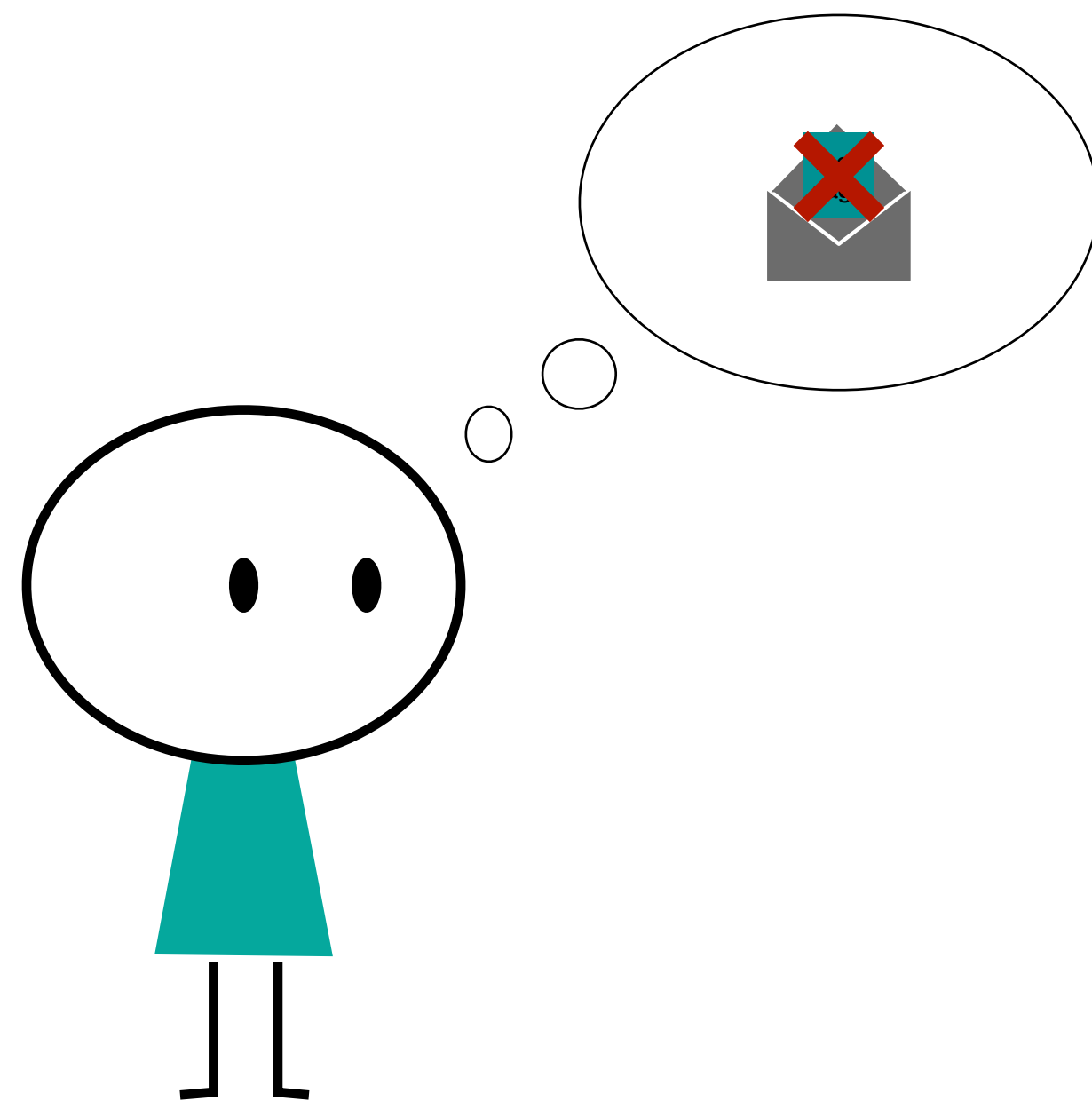
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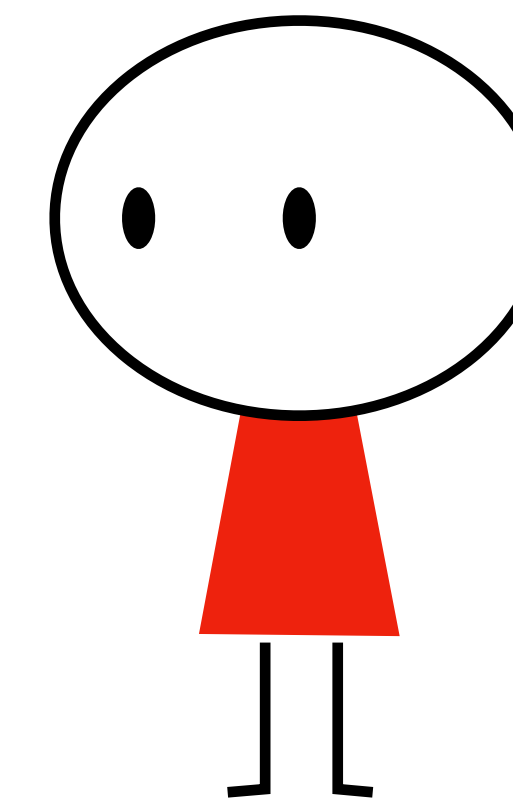
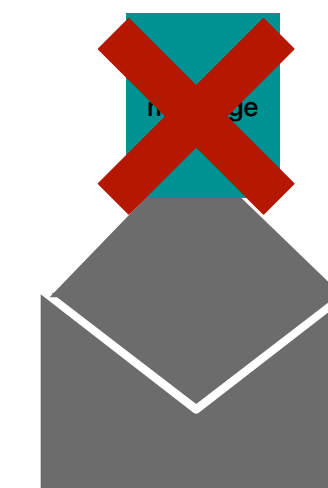
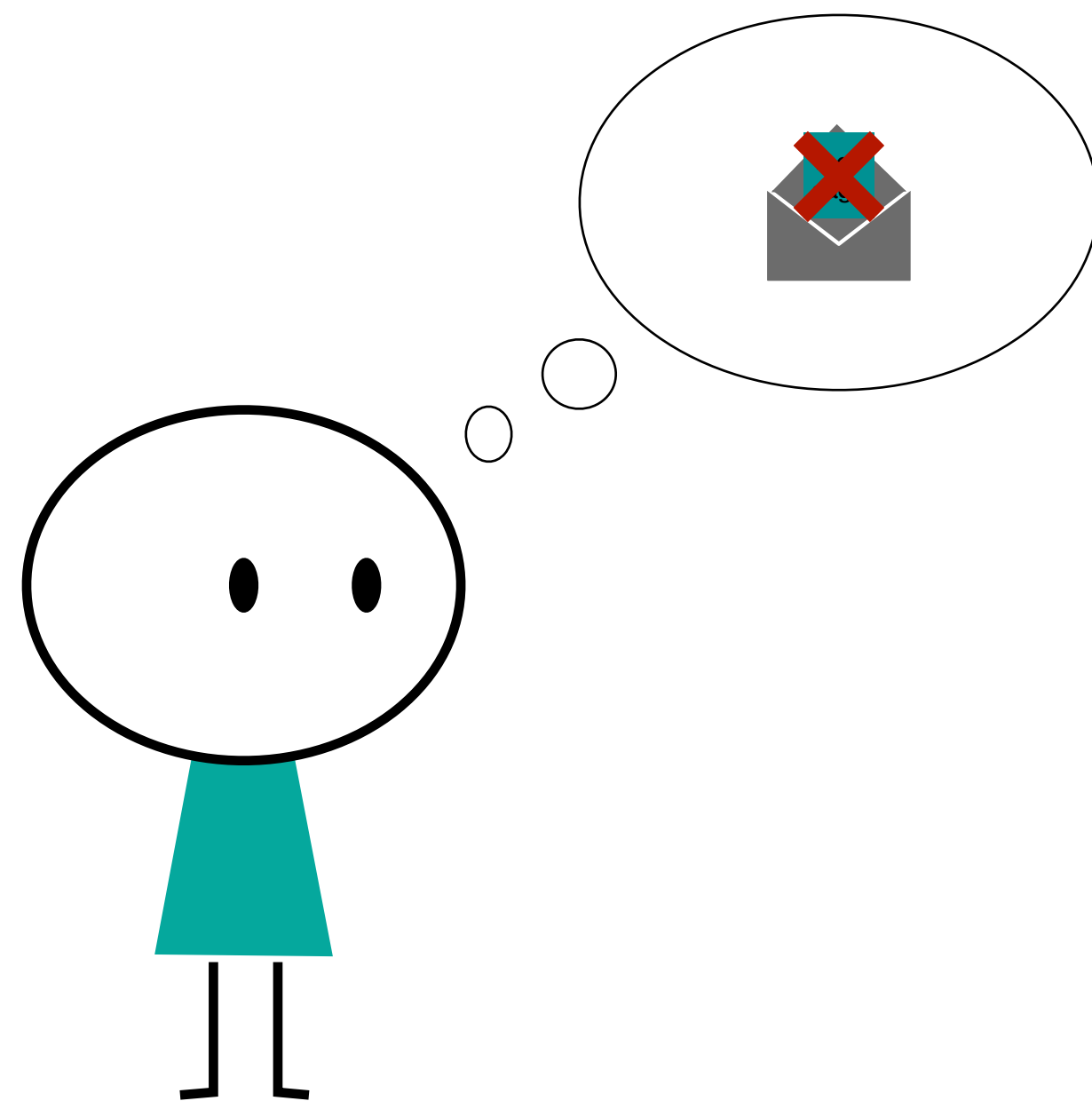
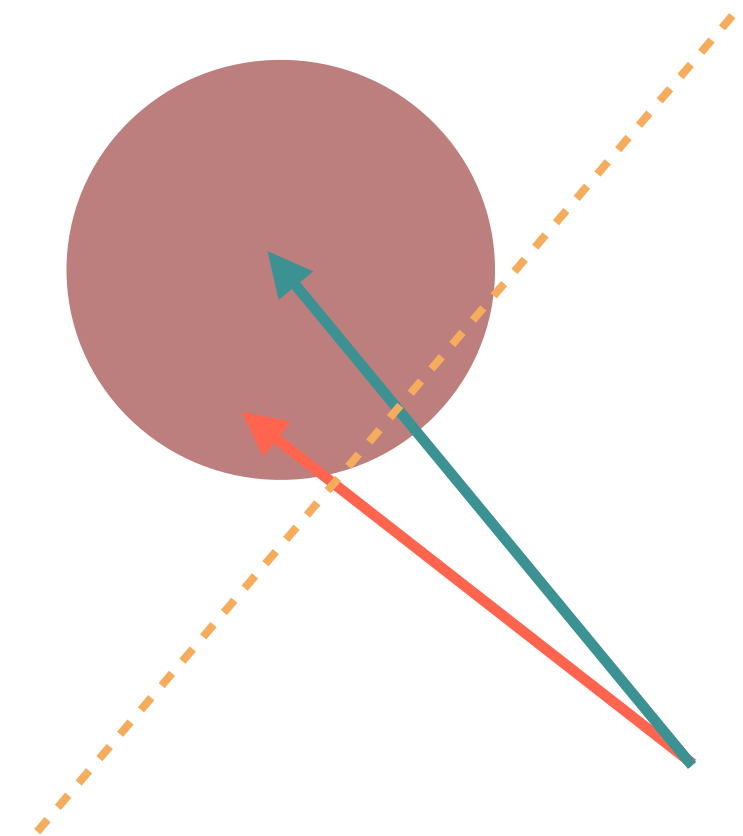
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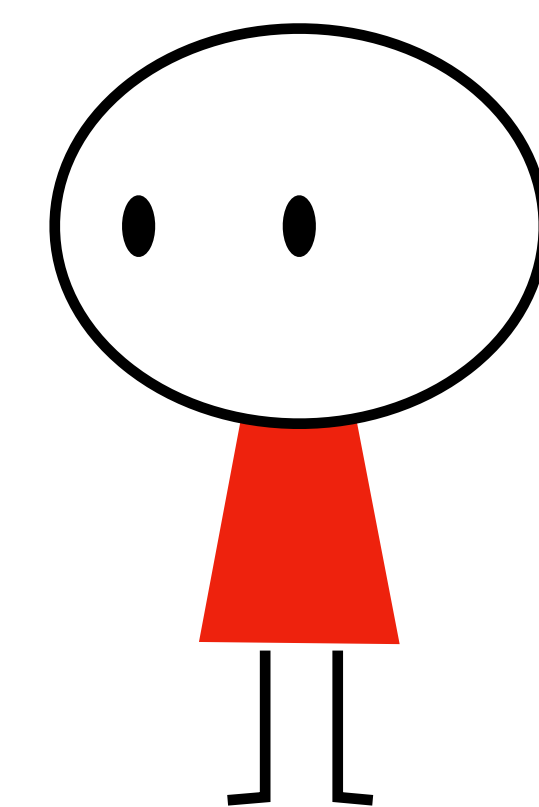
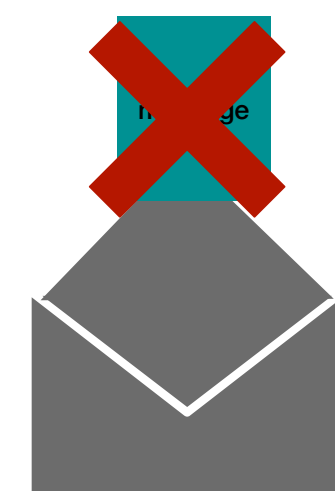
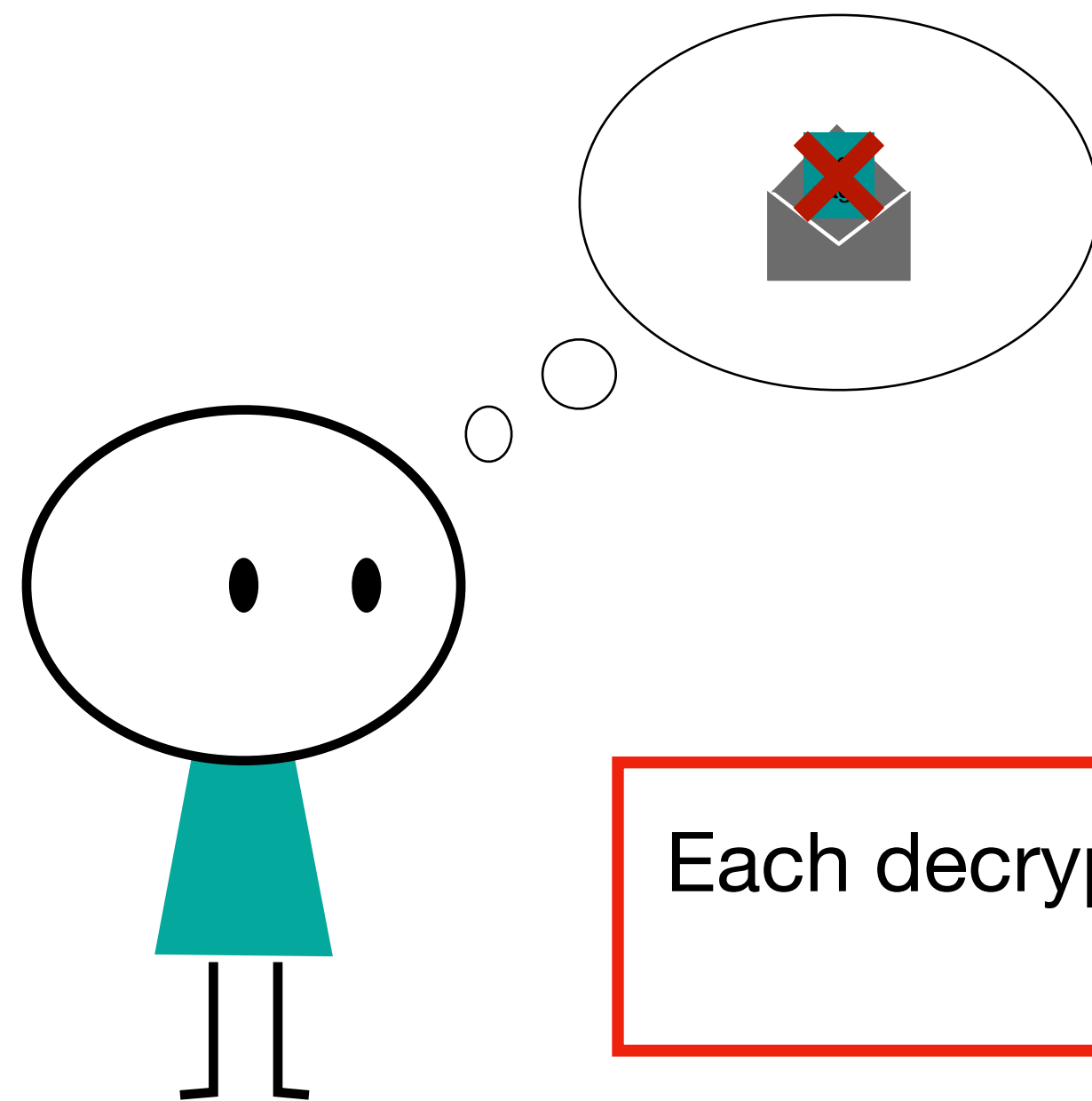
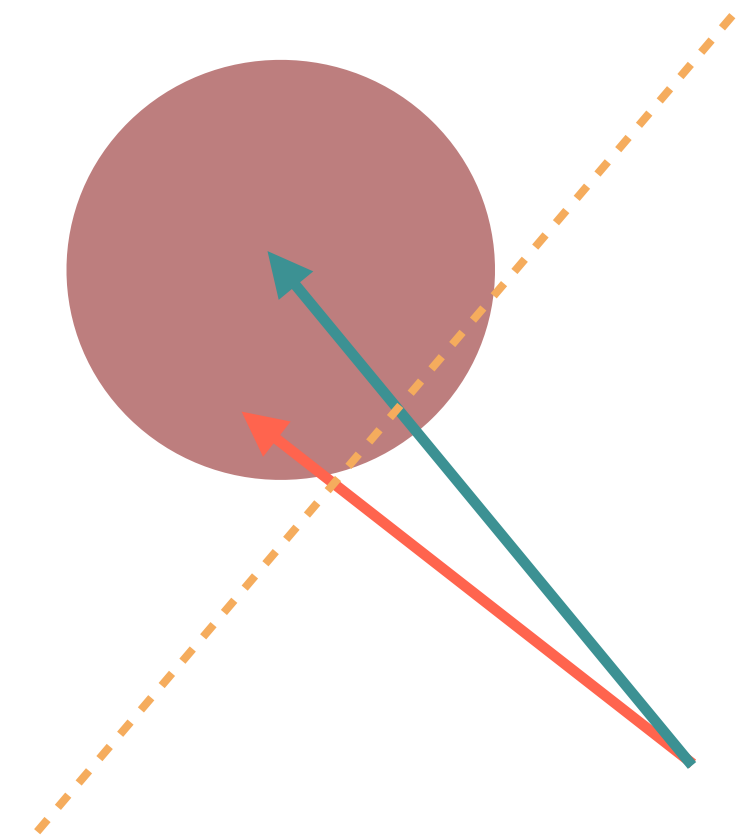
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Each decryption failure provides an equation on the direction of the secret

Decryption failure attacks

➔ Failure probability in IND-CCA setting

| | |
|------------|------------------|
| 0 | NTRU, NTRU Prime |
| 2^{-216} | NewHope |
| 2^{-206} | Three Bears |
| 2^{-199} | FrodoKEM |
| 2^{-164} | Kyber |
| 2^{-142} | LAC |
| 2^{-136} | Saber |
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Extremely unlikely in IND-CCA setting (protected by FO transform)

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The Fujisaki Okamoto transform can be bypassed :

- with timing measurement e.g. ▶ J.-P. D'Anvers, M. Tiepelt, F. Vercauteren, I. Verbauwhede. [TIS'2019](#)
- with power analysis e.g. ▶ R. Ueno, K. Xagawa, Y. Tanaka, A. Ito, J. Takahashi, N. Homma. [TCHES'2022](#)

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- with timing measurement e.g. ▶ J.-P. D'Anvers, M. Tiepelt, F. Vercauteren, I. Verbauwhede. [TIS'2019](#)
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➔ Open the door to crafting ciphertexts in order to create failures with high probability.

Likely in an IND-CPA setting (where the FO transform is bypassed)

Decryption failure attacks

This transform consists in recovering the encryption's random coins inside the decryption and checking honest generation by re-encryption.

➔ Failure probability in IND-CCA setting

| | |
|------------|------------------|
| 0 | NTRU, NTRU Prime |
| 2^{-216} | NewHope |
| 2^{-206} | Three Bears |
| 2^{-199} | FrodoKEM |
| 2^{-164} | Kyber |
| 2^{-142} | LAC |
| 2^{-136} | Saber |
| 2^{-117} | Round5 |

Extremely unlikely in IND-CCA setting (protected by FO transform)

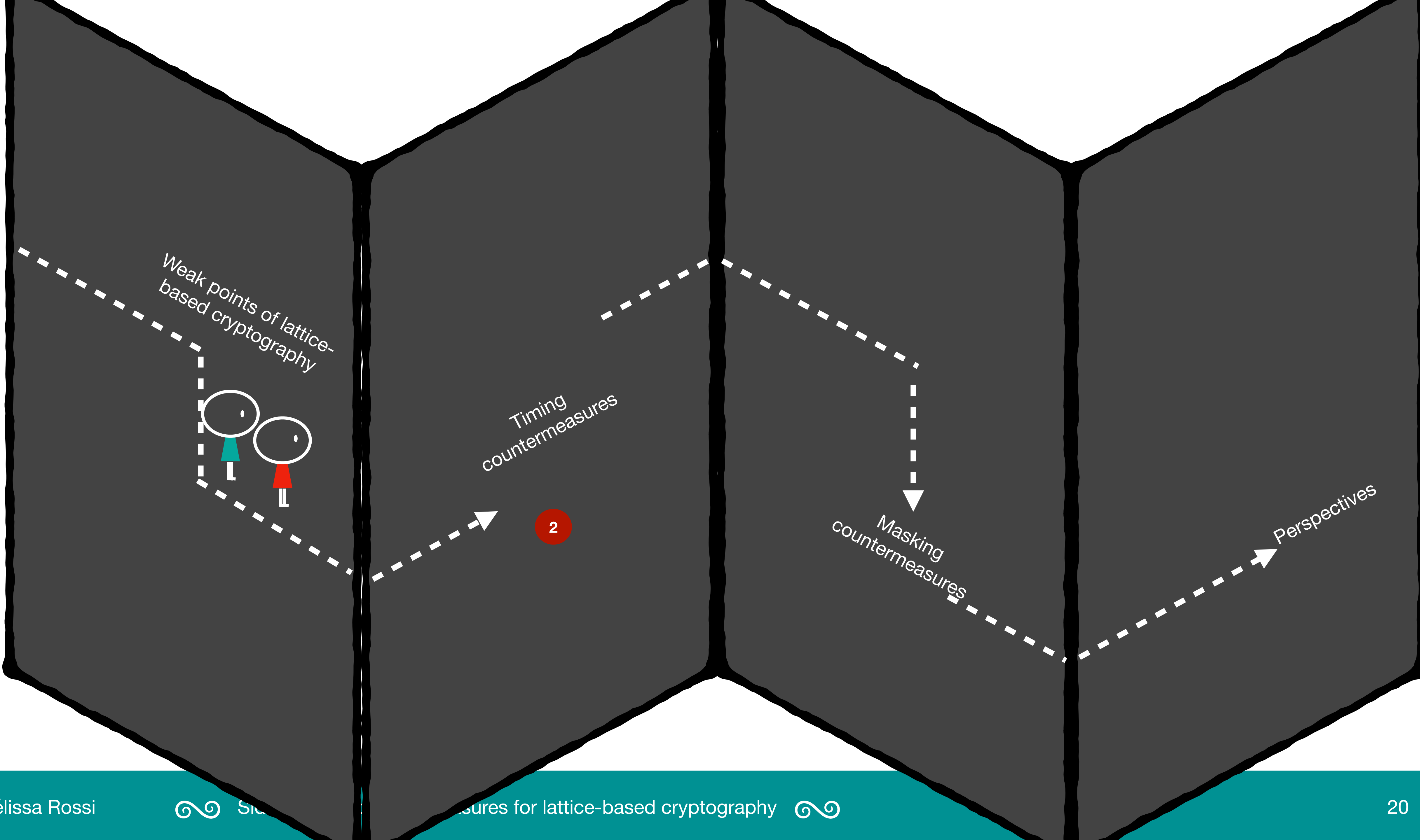
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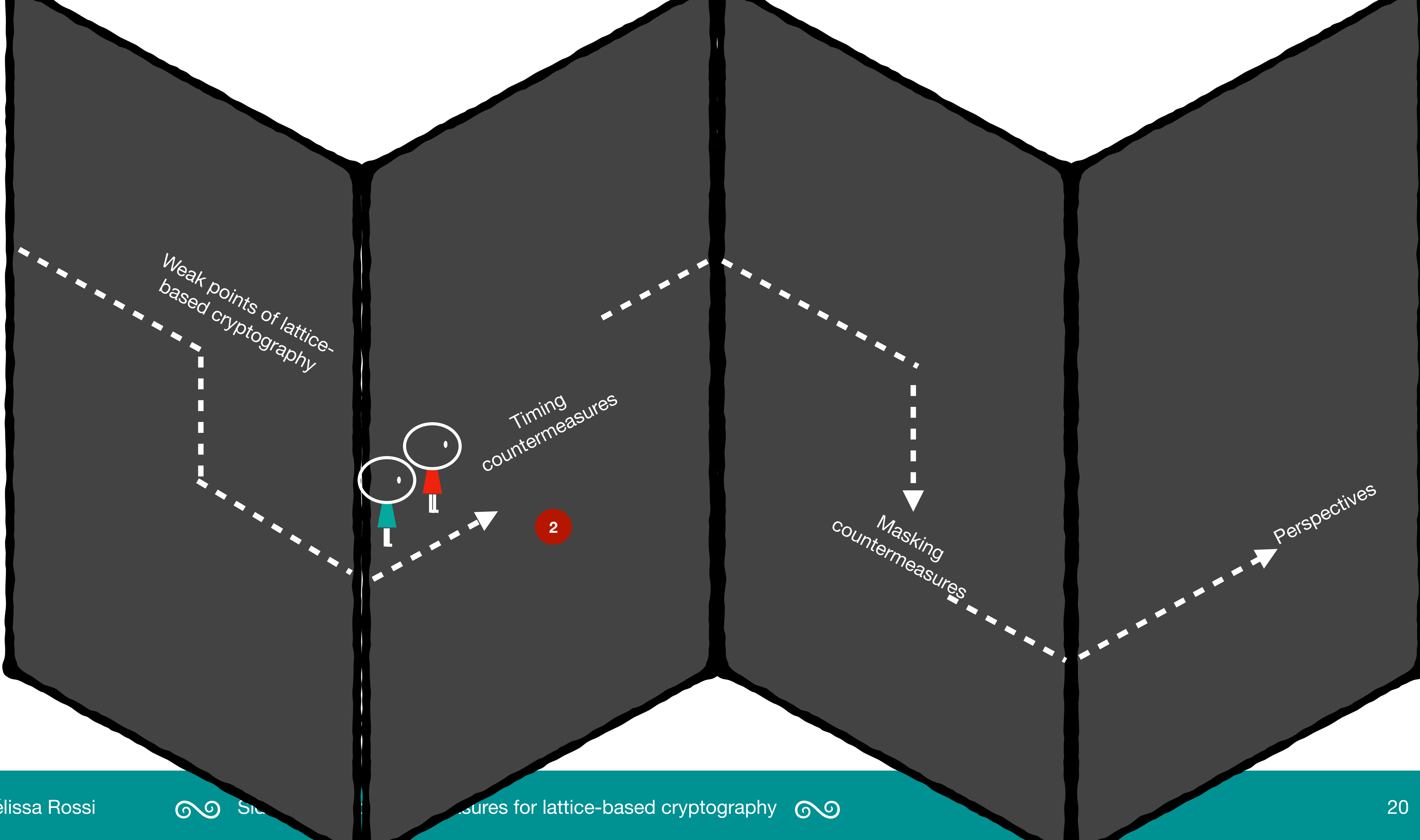
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Countermeasures are very important to avoid these side-channel assisted decryption failure attacks





How to remove timing attacks entry points

The entry points include:

- ◆ computer-science unfriendly distributions like Gaussians.
- ◆ secret-dependent internal distributions.
- ◆ numerous operations with the secret.
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Isochrony the execution time can vary but **its distribution should be independent from any sensitive data.**

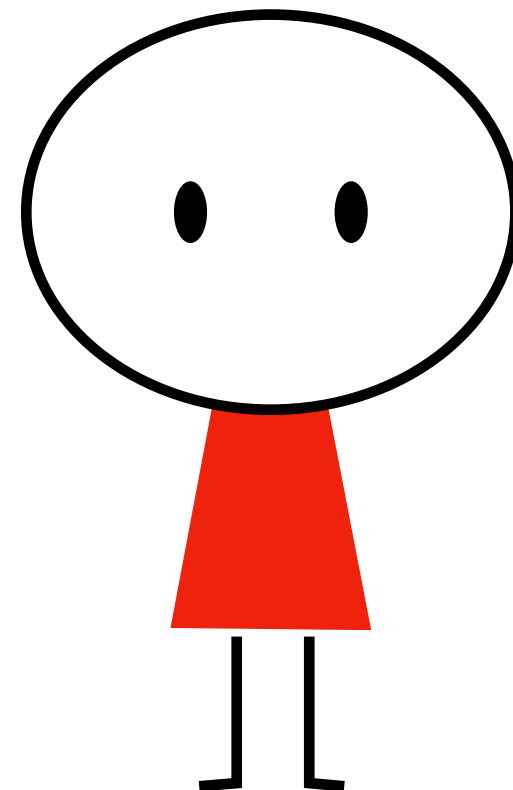
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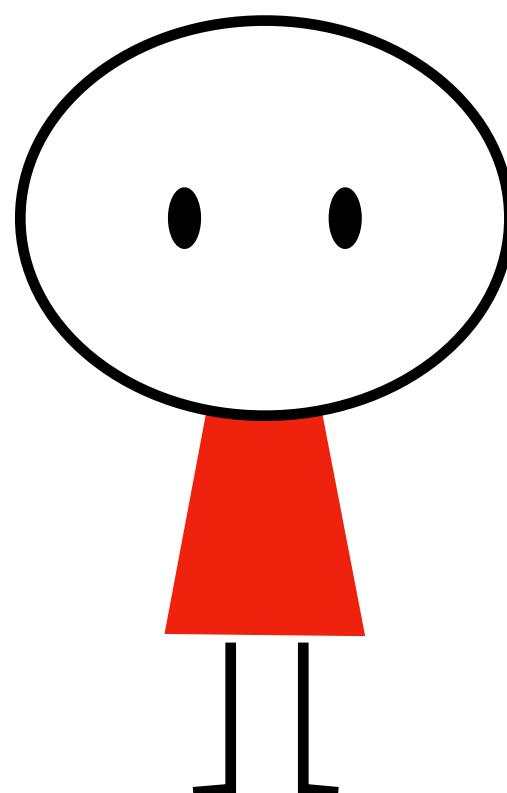
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Here are some provable countermeasure techniques:

- 1 Renyi divergence arguments
- 2 Polynomial approximations

1) Rényi divergence arguments

- ▶ S. Bai, A. Langlois, T. Lepoint, D. Stehlé, R. Steinfeld ASIACRYPT'15
- ▶ T. Prest ASIACRYPT'17

Distributions may be approximated/simplified because of the limited number of queries

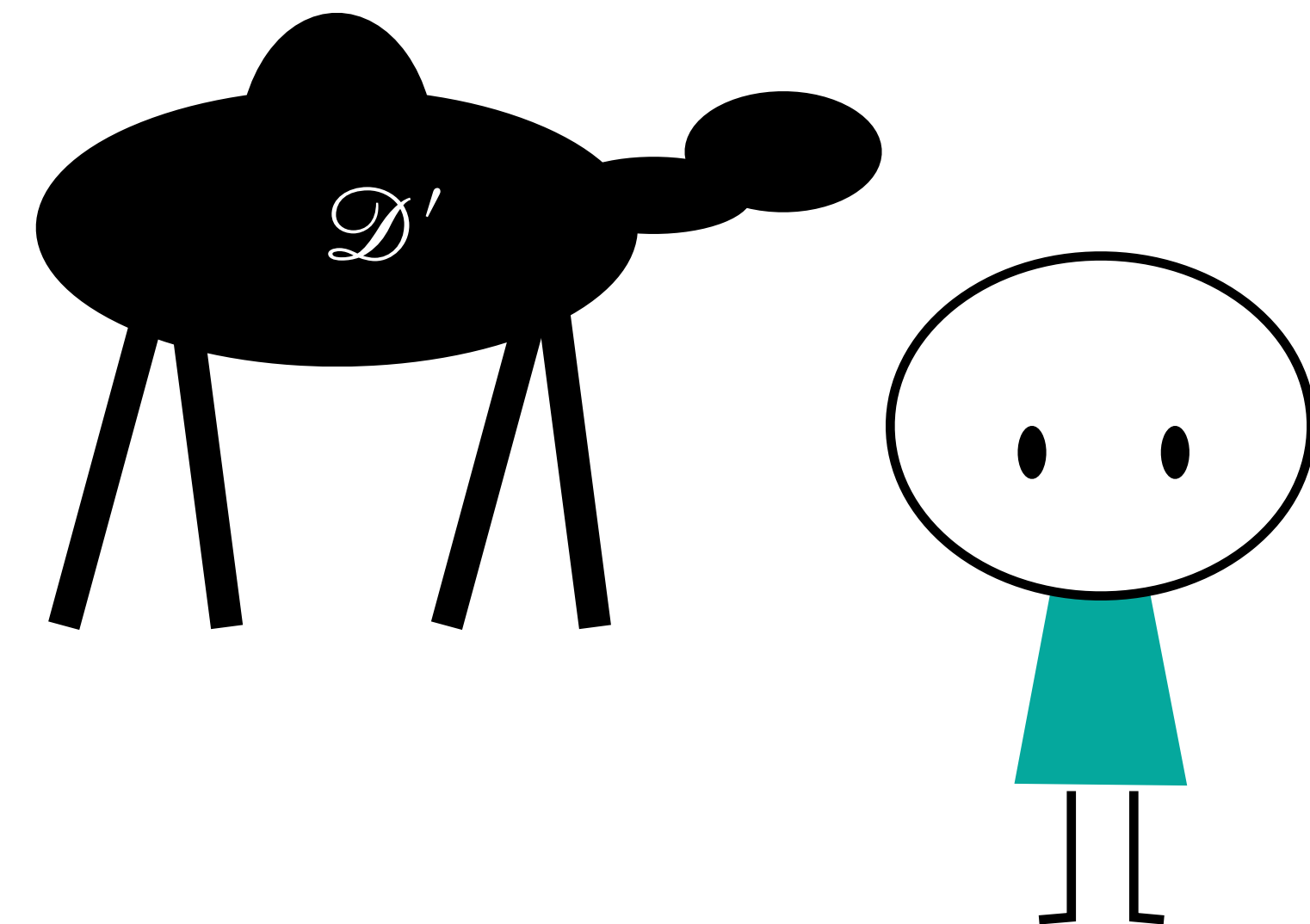
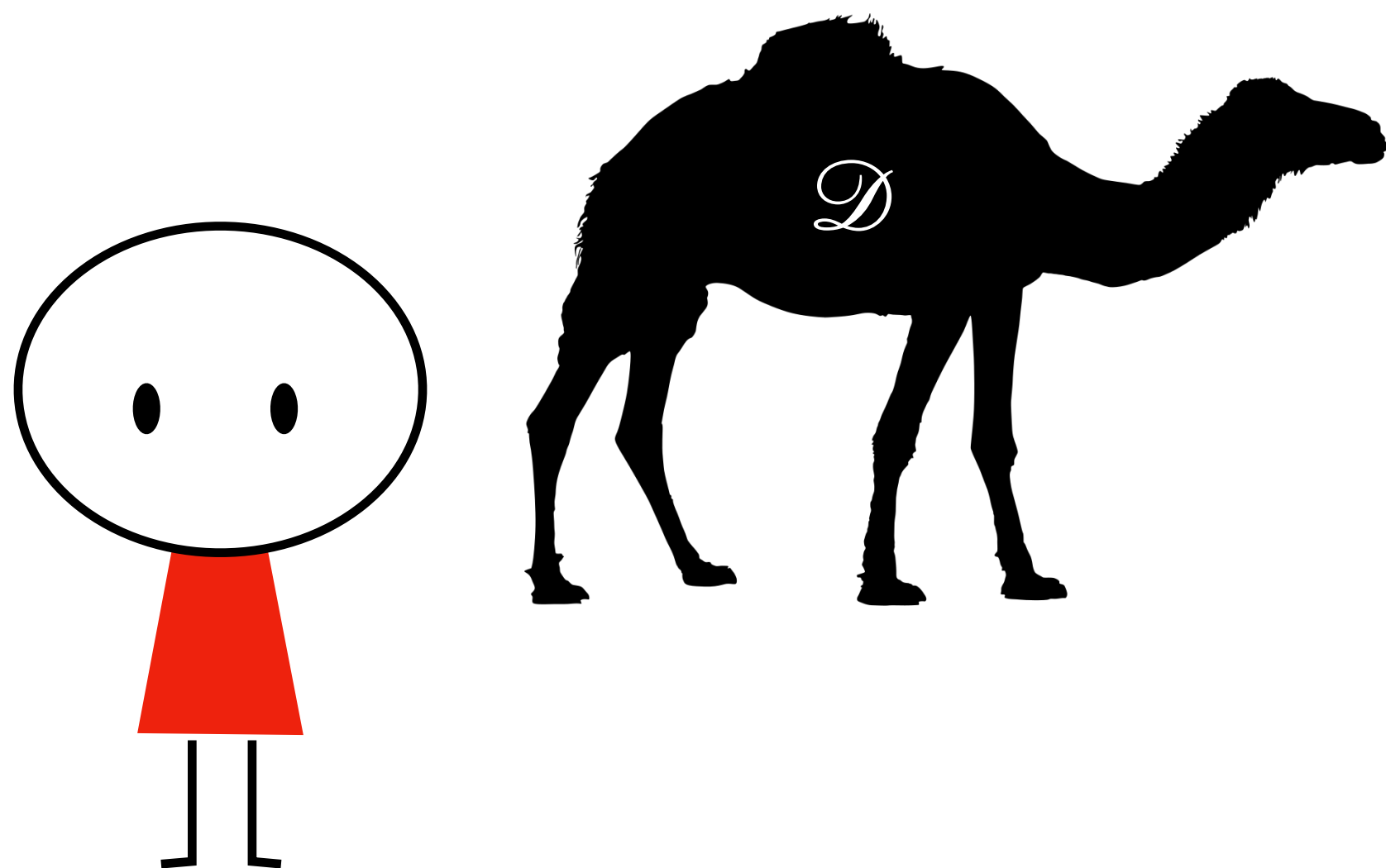
Take two cryptographic schemes

- One with distribution \mathcal{D}
- One with an approximate distribution \mathcal{D}' with the same support

Suppose that :

1. \mathcal{D} and \mathcal{D}' are close enough : $\left\| 1 - \frac{\mathcal{D}'}{\mathcal{D}} \right\|_{\infty} \leq 2^{-K}$
2. the number of sample queries is bounded

Then, the **bit security will remain almost the same.**



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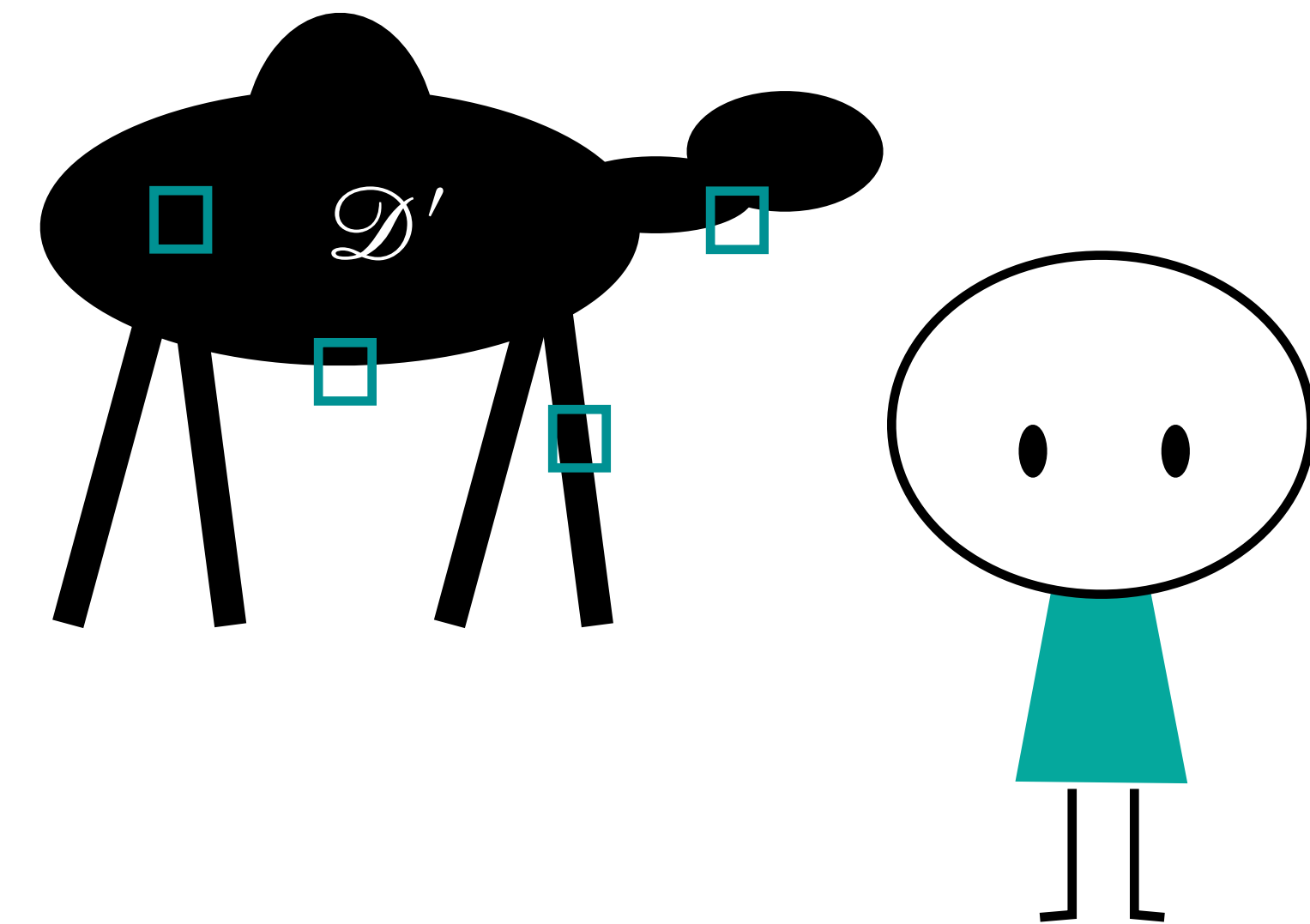
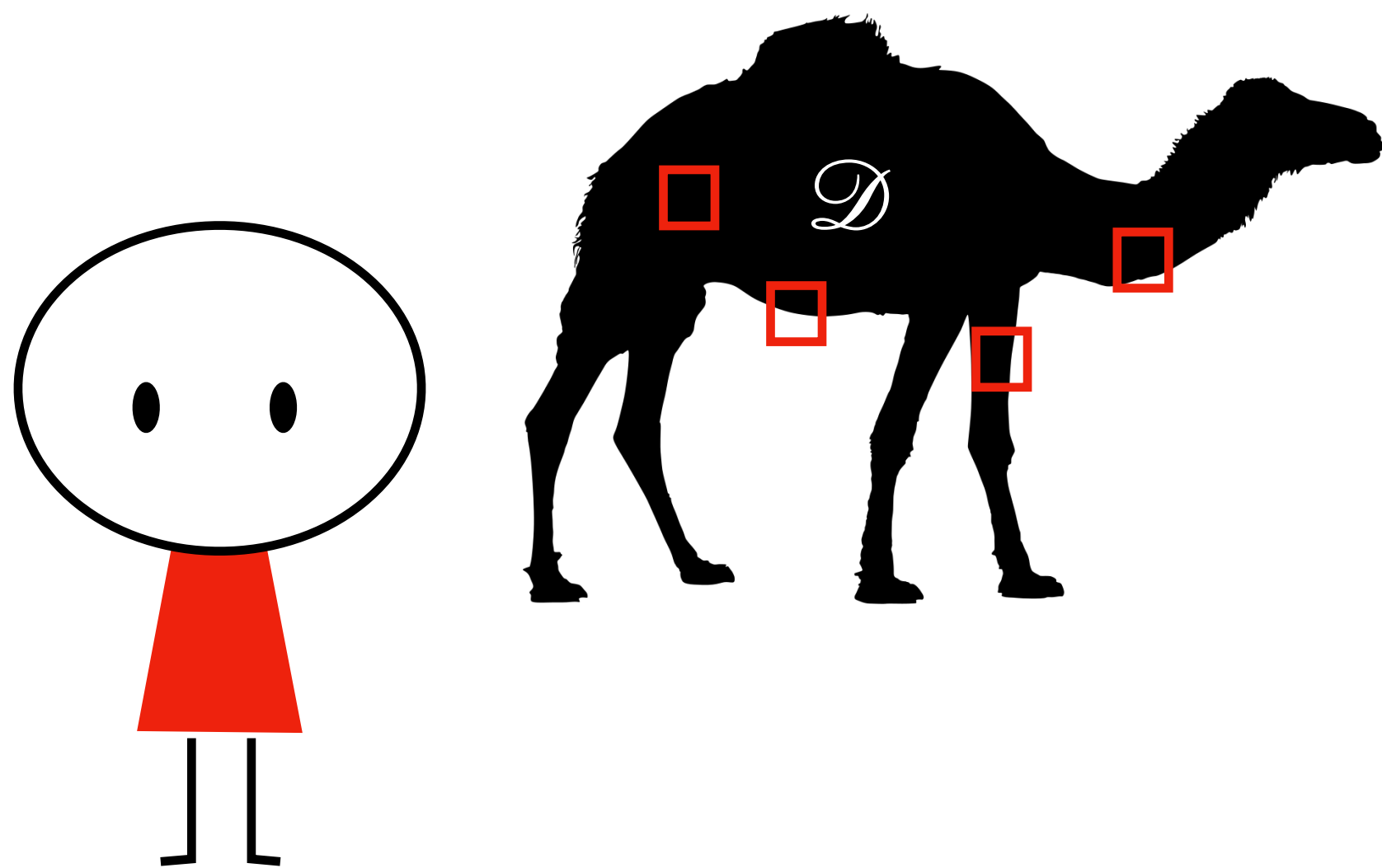
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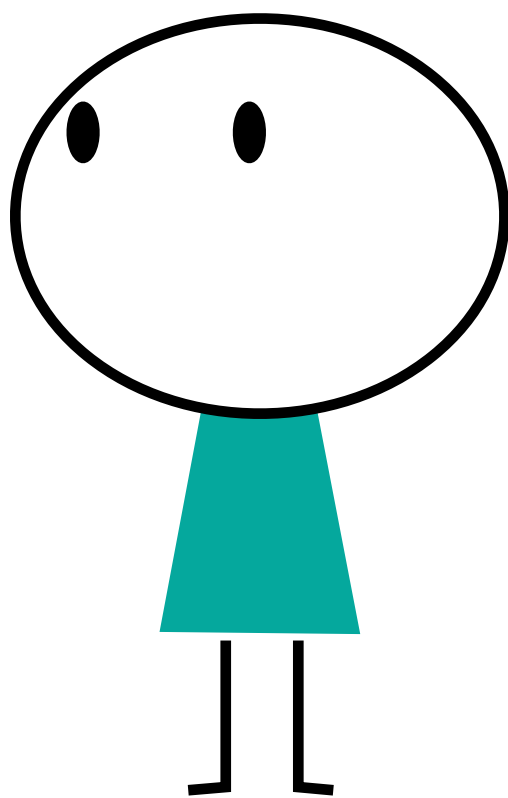
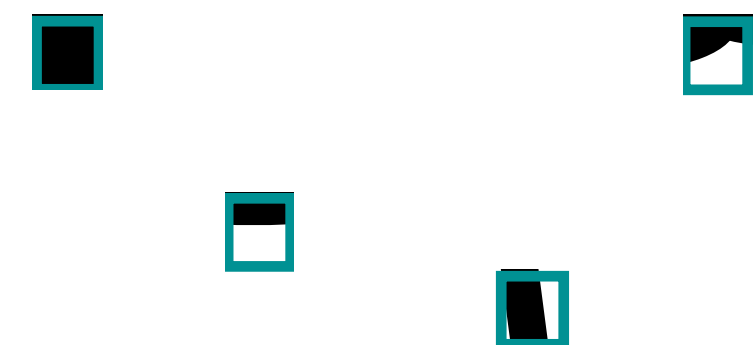
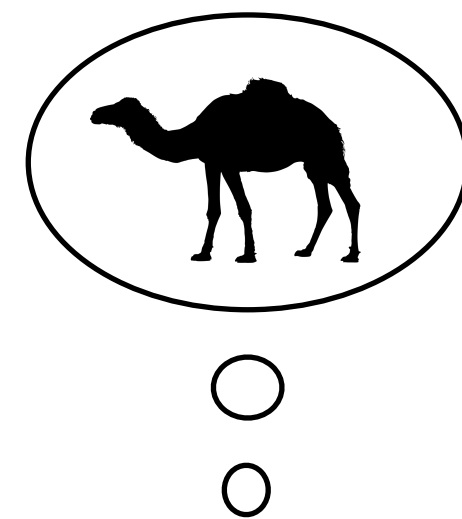
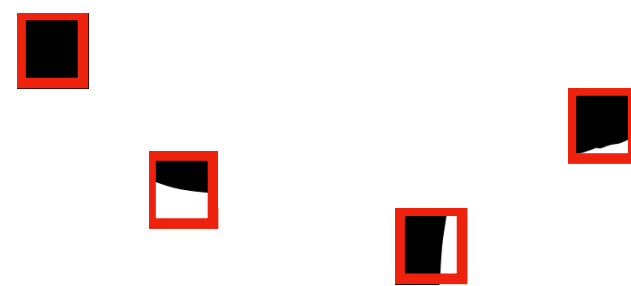
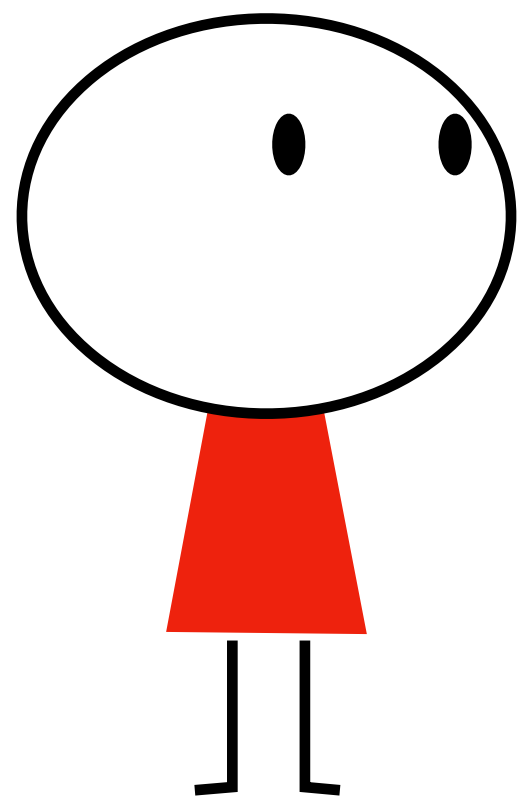
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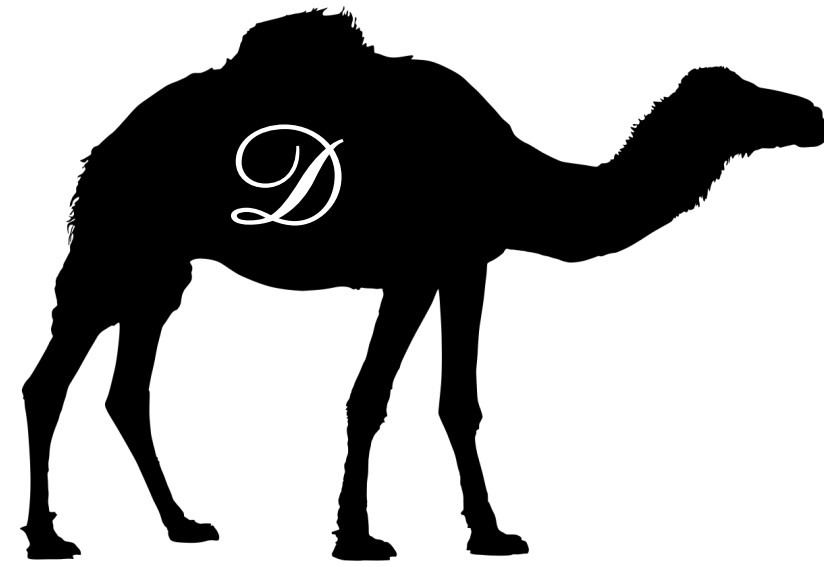
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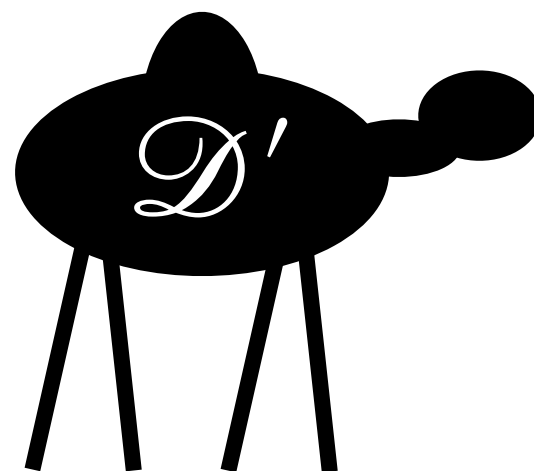


Transcendental
function

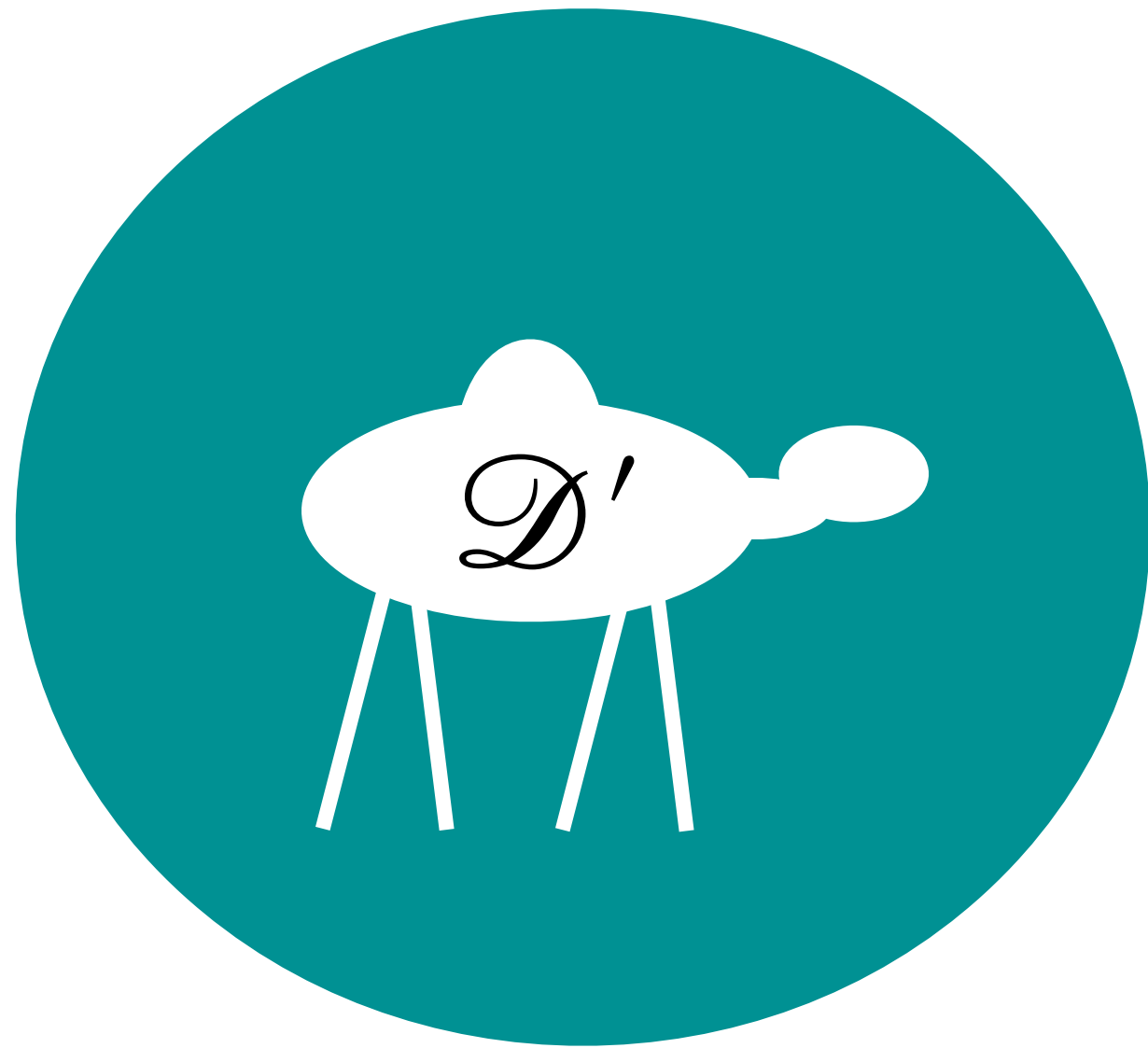


Polynomial approximation

Degree d polynomial in $\mathbb{Z}[x]$
with small coefficients

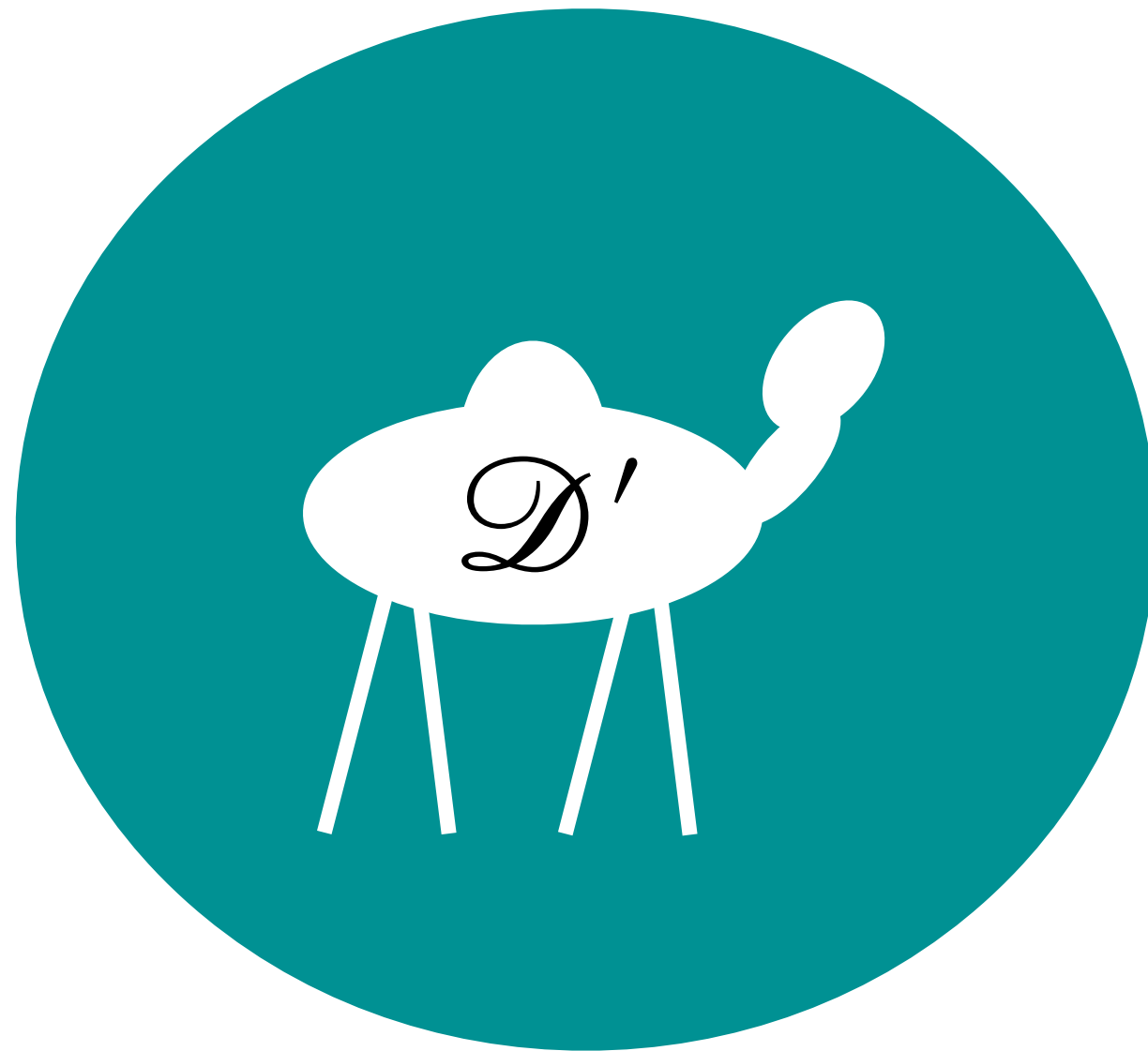


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• **Taylor expansion** $\mathcal{D}'(x) = \mathcal{D}(0) + \mathcal{D}'(1)(0) \cdot x + \dots + \frac{\mathcal{D}^{(d)}(0)}{d!} \cdot x^d$



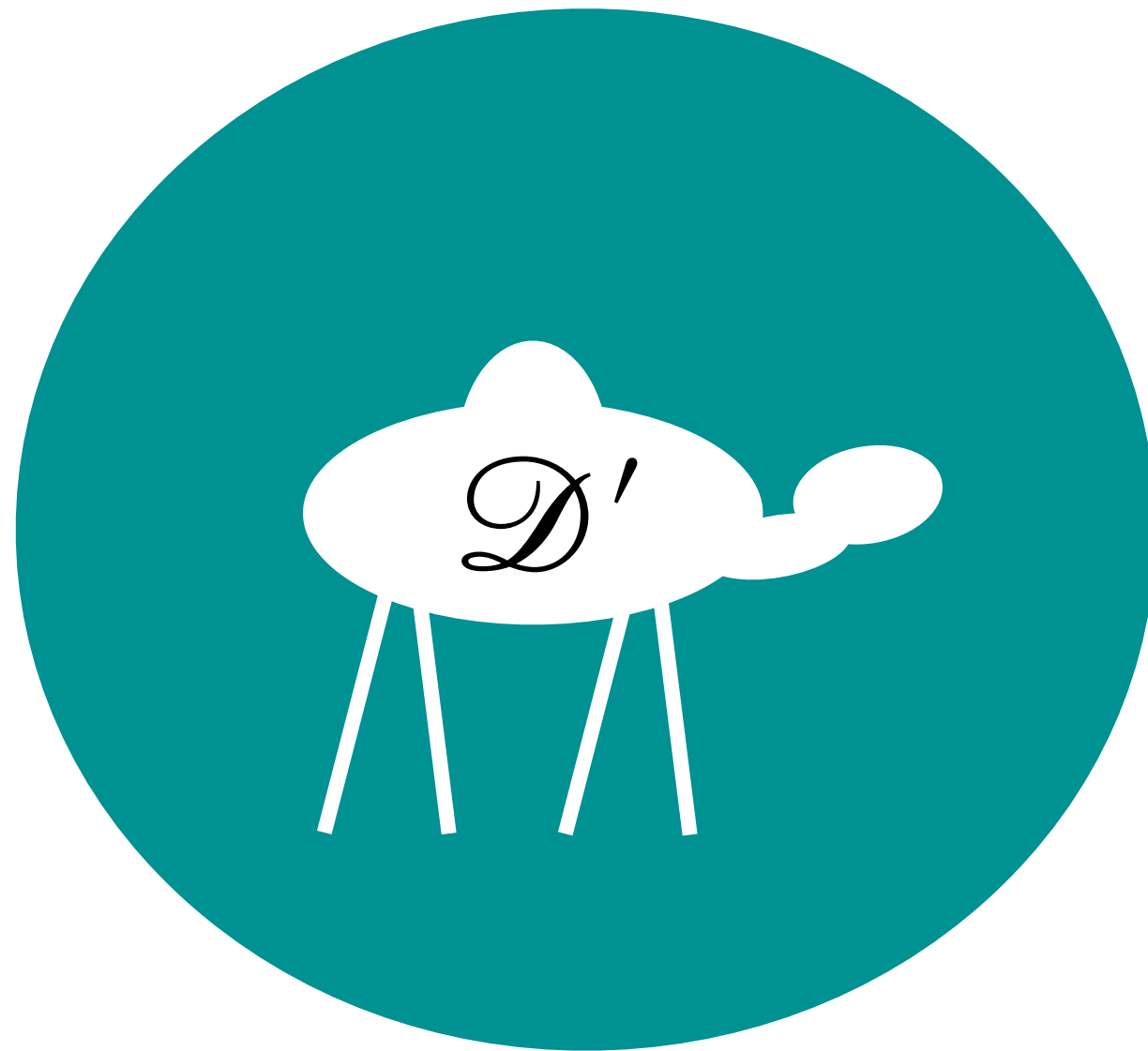
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▸ T. Prest [ASIACRYPT'17](#)

Two polynomials, higher degrees $\mathcal{D}'(x) = \frac{P(x)}{Q(x)}$



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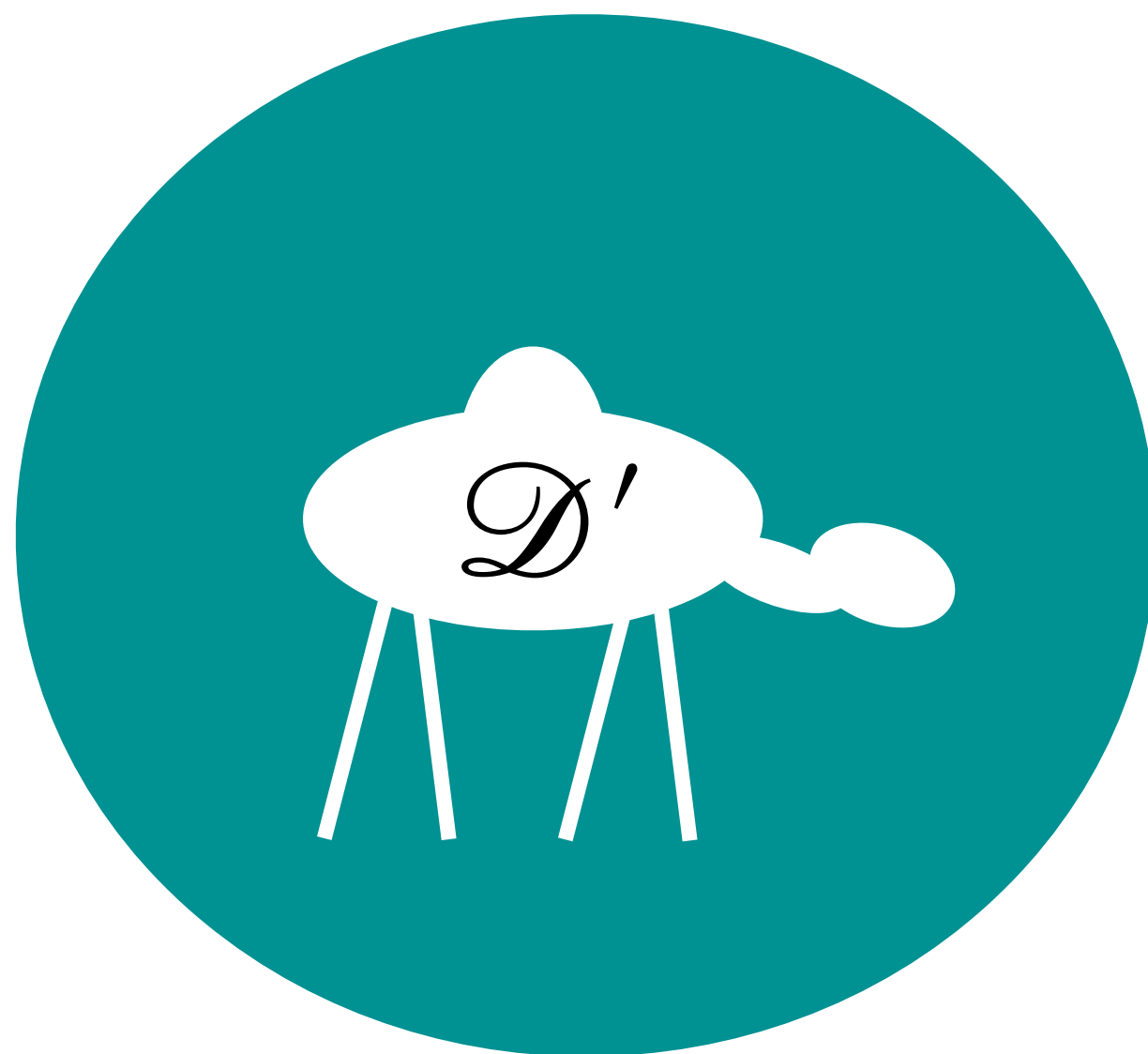
▸ N. Brisebarre and S. Chevillard [IEEE'07](#)

▸ S. Chevillard, M. Joldes and C. Q. Lauter [ICMS'10](#)

▸ R. Zhao, R. Steinfeld and A. Sakzad [IEEE'19](#)

Floating point arithmetics

$$\mathcal{D}' = \arg \min_{\deg(P) \leq d} \left(\sup_{x \in I} \left(1 - \frac{P(x)}{\mathcal{D}(x)} \right) \right)$$



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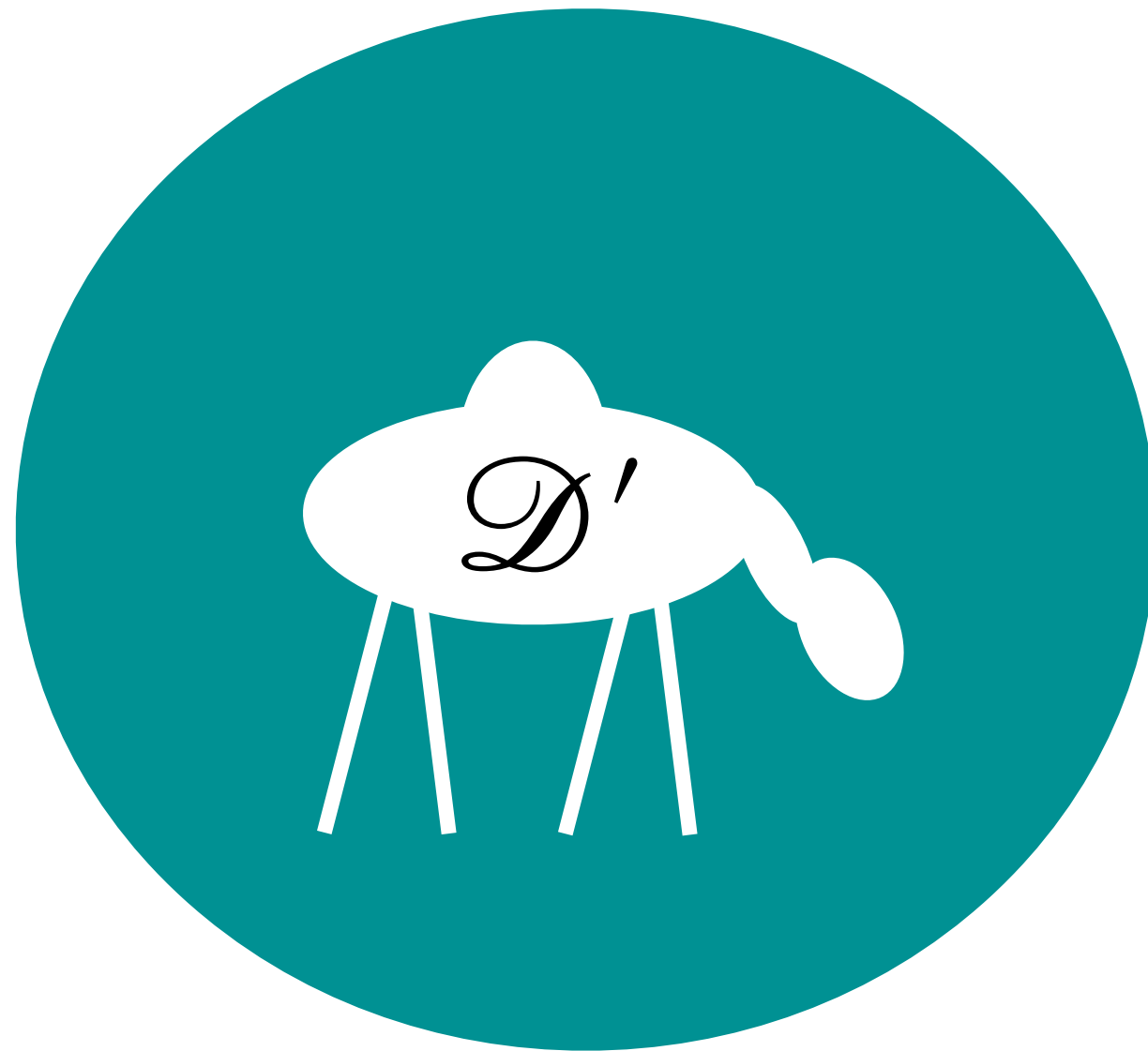
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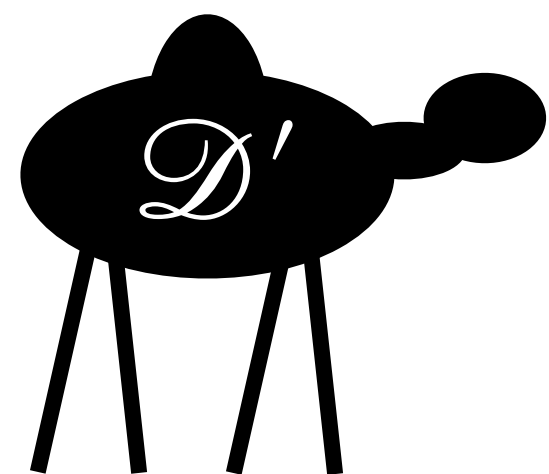
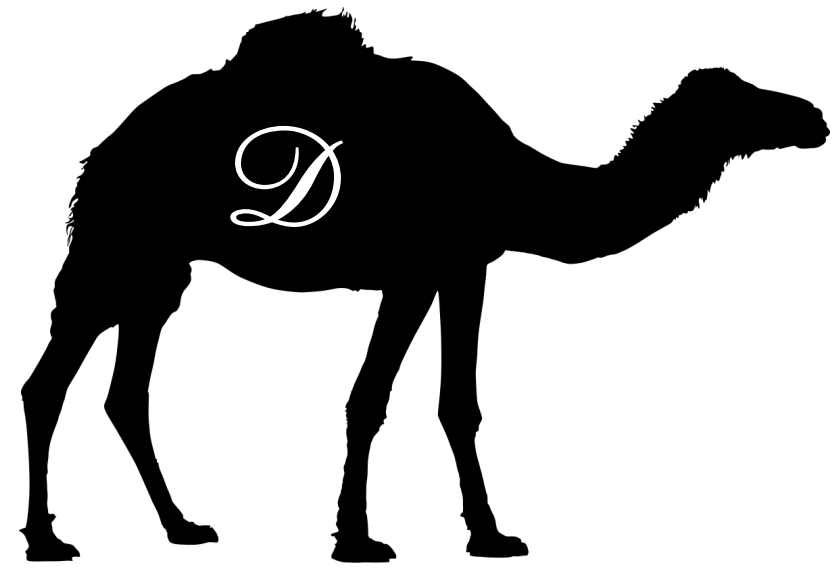
• **Projections with respect to the Sobolev Norm**

▶ [GALACTICS \[...\] ACM-CCS'2019](#). G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, M. Rossi and M. Tibouchi.

$$\|f\|_{\infty} \leq \sqrt{2} \cdot \|f\|_S$$



► *GALACTICS* [...] ACM-CCS'2019. G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, M. Rossi and M. Tibouchi.

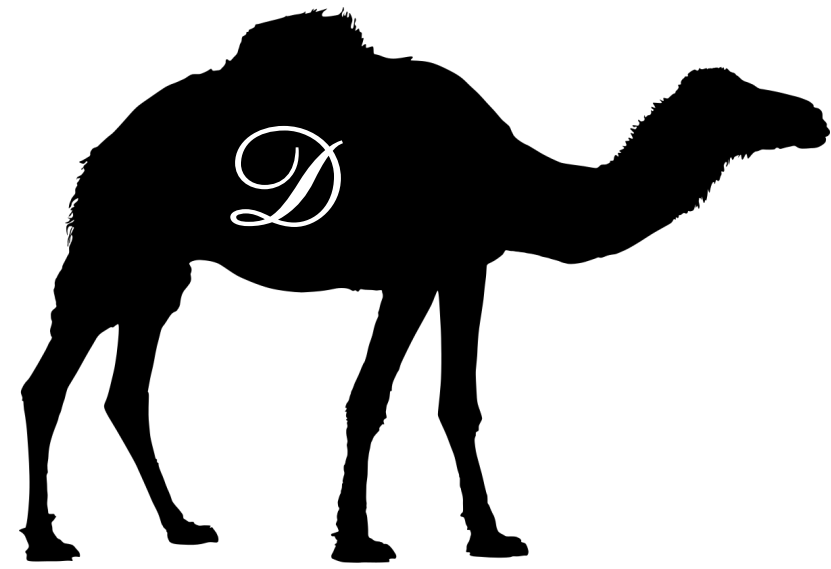


Polynomial approximation



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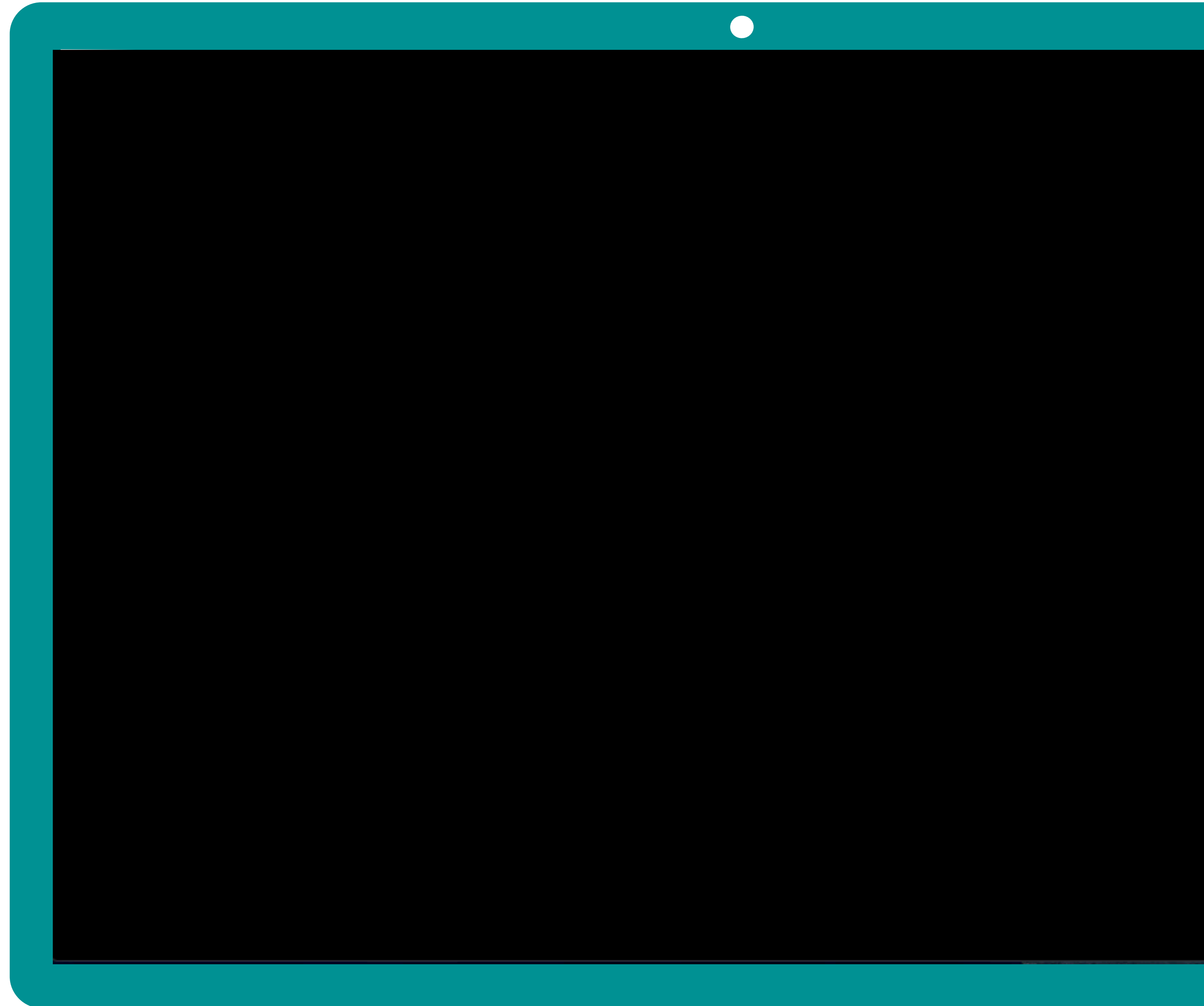
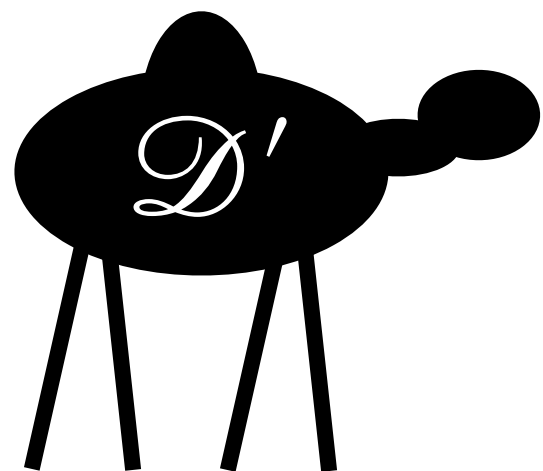
Transcendental
function



Degree d polynomial in $\mathbb{R}[x]$

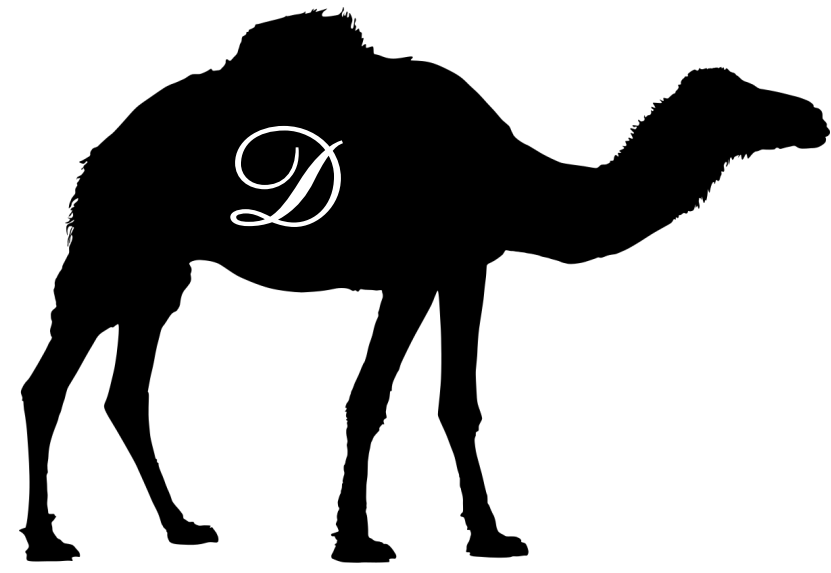


Degree d polynomial in $\mathbb{Z}[x]$
with η -bit coefficients



Polynomial approximation

► GALACTICS [...] ACM-CCS'2019. G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, M. Rossi and M. Tibouchi.



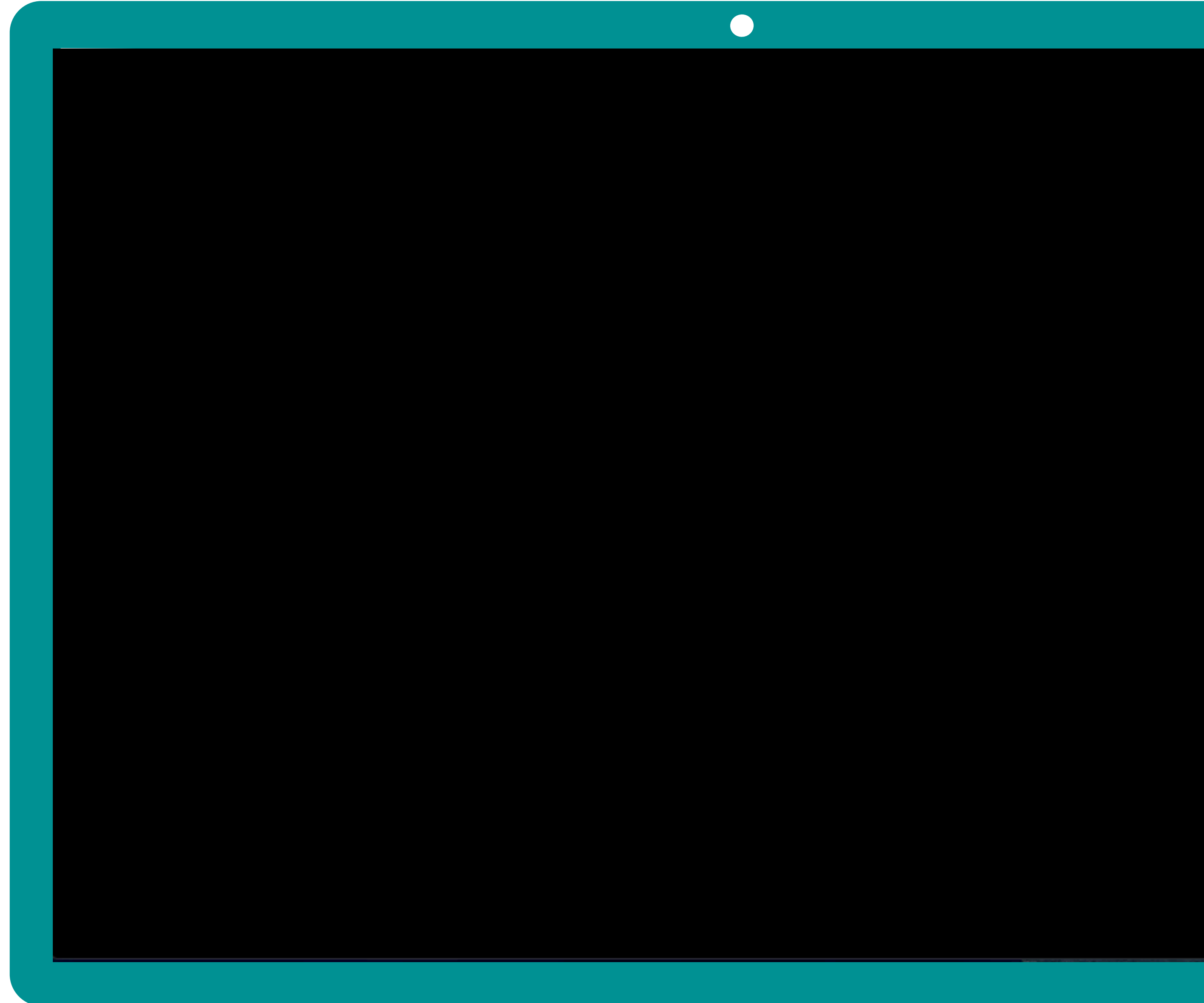
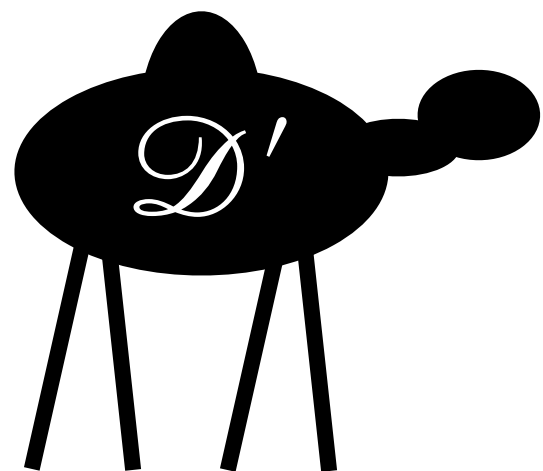
Transcendental
function

Successive orthogonal projections
in a polynomial space wrt the
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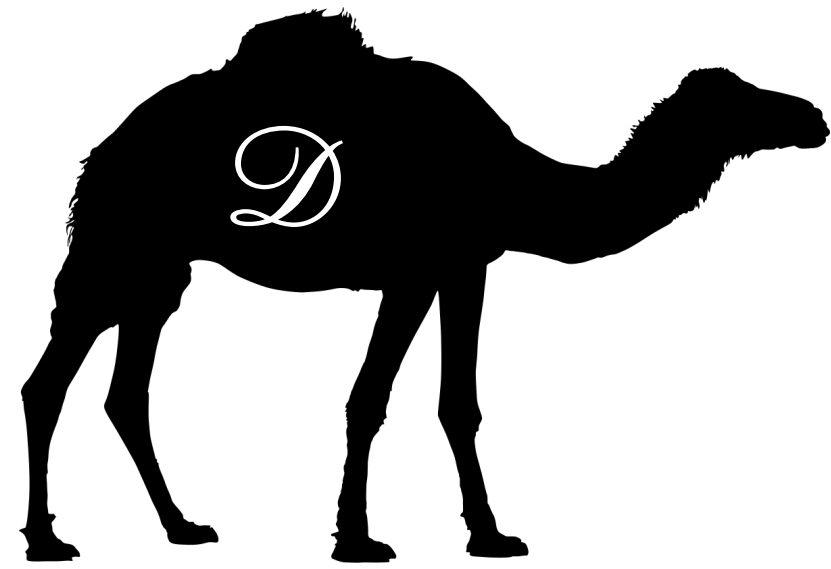
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Polynomial approximation



Transcendental
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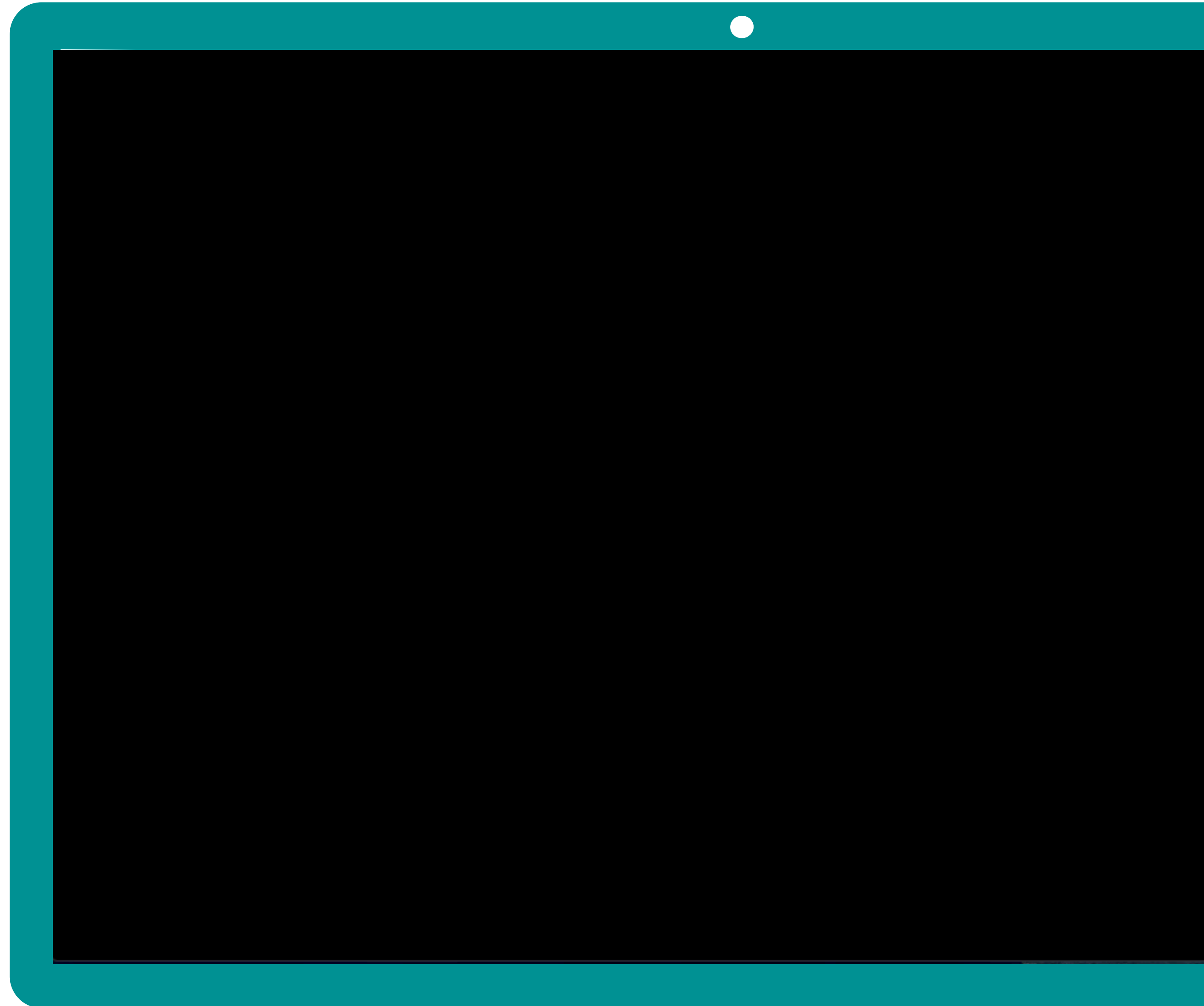
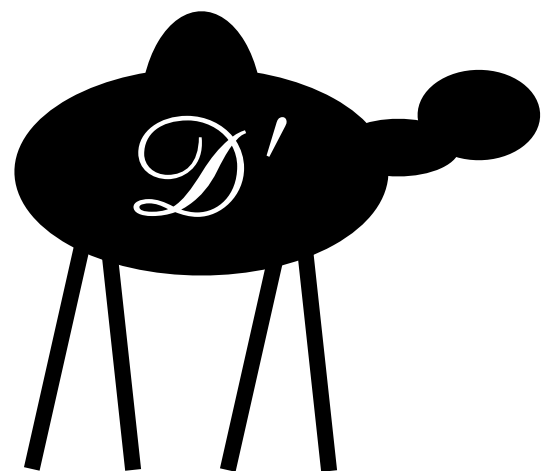
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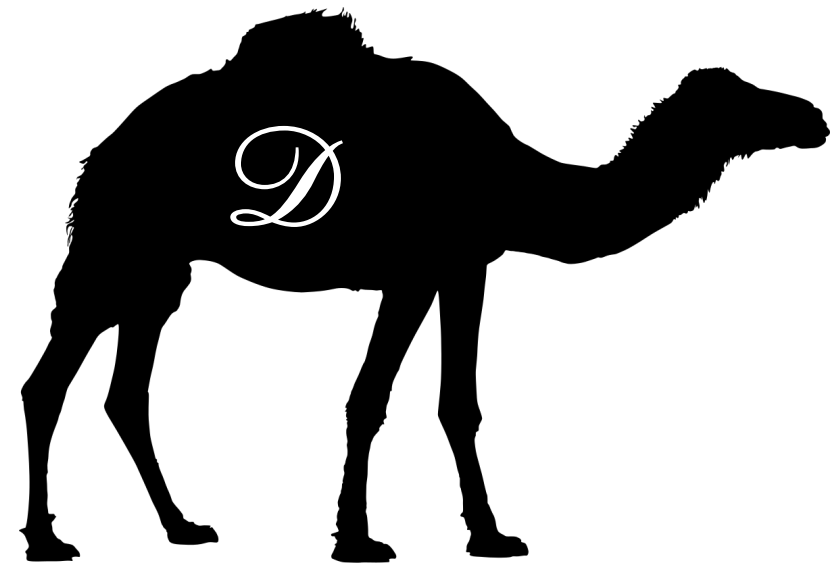
Degree d polynomial in $\mathbb{R}[x]$

LLL reduction
Babai rounding

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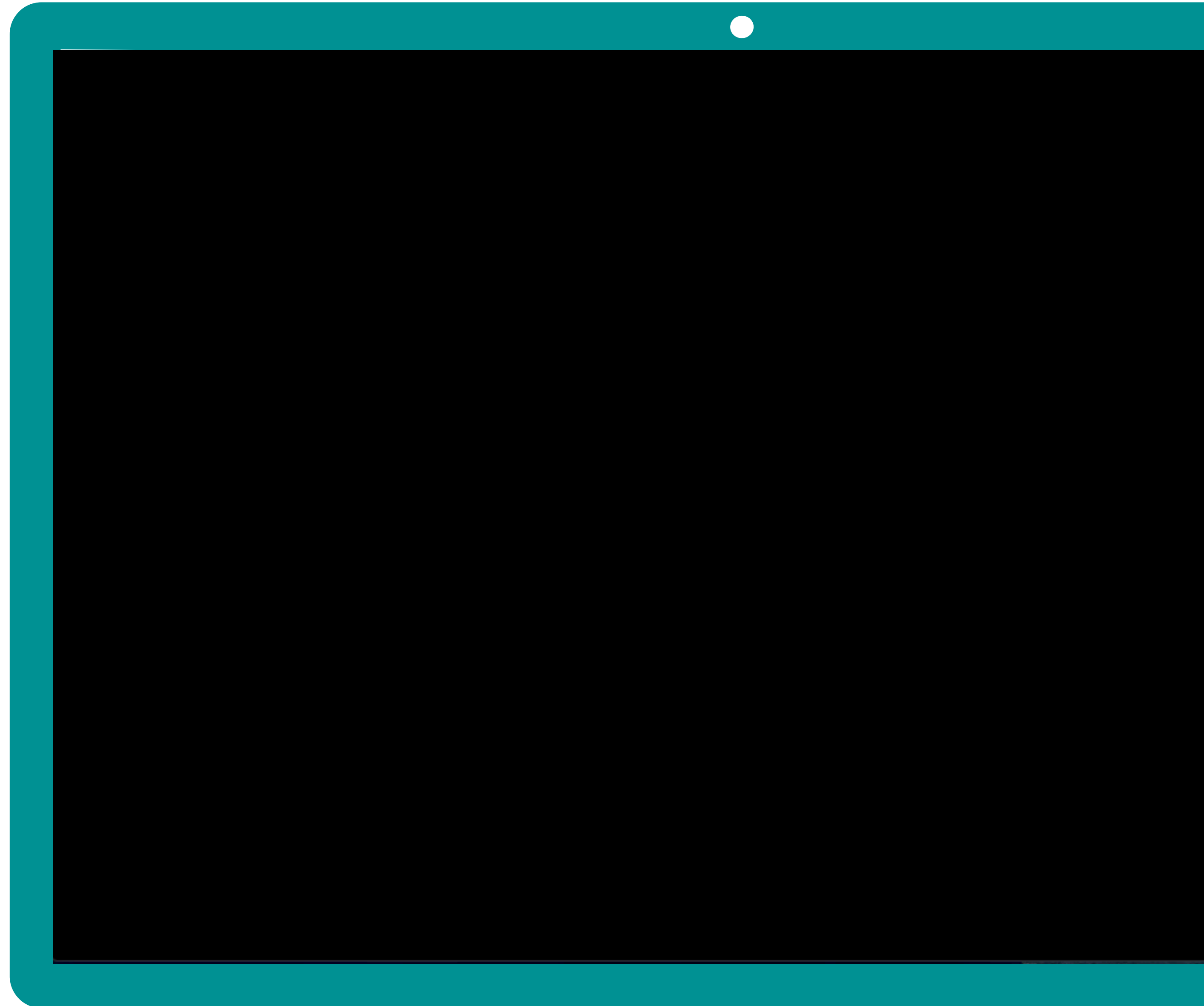
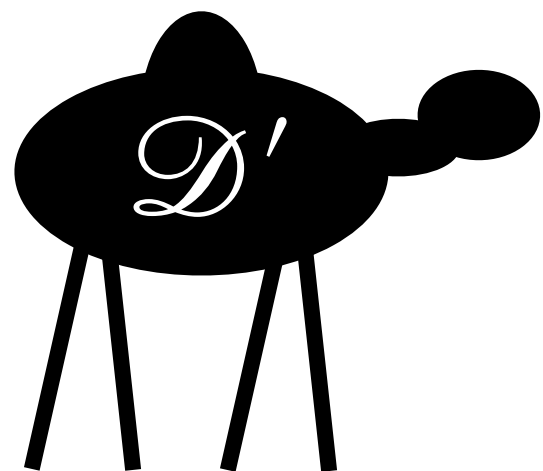
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An example with Fiat-Shamir with aborts

Signature algorithm:

1: do

2: $y \xleftarrow{\$} Y$

3: $c \leftarrow H(\mathbf{A}y, m)$

4: $\mathbf{z} \leftarrow c \cdot \mathbf{S} + y$

5: while Rejected($\mathbf{z}, c, \mathbf{S}$)

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For BLISS

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Signature algorithm:

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Isochronous

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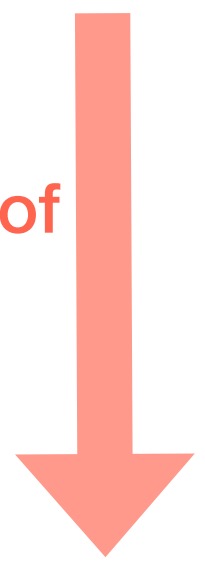
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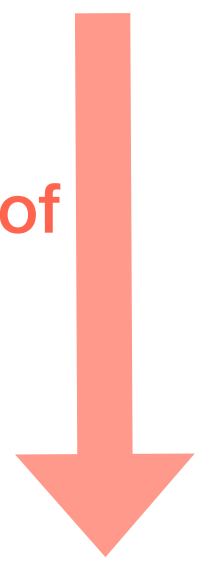
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Polynomial evaluation: simple and constant time (multiplications and additions)

➡ Would you say that it is more or less efficient?

Examples of application for proving isochrony

Falcon

Performance penalty factor :

+50 %

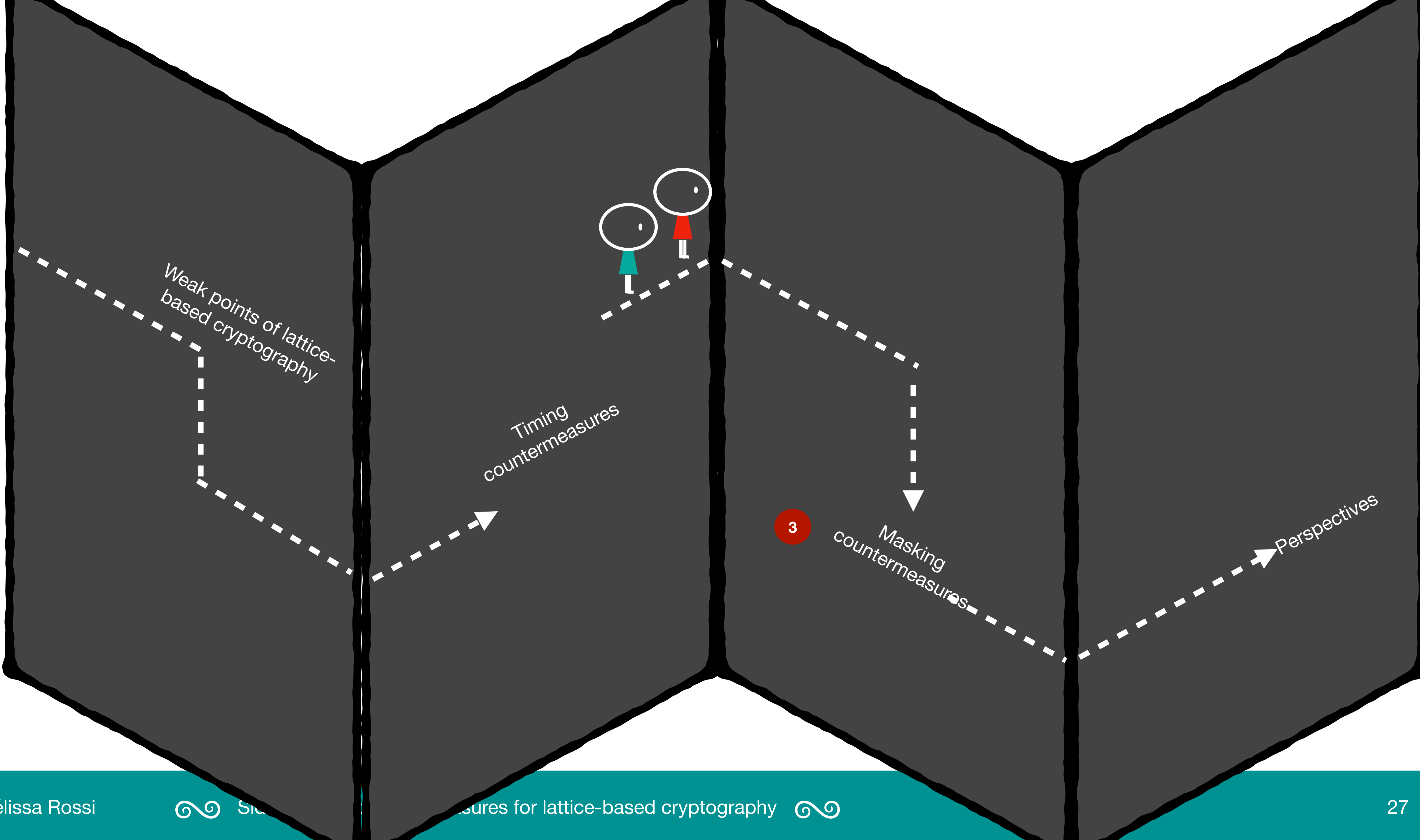
- ▶ J. Howe, T. Prest, T. Ricosset and M. Rossi. [PQ-CRYPTO'2020](#).
- ▶ T. Pornin <https://falcon-sign.info/falcon-impl-20190802.pdf>

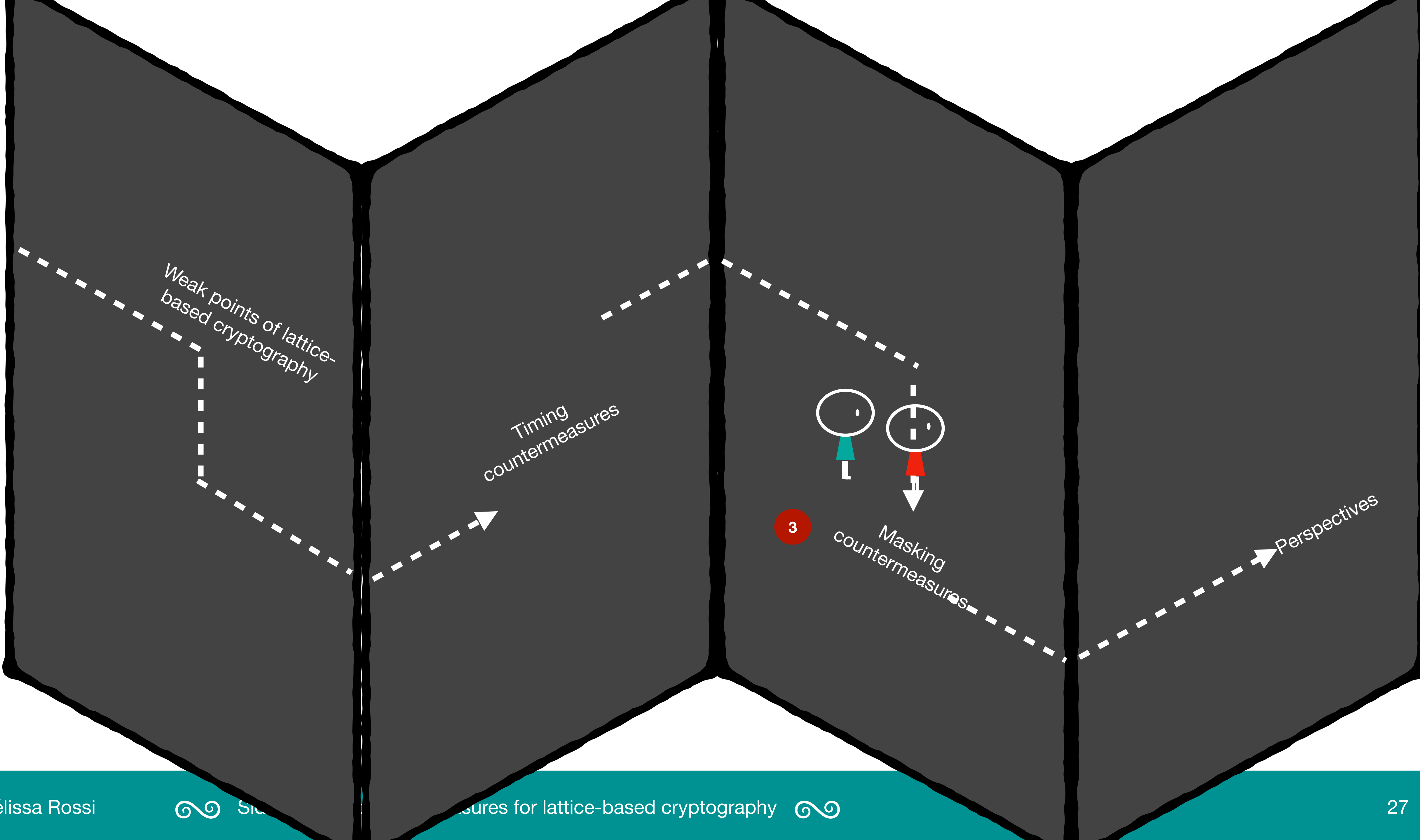
BLISS

Performance penalty factor :

+13 %

- ▶ G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, M. Rossi and M. Tibouchi. [ACM-CCS'2019](#).





Weak points of lattice-based cryptography

Timing countermeasures

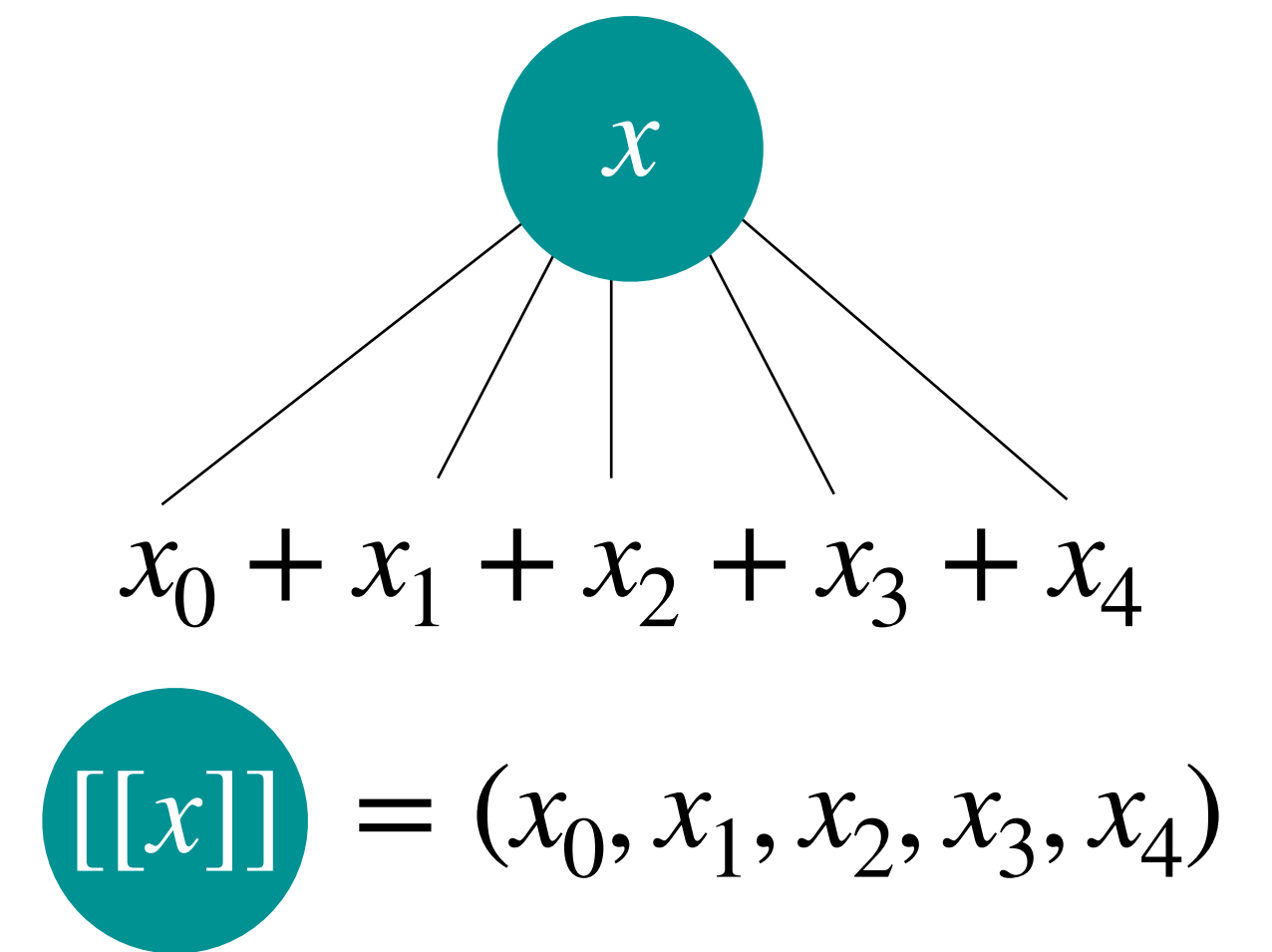
3

Masking countermeasures

Perspectives



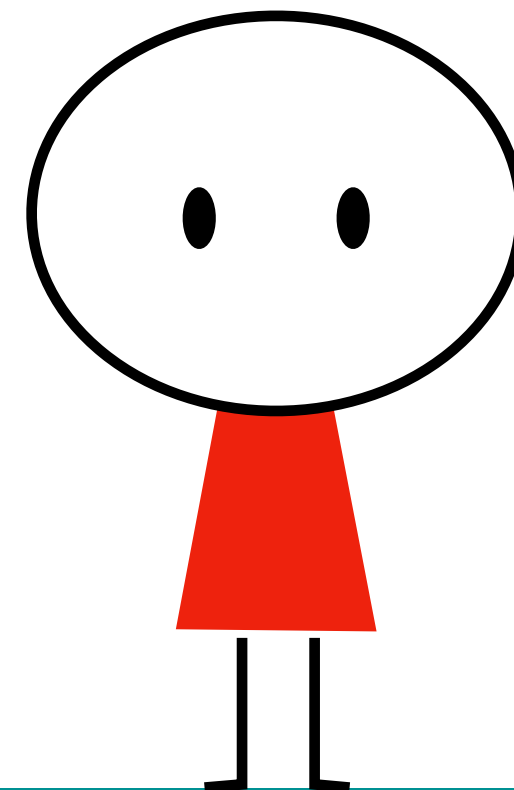
Masking



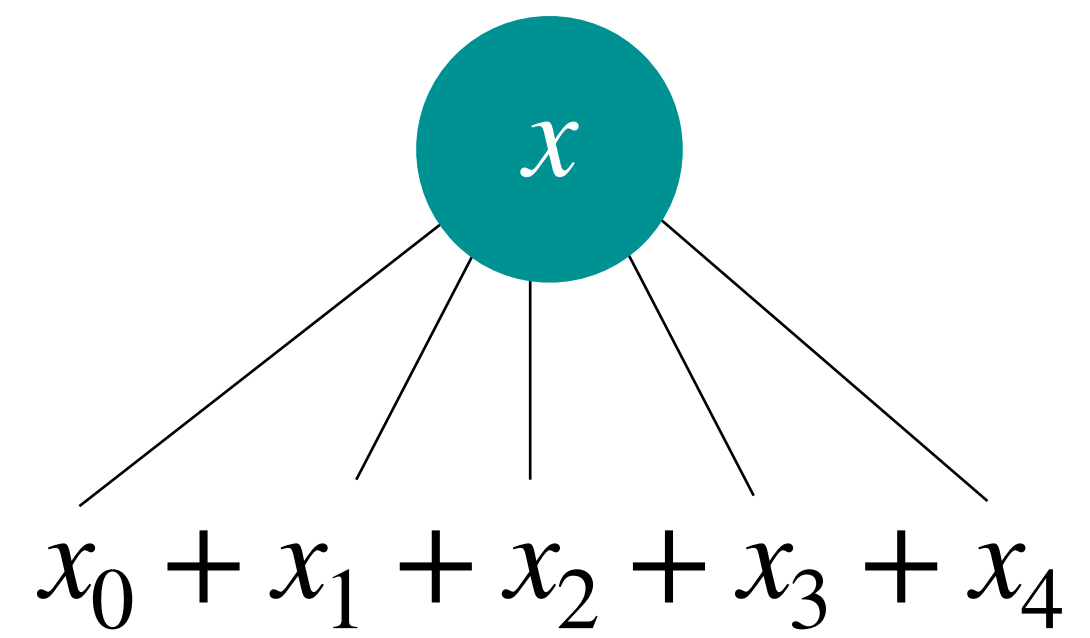
Each share looks random.

The only way to recover x is to know all of them.

Masking order : $d = 4$.



Masking



$$[[x]] = (x_0, x_1, x_2, x_3, x_4)$$

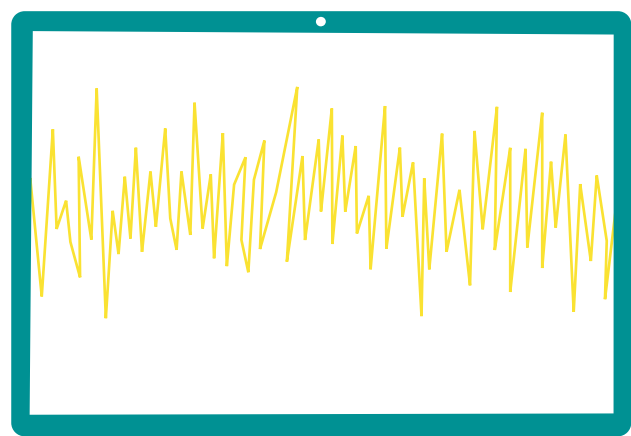
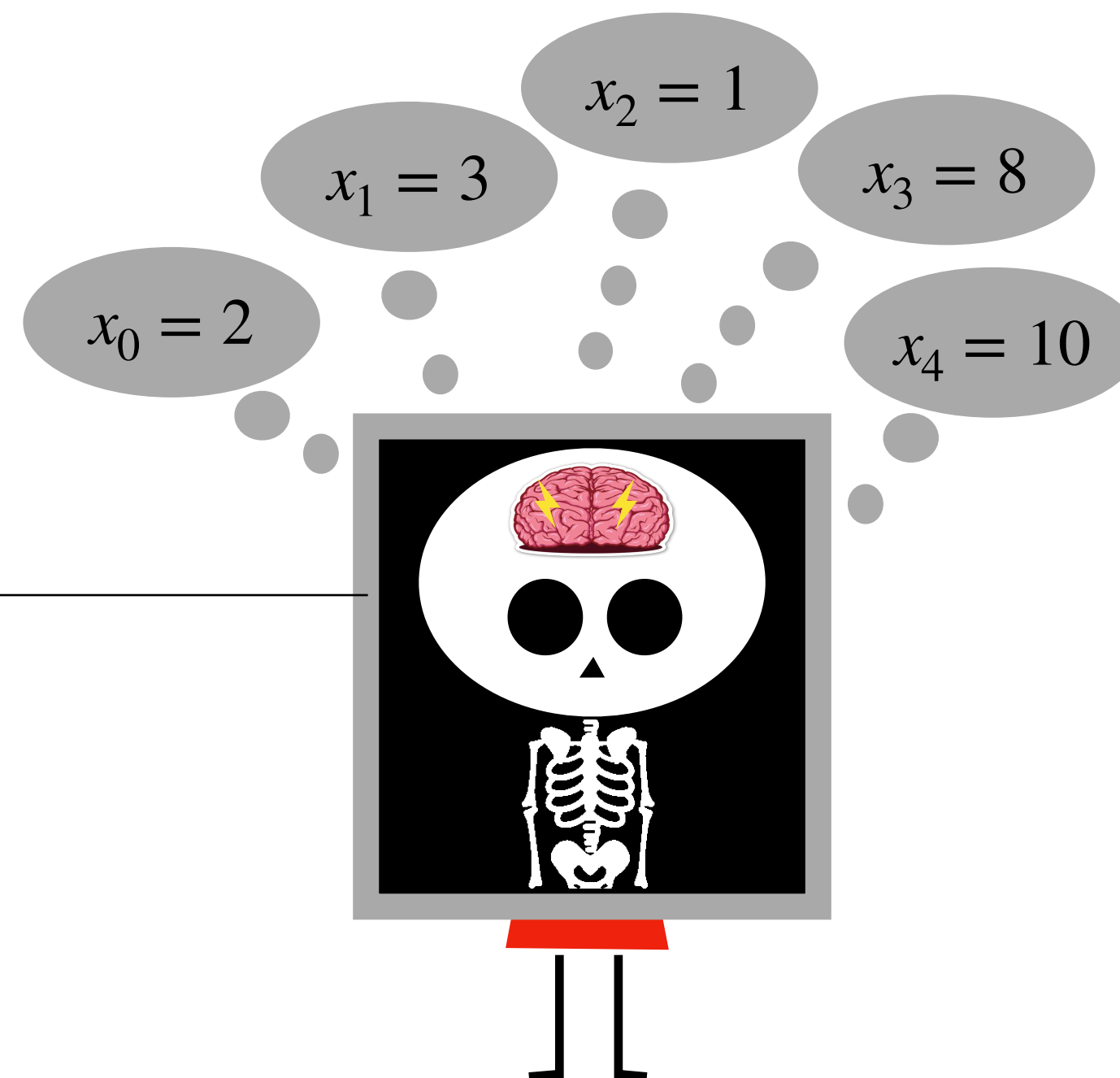
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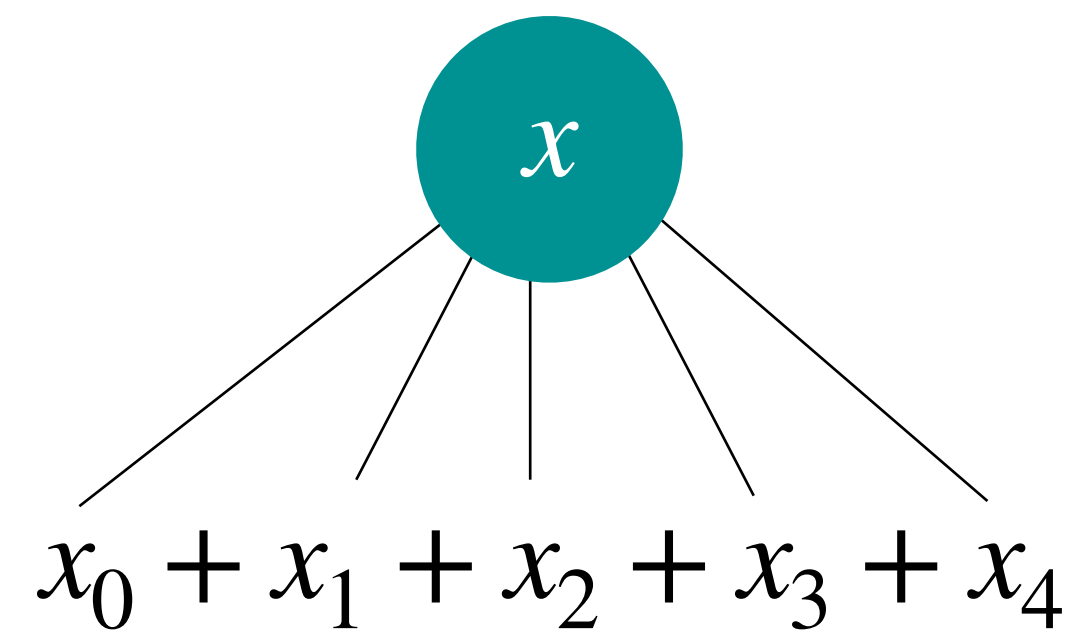
The real secret value is

$$\begin{aligned} x &= 2 + 3 + 1 + 8 + 10 \\ &= 24 \end{aligned}$$



Masking

➔ Increase of the noise: Highly complicates the dependancies between the secret and the measurement



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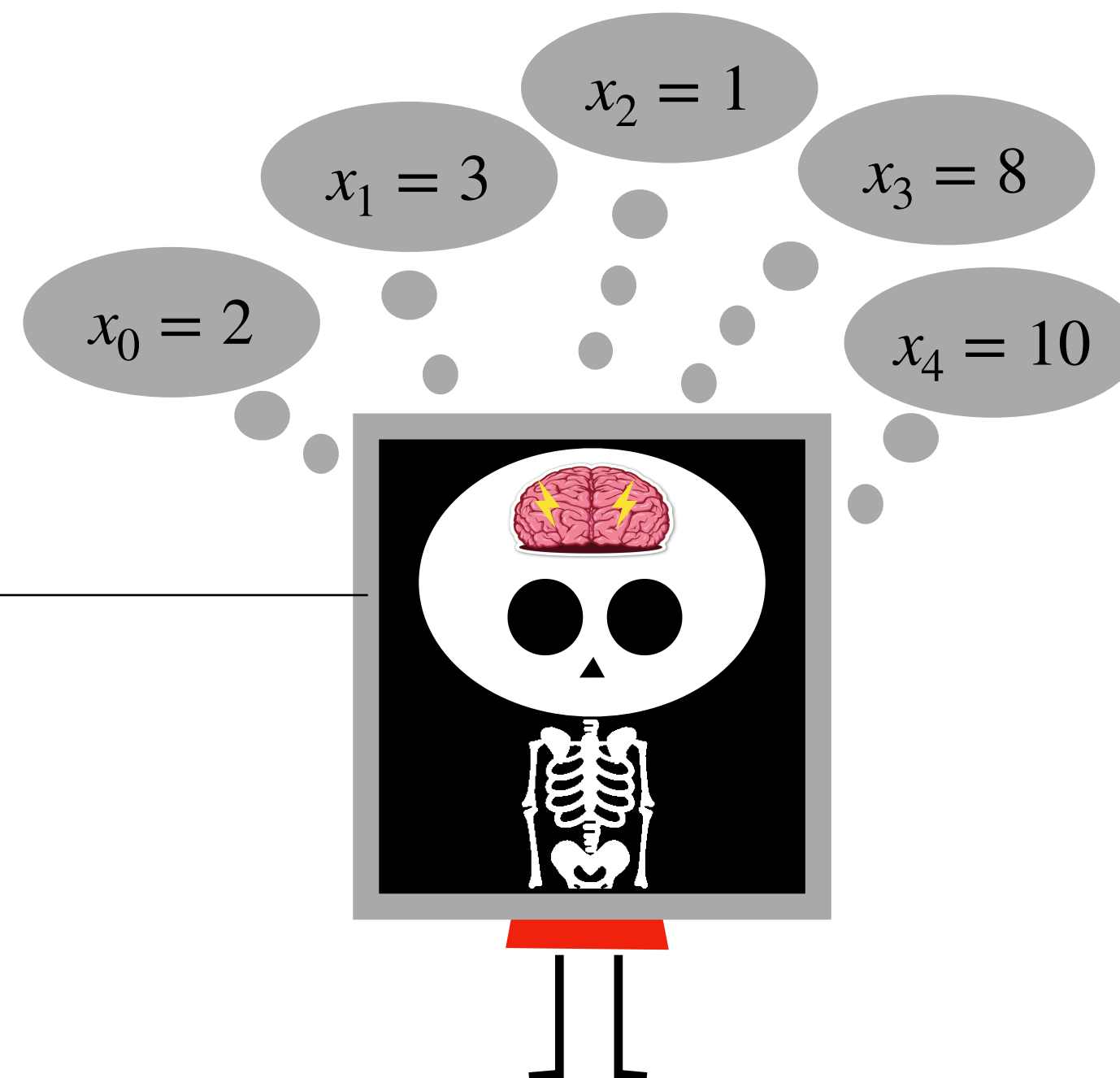
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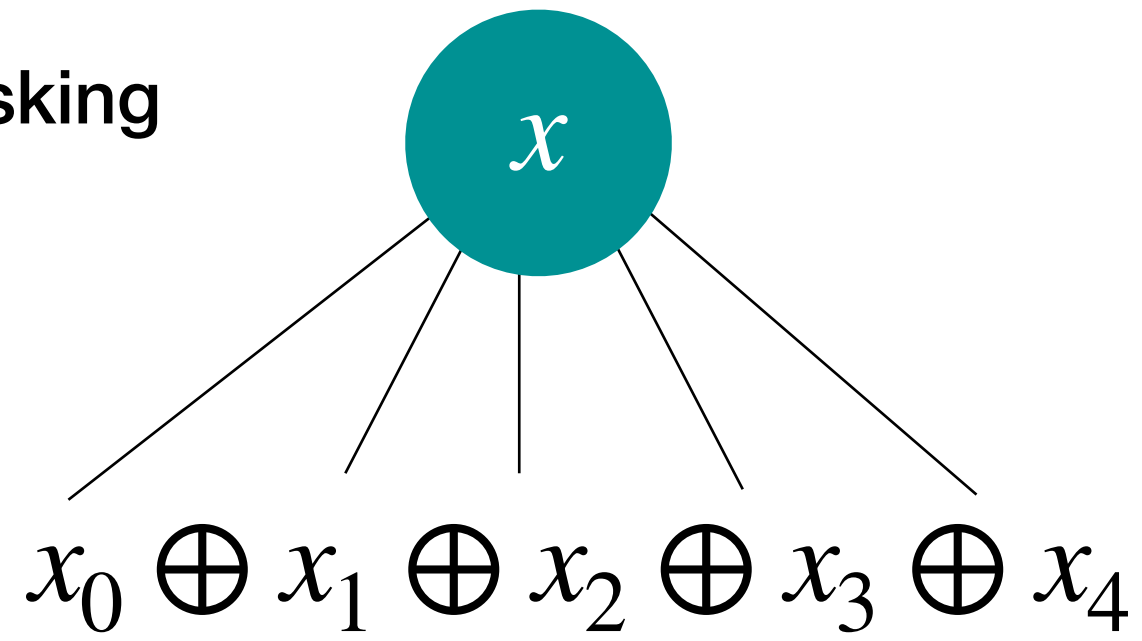
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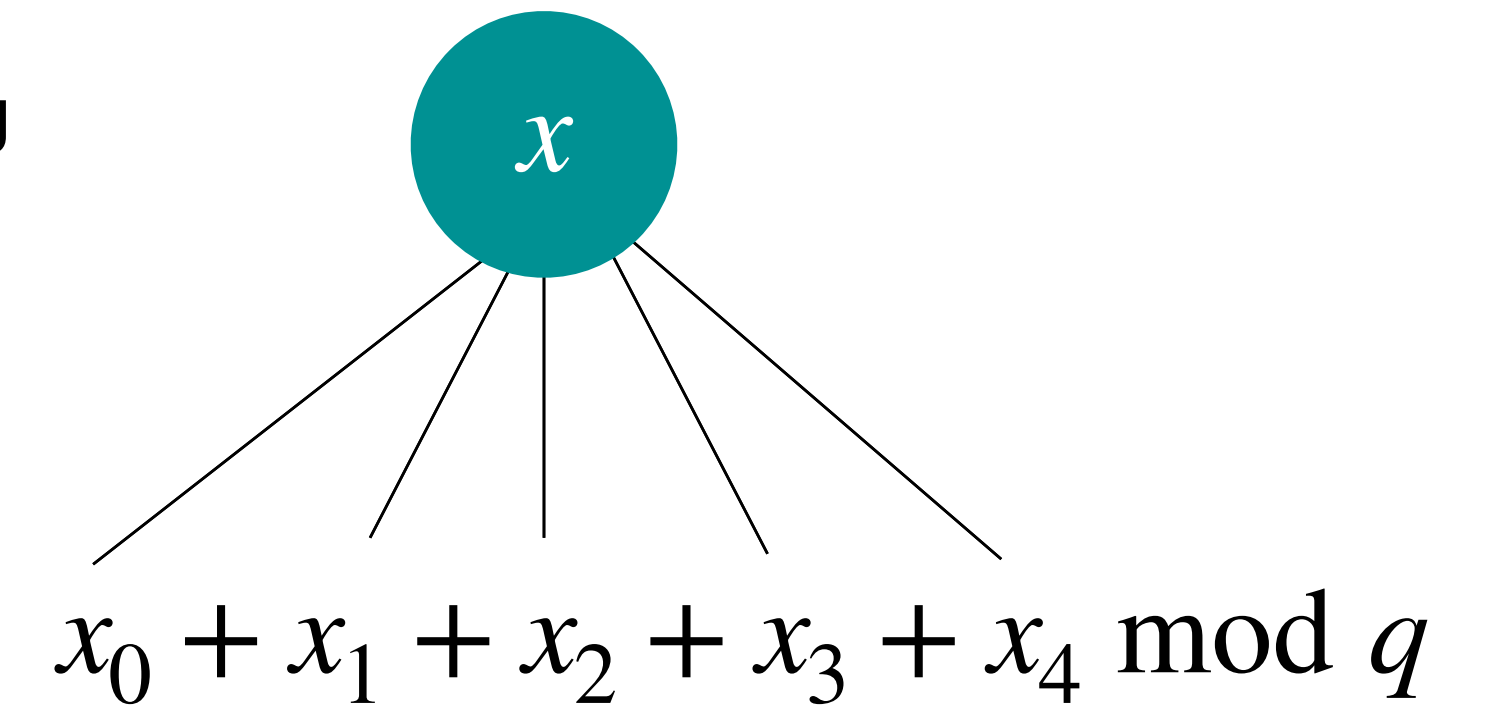
Boolean and arithmetic masking

Boolean masking



$$[[x]] = (x_0, x_1, x_2, x_3, x_4)$$
$$[[y]] = (y_0, y_1, y_2, y_3, y_4)$$

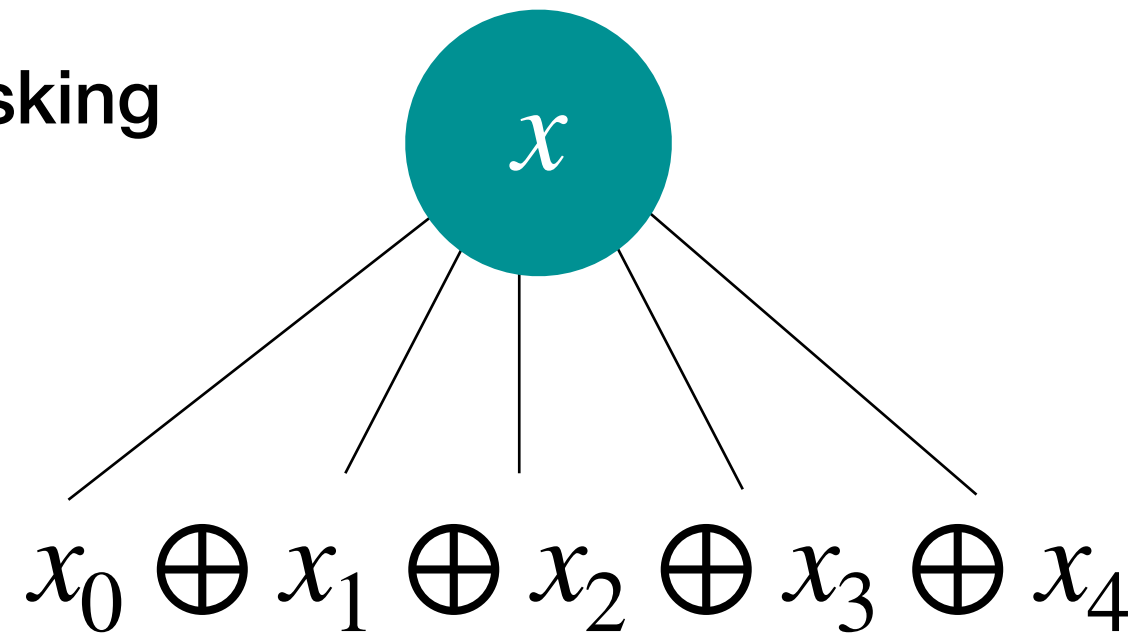
Arithmetic masking



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Boolean and arithmetic masking

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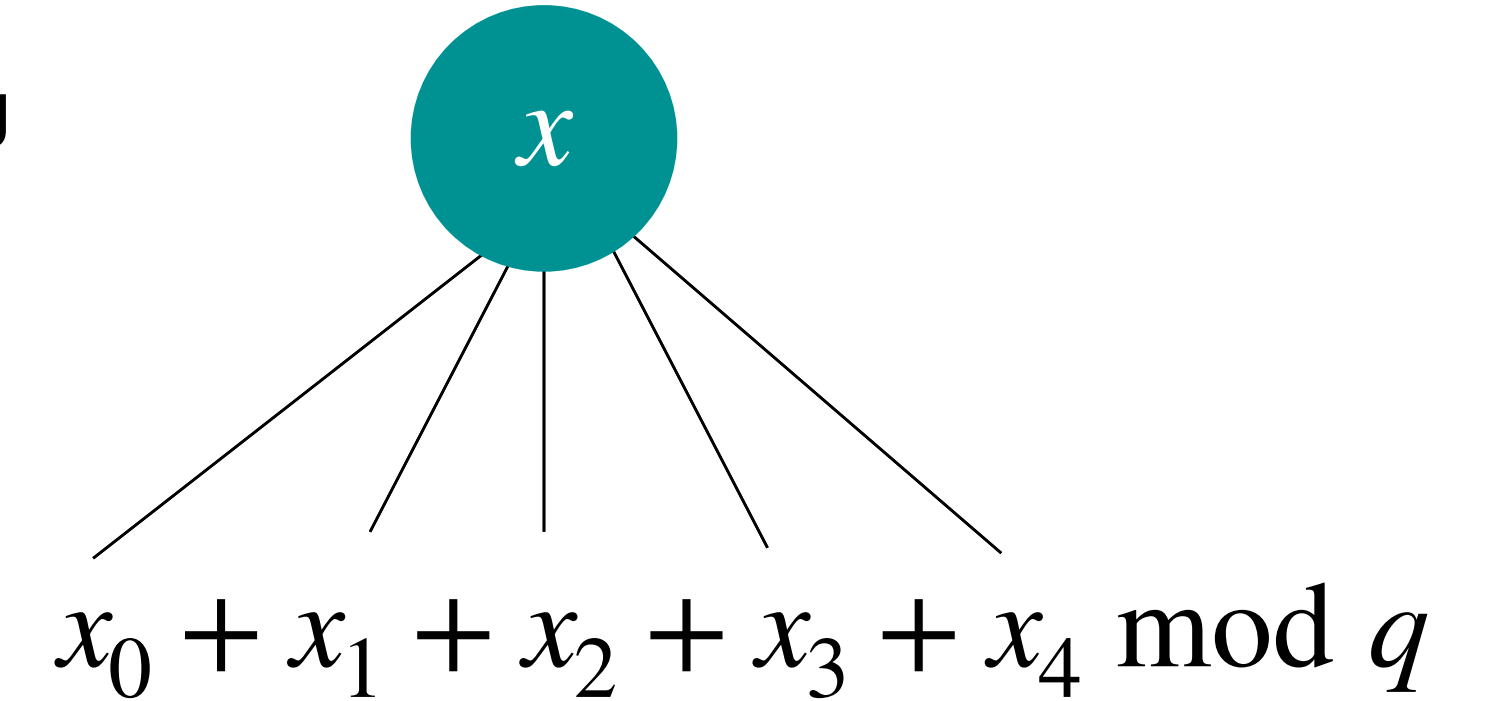
$$[[x]] = (x_0, x_1, x_2, x_3, x_4)$$

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\mathbb{F}_2 -linear operations:

$$[[x]] \oplus [[y]] = (x_0 \oplus y_0, x_1 \oplus y_1, x_2 \oplus y_2, x_3 \oplus y_3, x_4 \oplus y_4)$$

Arithmetic masking



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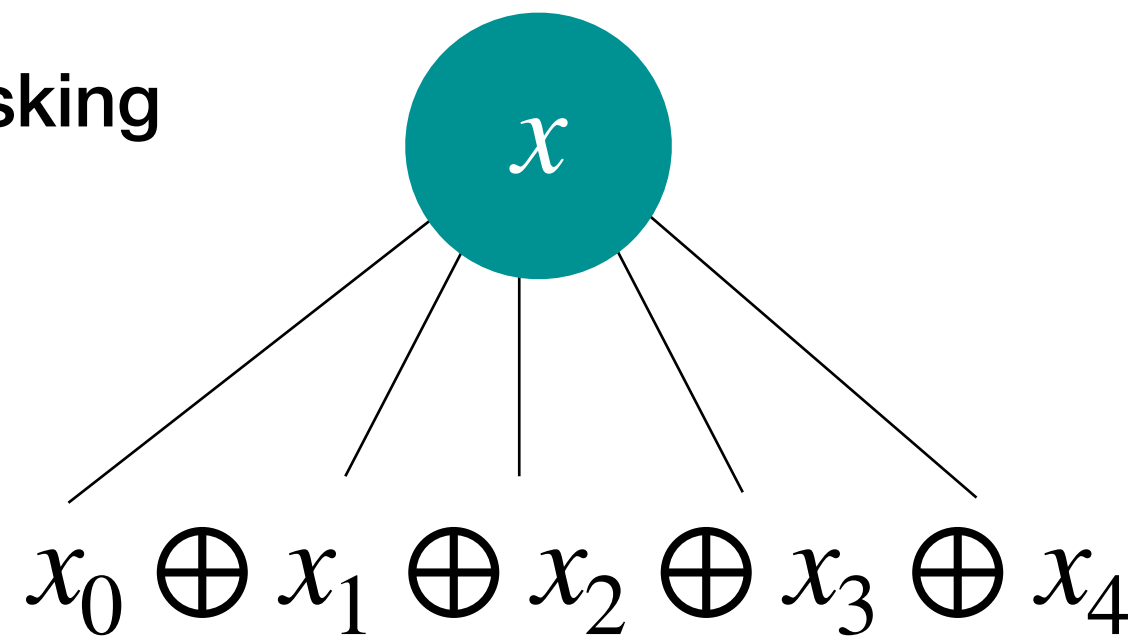
$$[[y]] = (y_0, y_1, y_2, y_3, y_4)$$

\mathbb{F}_q -linear operations:

$$[[x]] + [[y]] \bmod q = (x_0 + y_0 \bmod q, \dots, x_4 + y_4 \bmod q)$$

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Boolean masking



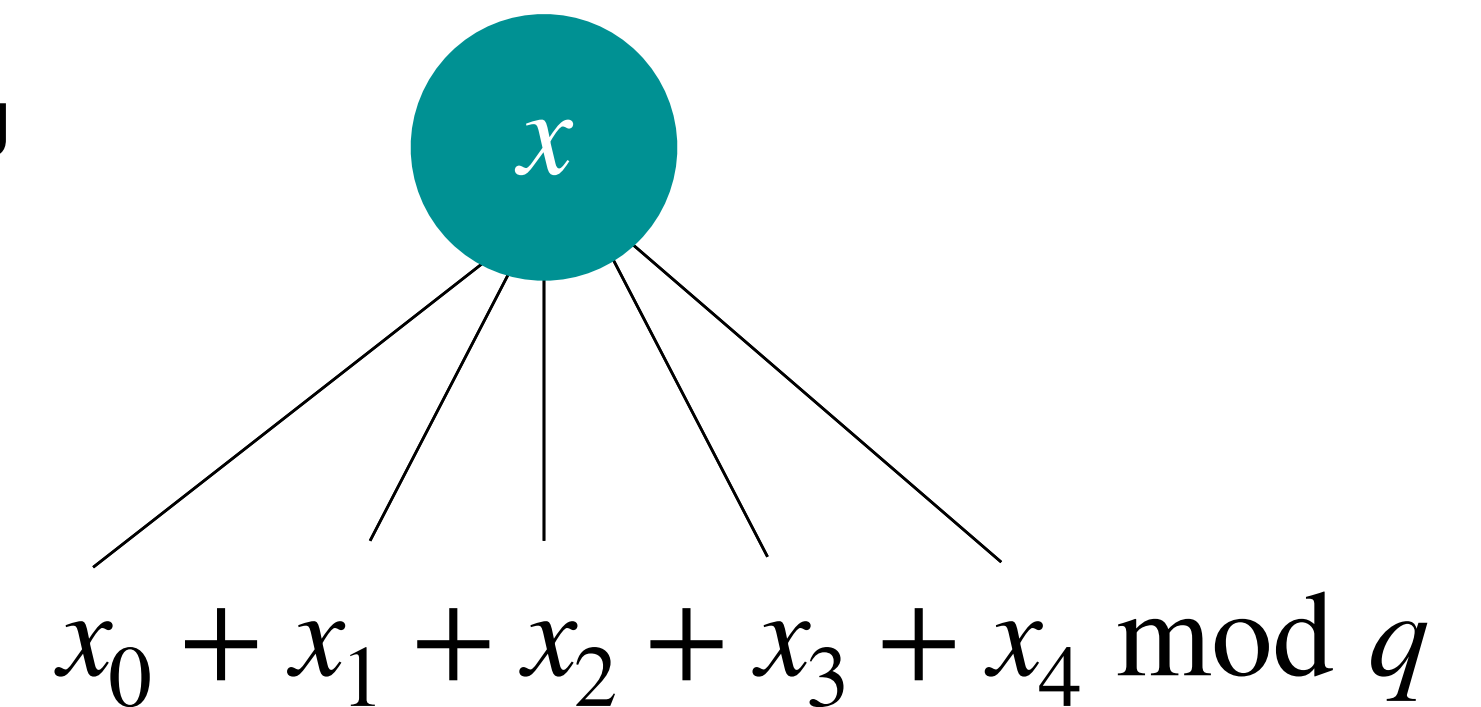
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What about non linear operations?

➡ Need for extra randomness to mix shares without introducing any biases.

Designs for the multiplication of two shared values ▶ L. Goubin and J. Patarin [CHES'1999](#)

▶ S. Chari, C. Jutla, J. Rao and P. Rohatgi [CRYPTO'1999](#)

More information in J.S. Coron's presentation

How to combine many operations?

- ▶ Y. Ishai, A. Sahai and D. Wagner [CRYPTO'2003](#)
- ▶ G. Barthe, S. Belaid, F. Dupressoir, P.-A. Fouque, B. Grégoire, P.-Y. Strub, and R. Zucchini. [ACM-CCS'2016](#)

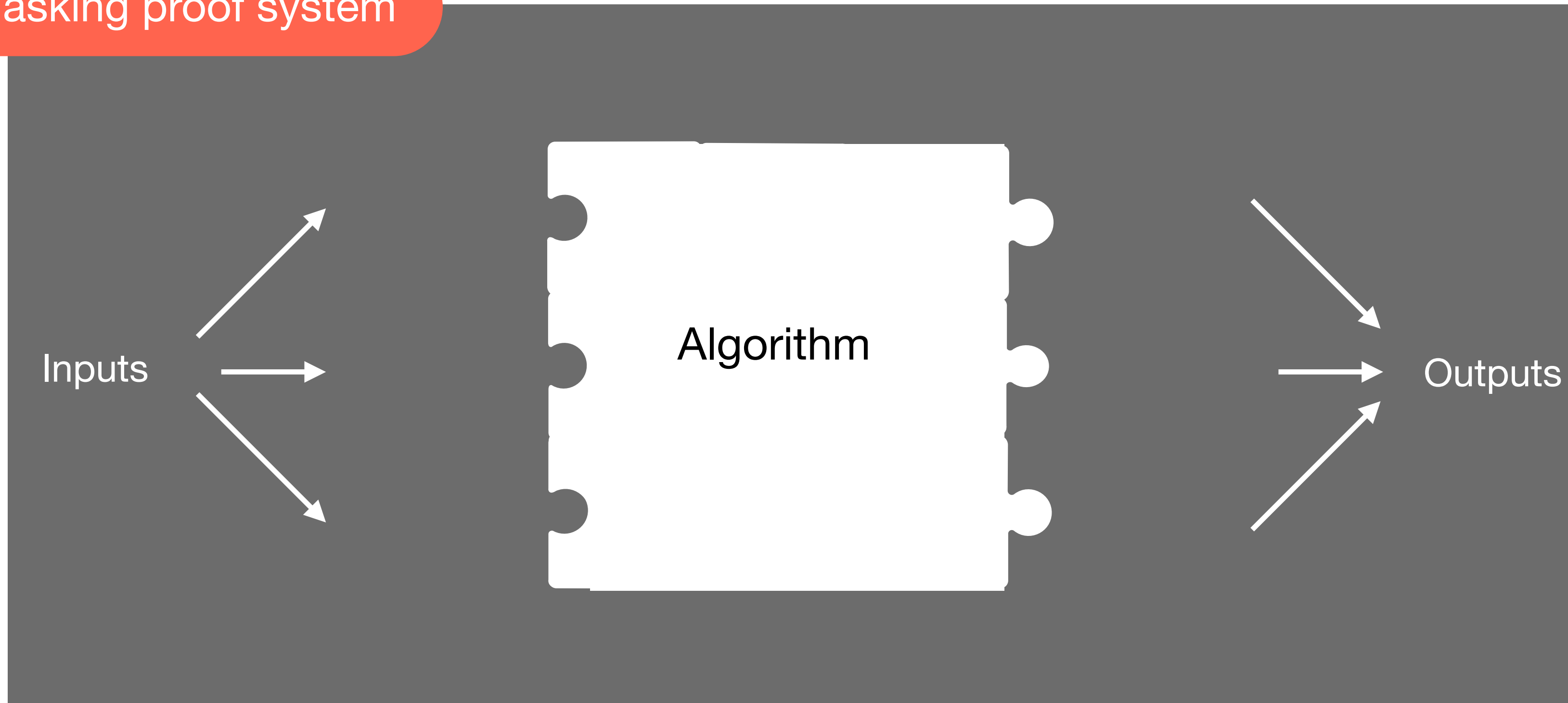
Masking proof system

Algorithm

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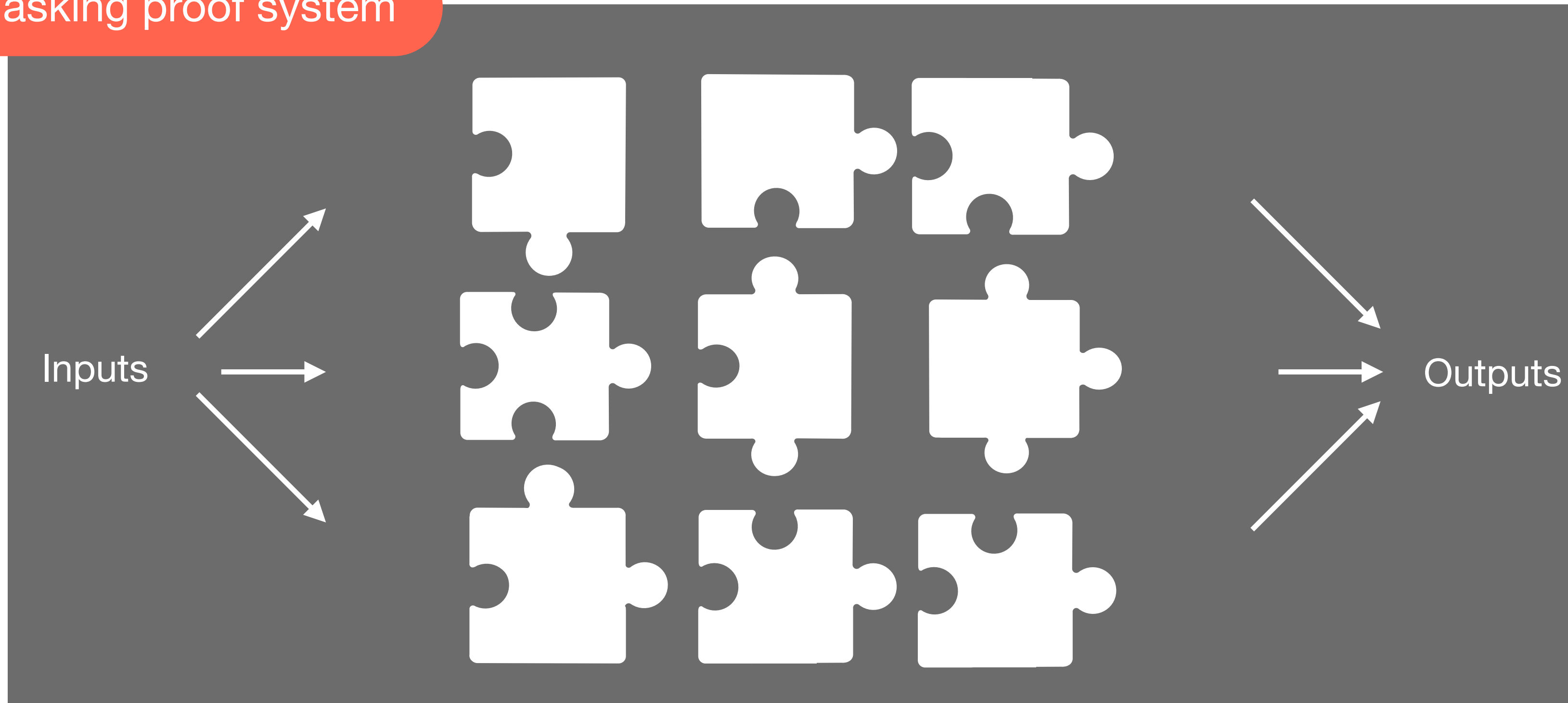


Proofs of masking for each gadget
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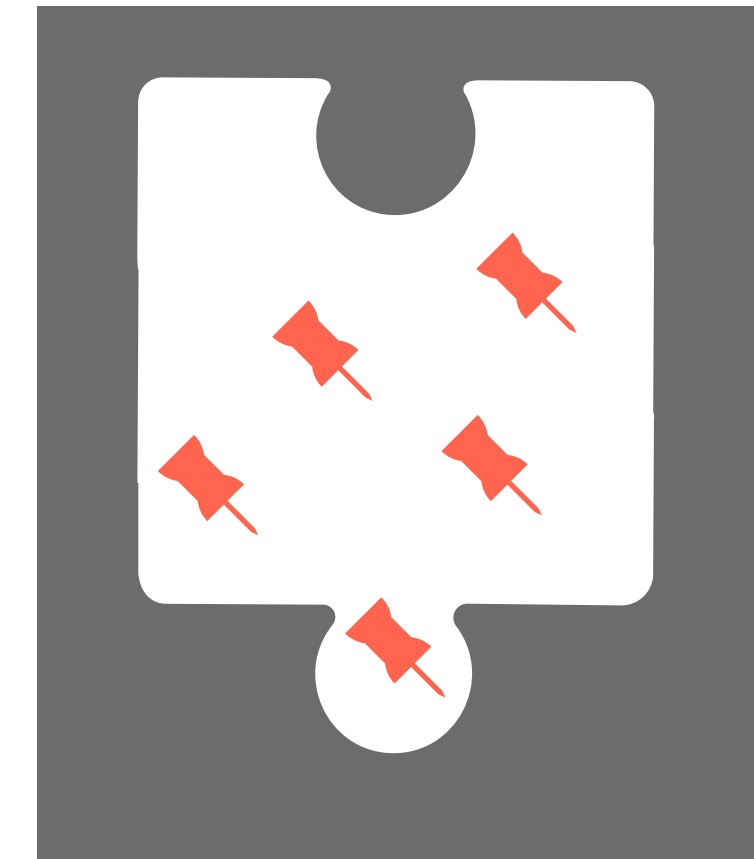
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Non interference

▸ G. Barthe, S. Belaid, F. Dupressoir, P.-A. Fouque, B. Grégoire, P.-Y. Strub, and R. Zucchini. [ACM-CCS'2016](#)

Non Interference

A gadget is d -non-interfering (NI) iff any set of at most d observations can be perfectly simulated from at most d shares of each input.

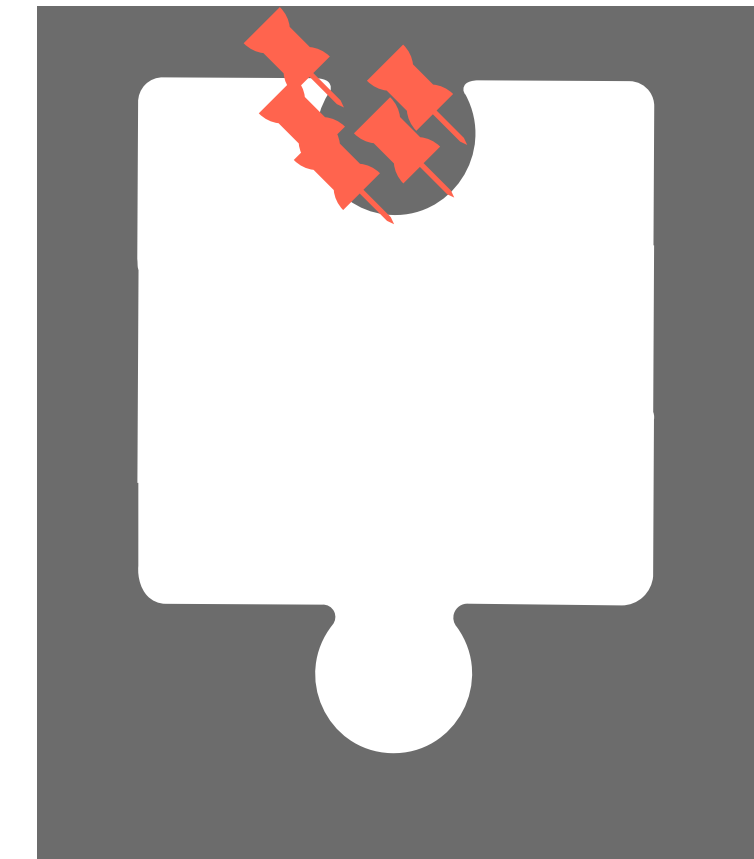


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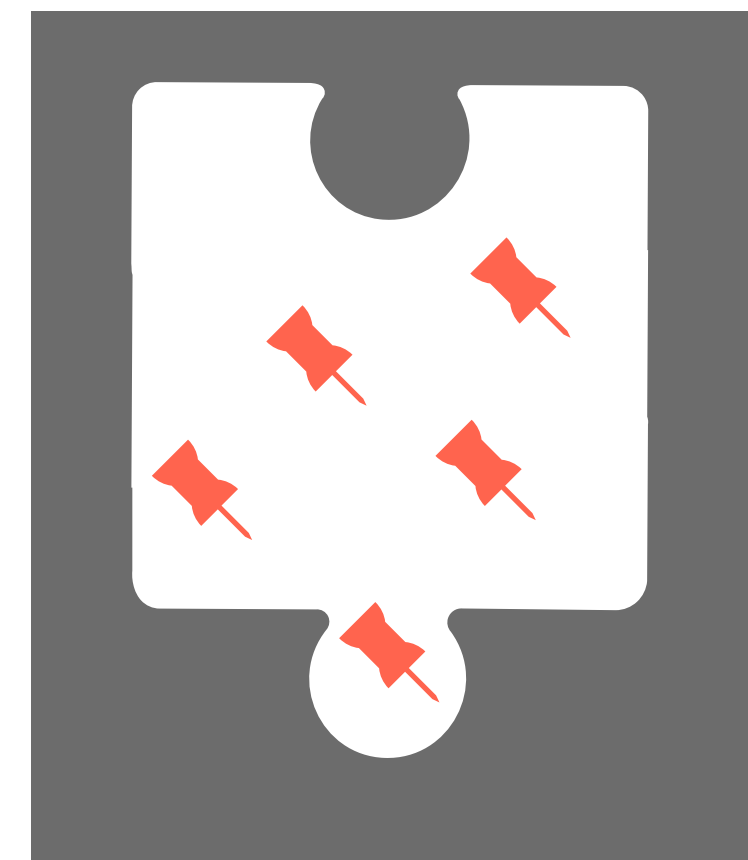
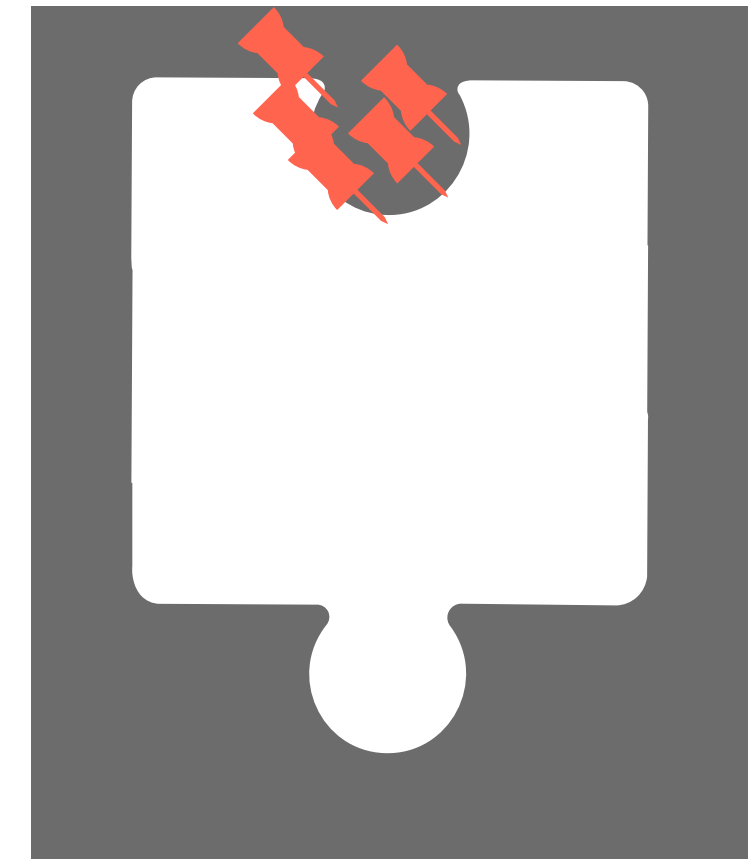
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Non interference

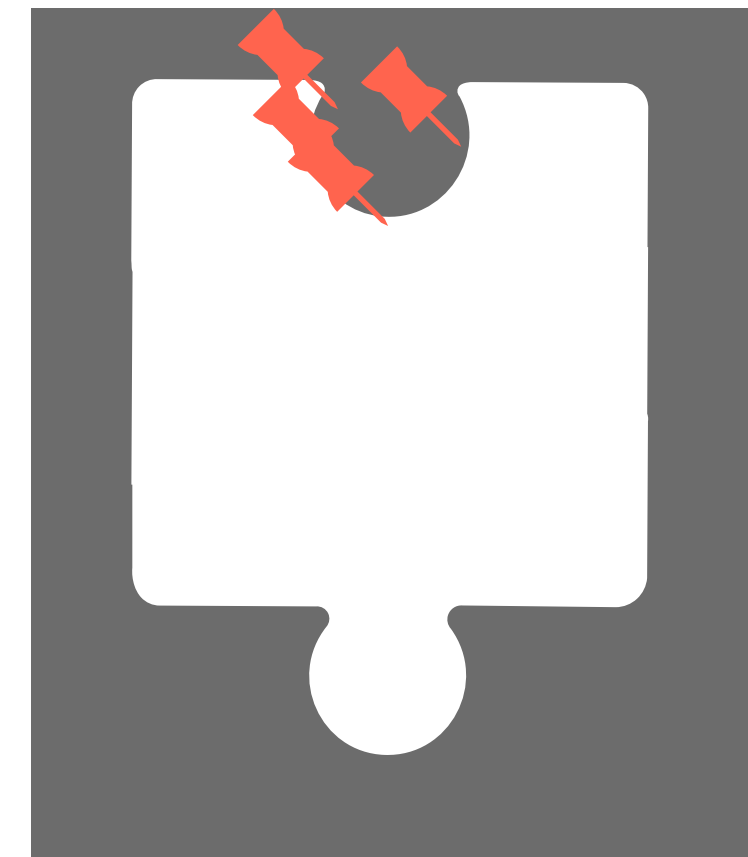
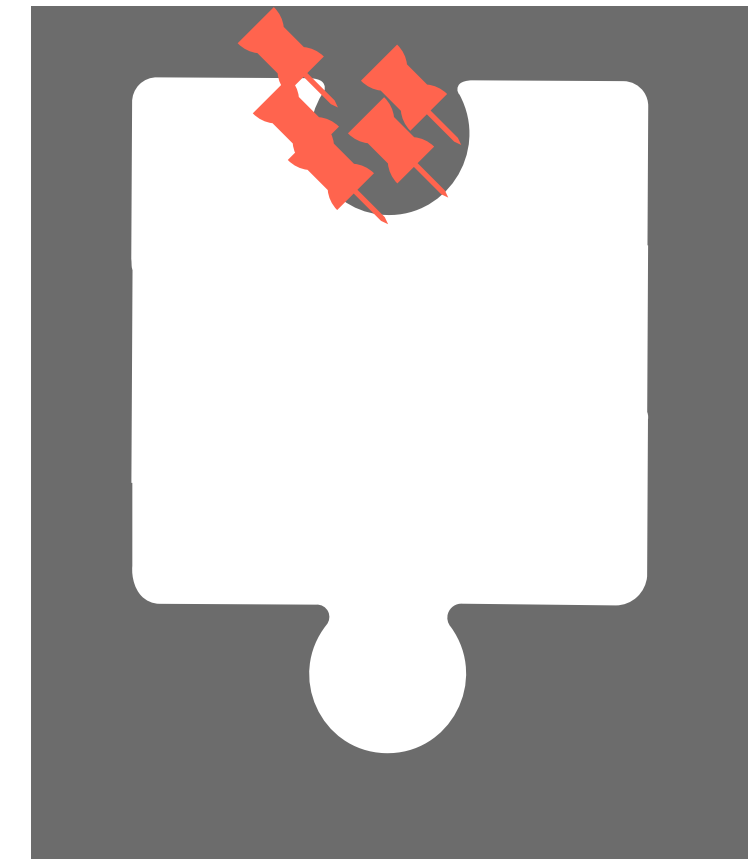
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Non interference with public outputs

Signature algorithm:

1: do

2: $y \xleftarrow{\$} Y$

3: $c \leftarrow H(\mathbf{A}y, m)$

4: $\mathbf{z} \leftarrow c \cdot \mathbf{S} + y$

5: while Rejected($\mathbf{z}, c, \mathbf{S}$)

6: return (\mathbf{z}, c)

Non interference with public outputs

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\approx

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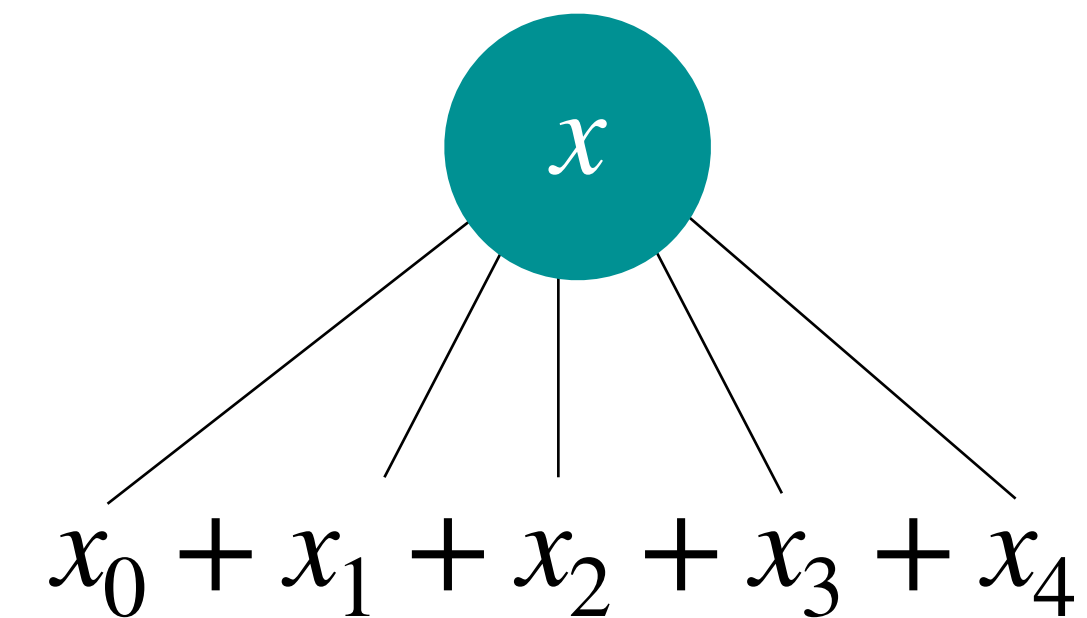
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Masking for lattice-based cryptography

Need for lattice adapted gadgets

-  Small uniform random generation in $\mathbb{Z}/q\mathbb{Z}$
-  Gaussian generation
-  Rejection sampling

The constructions must use mask conversions



- ▶ J.-S. Coron, J. Großschädl and P. K. Vadnala [CHES'2014](#)
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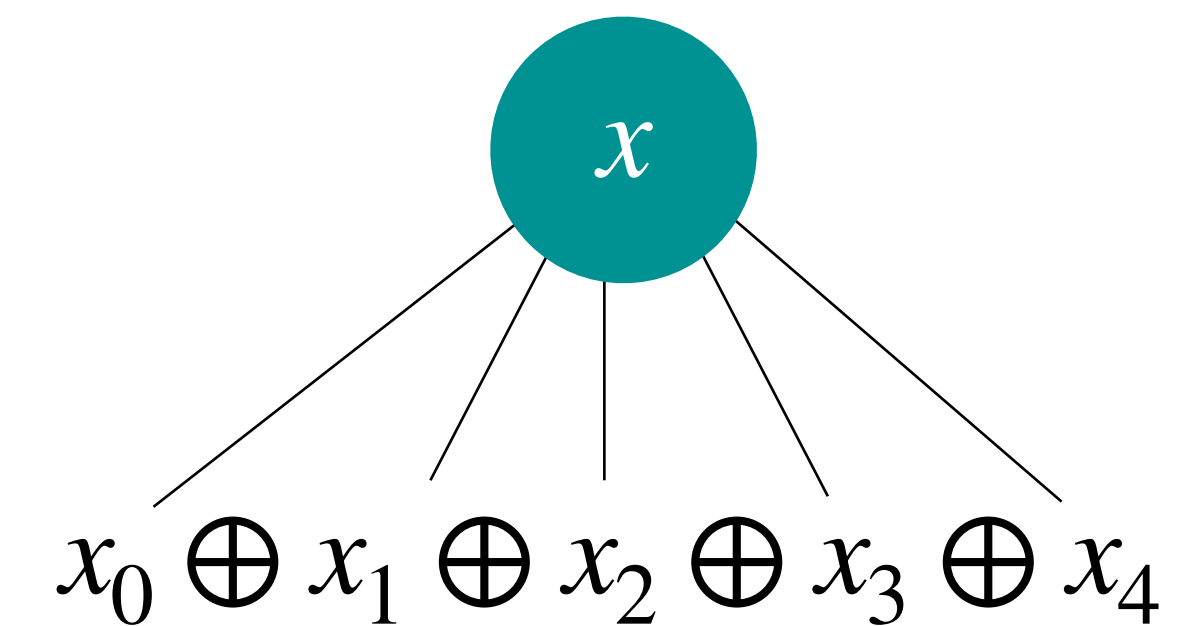
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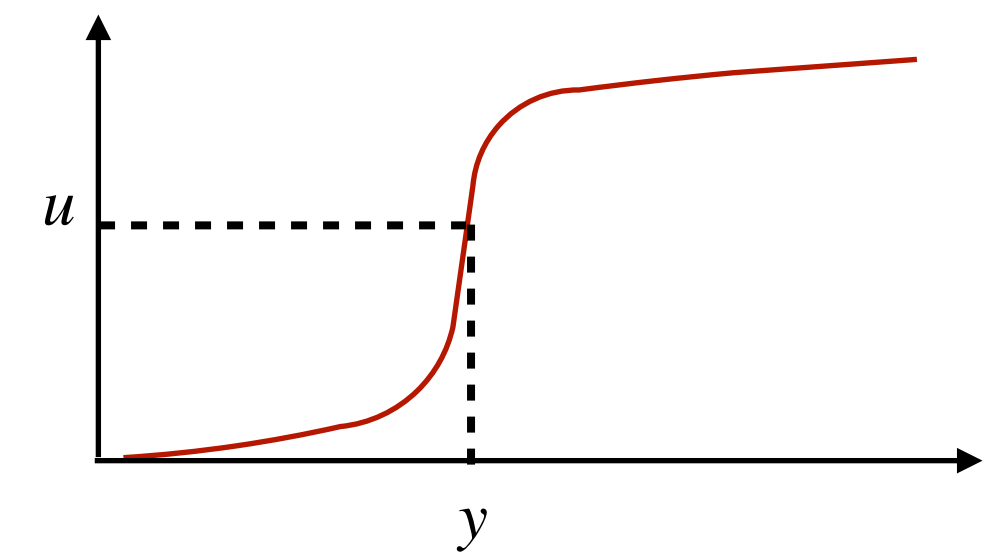
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Masking Gaussian sampling

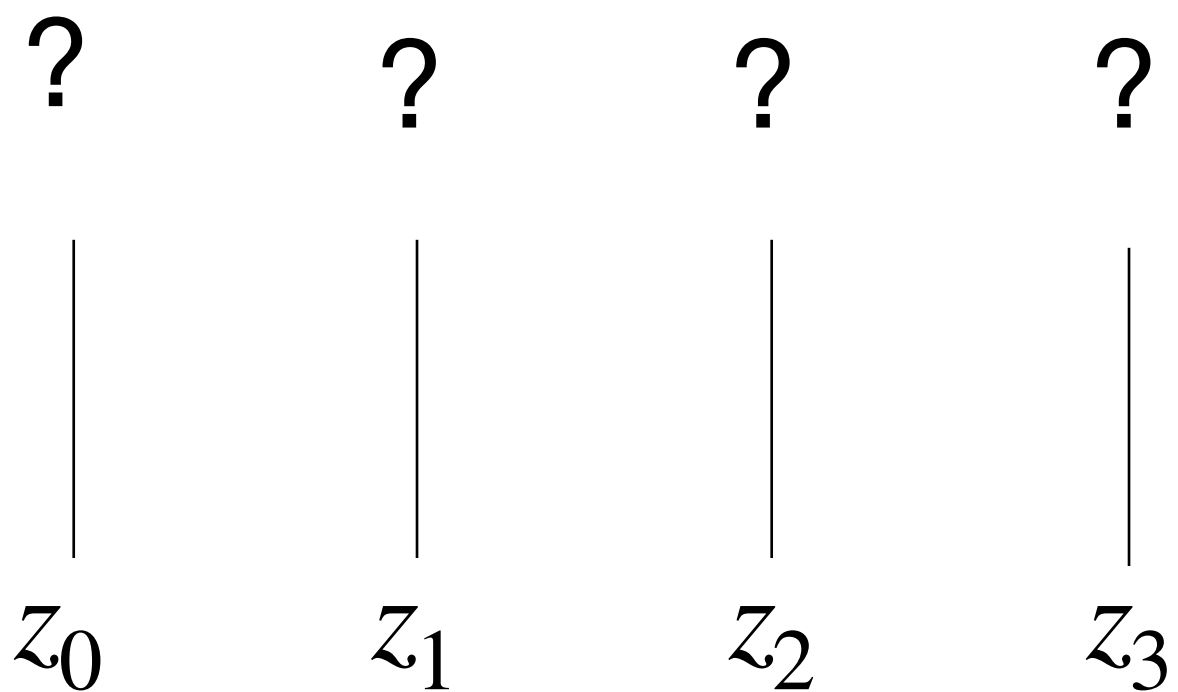
How to mask Gaussian generation?

| | Fixed center | Masked variable center |
|------------------------------------|--|----------------------------------|
| Fixed standard deviation | Masking the CDT sampling | Mitaka's share by share sampling |
| Masked variable standard deviation | Mask the existing convolution and rejection sampling techniques. | |





Gaussian share by share sampling

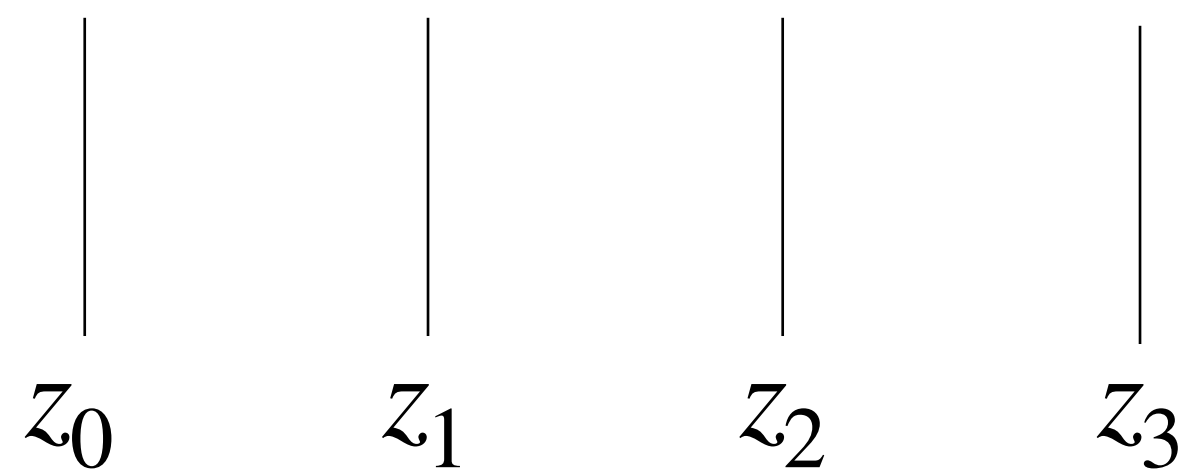


such that $\sum z_i \bmod q \sim D_{\mathbb{Z}, \sum c_i \bmod q, \sigma}$



Gaussian share by share sampling

$$D_{\mathbb{Z},c_0,\sigma} \quad D_{\mathbb{Z},c_1,\sigma} \quad D_{\mathbb{Z},c_2,\sigma} \quad D_{\mathbb{Z},c_3,\sigma}$$

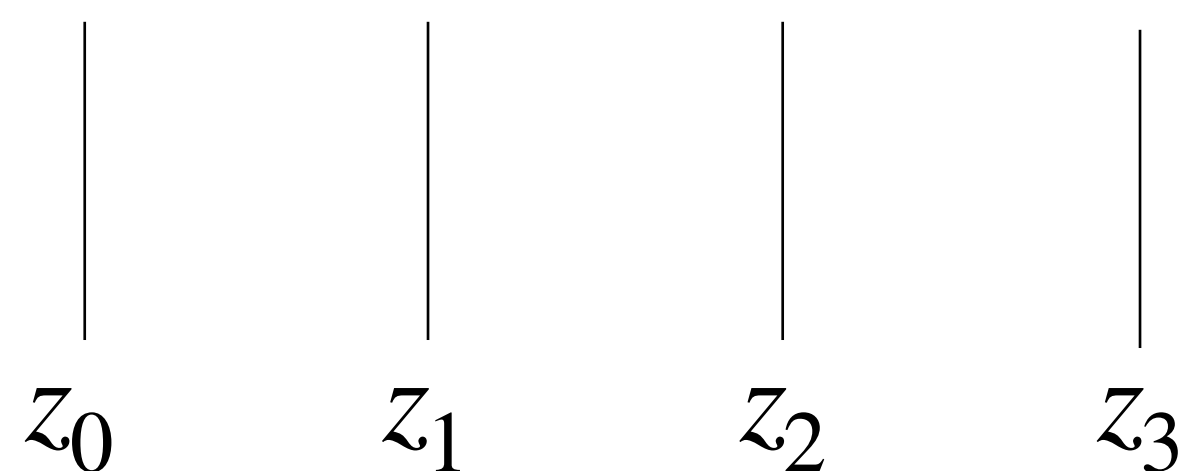


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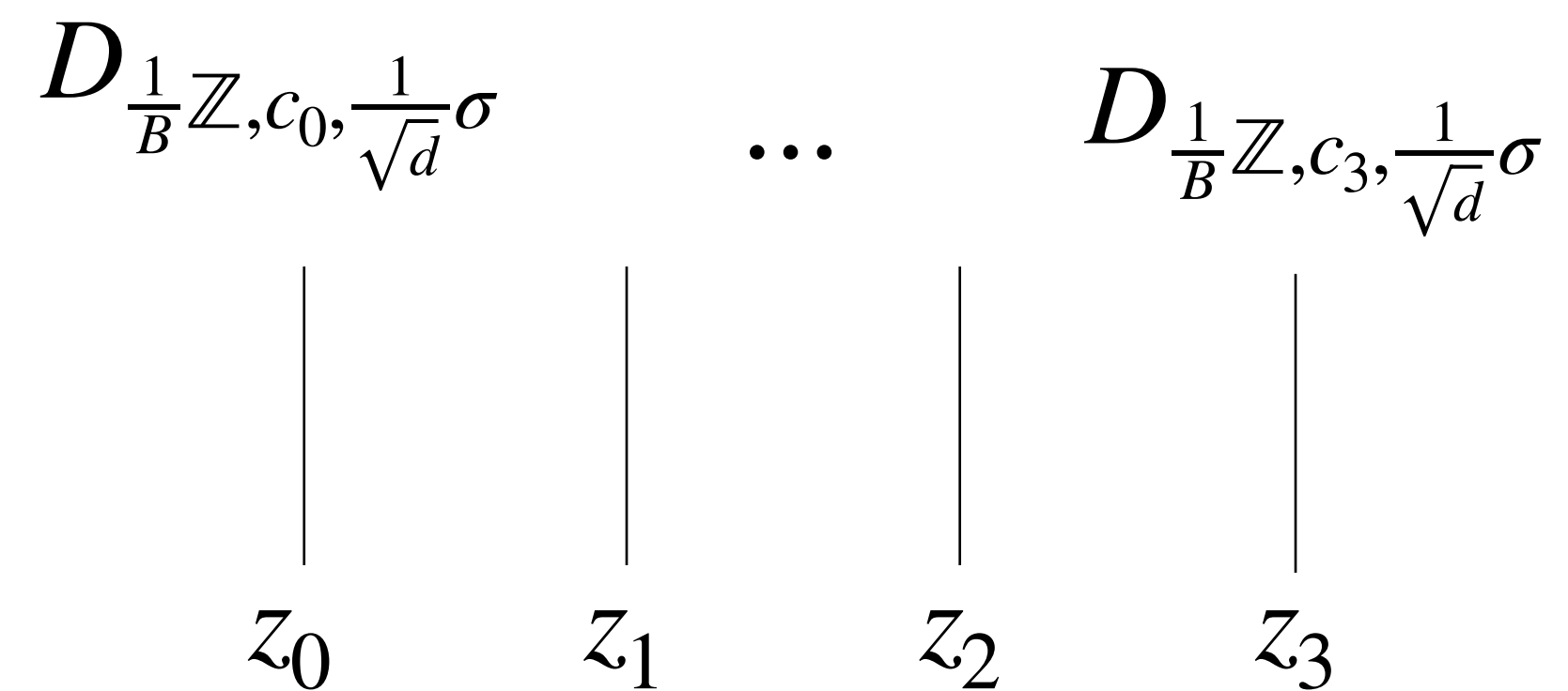
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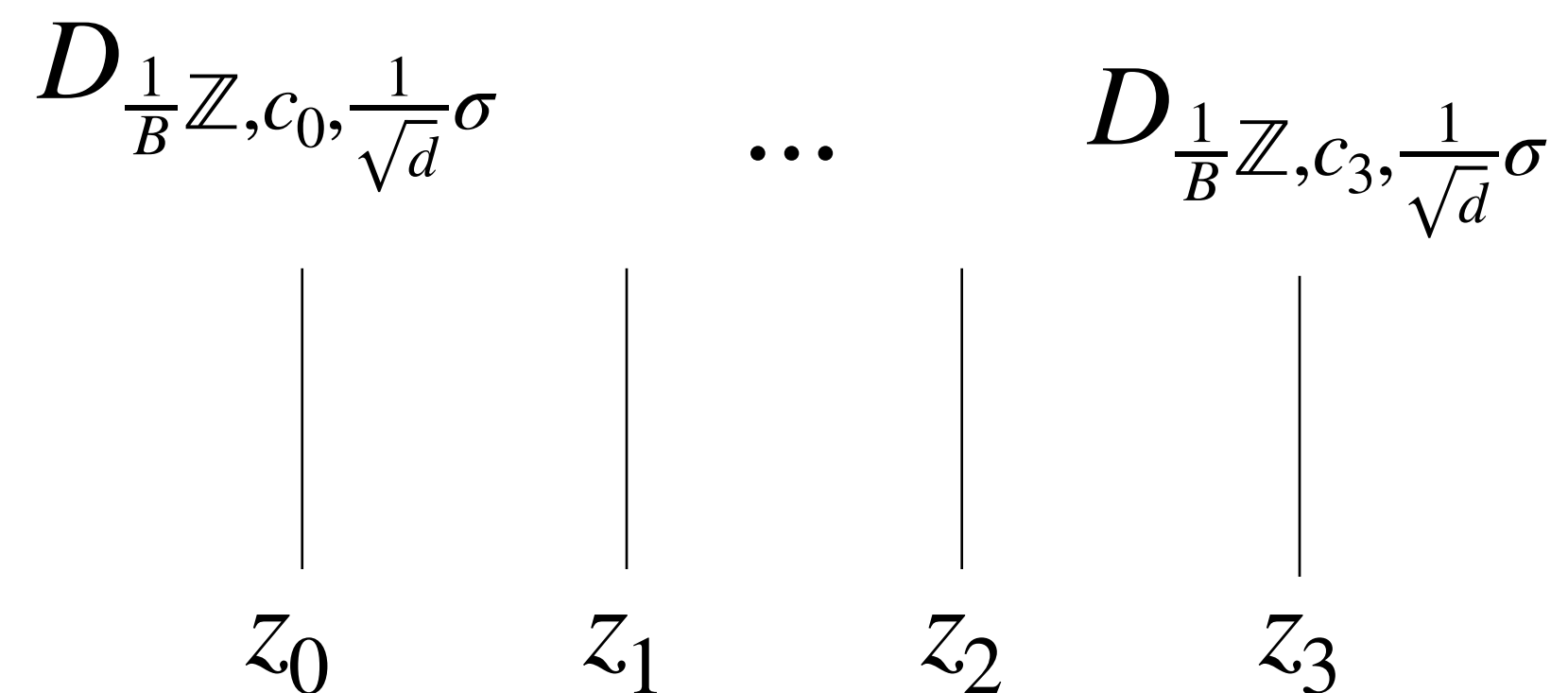
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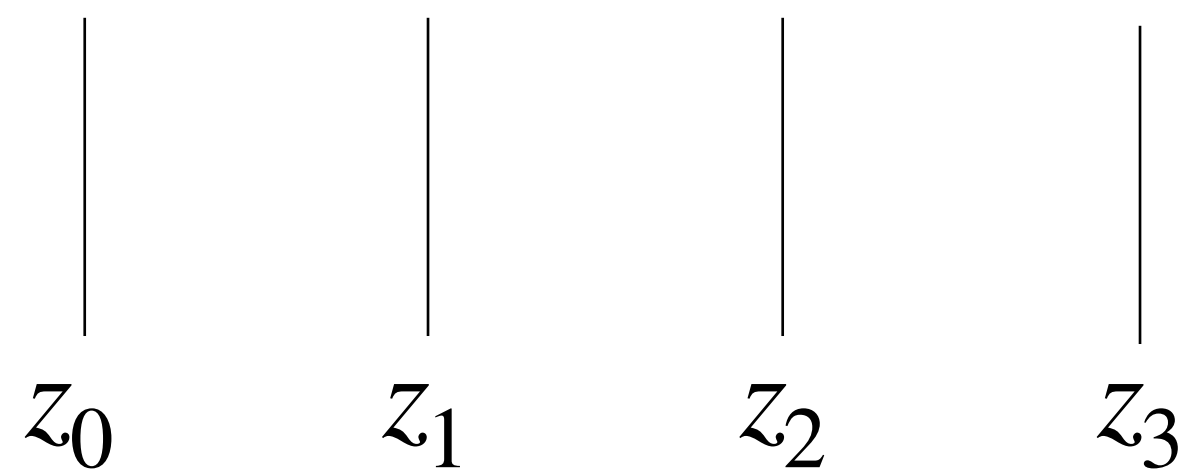


such that $\sum z_i \bmod q \sim D_{\frac{1}{B}\mathbb{Z}, \sum c_i \bmod q, \sigma}$



Gaussian share by share sampling

$$D_{\frac{1}{B}\mathbb{Z},c_0,\frac{1}{\sqrt{d}}\sigma} \quad \dots \quad D_{\frac{1}{B}\mathbb{Z},c_3,\frac{1}{\sqrt{d}}\sigma}$$



Reject if $\sum (z_i \bmod 1) \neq 0$

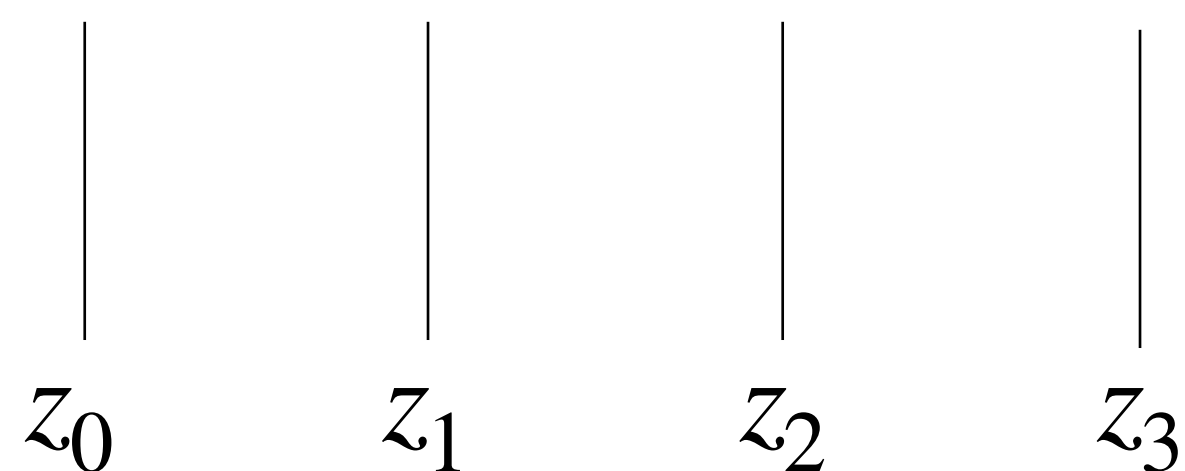
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Gaussian share by share sampling

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Gauss share by share

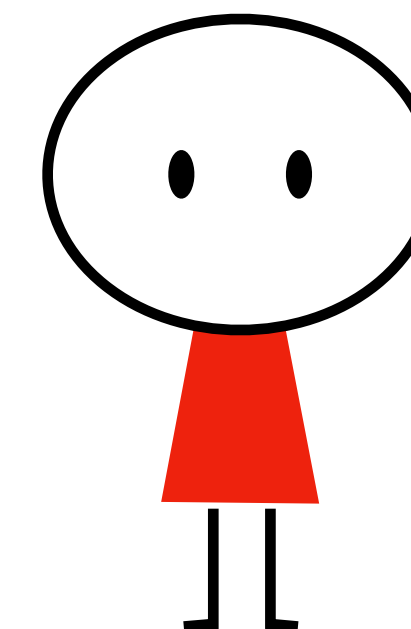
This Gaussian share by share sampling is correct and secure.

- ▶ « Mitaka: A Simpler, Parallelizable, Maskable Variant of Falcon »
T. Espitau, P.-A. Fouque, F. Gérard, M. Rossi, A. Takahashi, M. Tibouchi, A. Wallet, Y. Yu [EUROCRYPT'2022](#)

Example of performance

Examples of overhead on the number of cycles for qTesla signature scheme

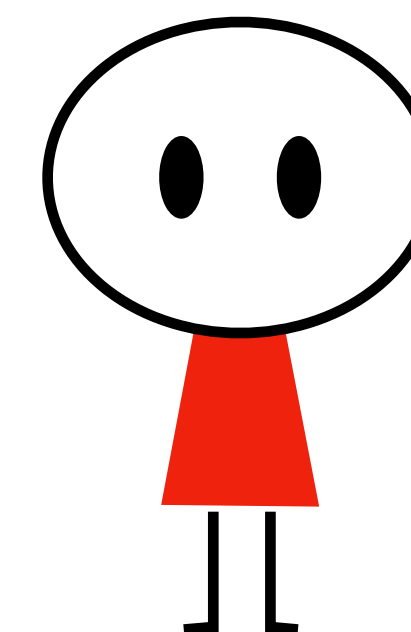
| Unmasked | Order 1 | Order 2 | Order 3 | Order 4 |
|----------|---------|---------|---------|---------|
| 1 | × 4 | × 21 | × 37 | × 60 |

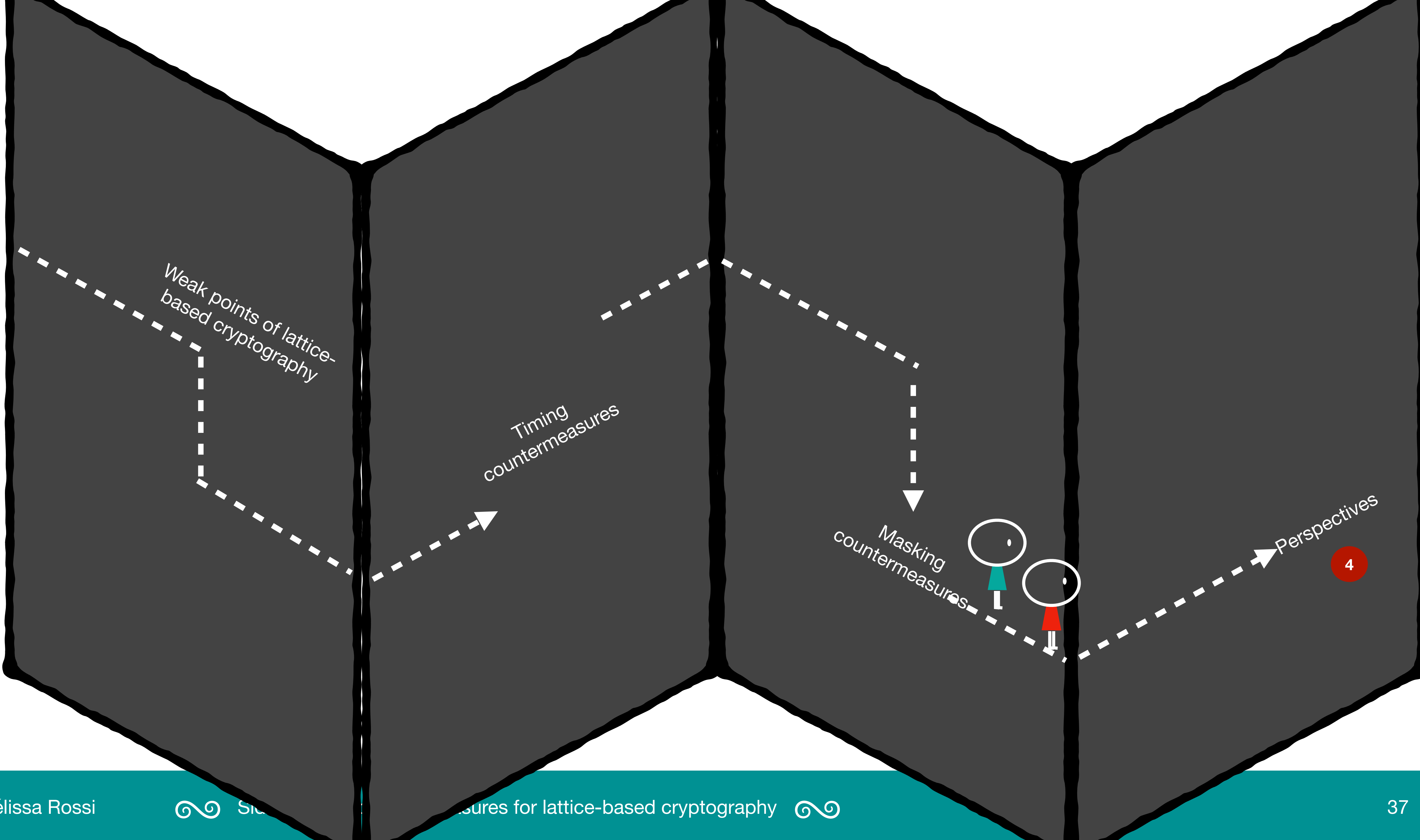


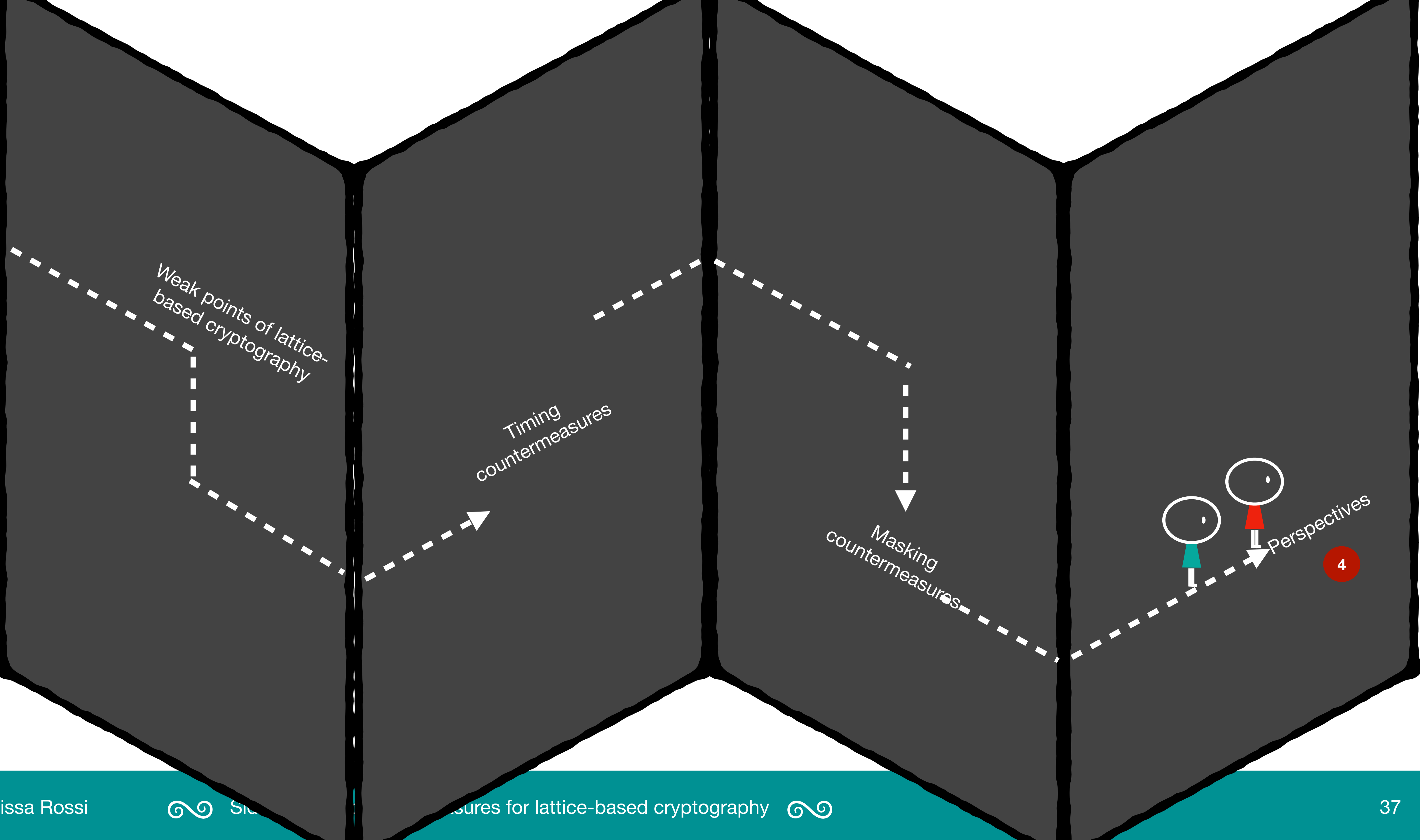
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Provable countermeasures are not infallible



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Besides proofs, how to verify automatically the **isochrony** of lattice-based crypto?

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Intuition:

- generate two random keys
- sign many messages or decrypt many ciphertexts with either of the two keys
- look for statistical differences in the timing among the two keys

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Challenges for lattice-based crypto :

- How to handle inherent variable execution time ?
- The sensitive values are not only the keys but many intermediate randomness are sensitive

Automated verification of side-channel protection in the lattice domain

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Challenges:

Arithmetic and Boolean masking

Conversions

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Partial resolution:

- ▶ « Formal verification of Arithmetic Masking in Hardware and Software »
B. Gigerl, R. Primas, S. Magnard eprint.iacr.org/2022/849

Modeling arithmetic expression with Boolean logic

Applied to A2B and B2A

Other perspectives

Masking friendly design

The designs contain many « masking unfriendly » features: Gaussian distributions, uniform small distributions, comparison of sensitive values, rejection, prime modulus...

➔ Schemes designs that minimize the masking overhead at a cost of less efficient unmasked version.

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Fujisaki-Okamoto transform

This transform is needed because it protects against active attacks (IND-CCA2 security) but it highly increases the attack surface and introduces new attack entry points.

- ➔ Is re-encryption (or similar tests) inevitable?
- ➔ Is it possible to design a fully protected generic Fujisaki-Okamoto transform?