Masking the GLP Lattice-Based Signature Scheme at Any Order

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2 Power Analysis Attacks and Masking

3 Contribution: Higher-Order Masking of GLP

4 Implementation of the Countermeasure

5 Conclusion

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Context

- NIST postquantum competition
- demands for practical implementation

Lattice-Based Signatures

So far

- Numerous physical attacks against lattice-based schemes (Gaussian distributions, rejection sampling)
- Few countermeasures exist

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Power Analysis Attacks



Masking

- sound countermeasure which splits every sensitive variable x into d + 1 shares $(x_i)_{0 \le i \le d}$ such that
 - ▶ for every $1 \le i \le d$, x_i is picking uniformly at random
 - $x_0 \leftarrow x \oplus x_1 \oplus \cdots \oplus x_d$
- \blacksquare any strict subvector of at most d shares is independent from \pmb{x}
- d is called masking order or security order

Leakage Models

Probing model by Ishai, Sahai, and Wagner (Crypto 2003)

a circuit is *d*-probing secure iff any set composed of the exact values of at most *d* intermediate variables is independent from the secret



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- Reduction by Duc, Dziembowski, and Faust (EC 2014)
 - \blacktriangleright $d\text{-probing security} \Rightarrow$ security in the noisy leakage model for some level of noise

Probing Model

variables: secret, shares, constant

• masking order d = 3

 $\frac{\text{function Ex-t3}(x_0, x_1, x_2, x_3, c):}{(* x_0, x_1, x_2 = \$)} \\
\xrightarrow{(* x_3 = x + x_0 + x_1 + x_2 *)} \\
r_0 \leftarrow \$ \\
r_1 \leftarrow \$ \\
y_0 \leftarrow x_0 + r_0 \\
y_1 \leftarrow x_3 + r_1 \\
t_1 \leftarrow x_1 + r_0 \\
t_2 \leftarrow (x_1 + r_0) + x_2 \\
y_2 \leftarrow (x_1 + r_0 + x_2) + r_1 \\
y_3 \leftarrow c + r_1 \\
\text{return}(y_0, y_1, y_2, y_3)$

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a circuit is d-SNI iff any set of d intermediate variables, whose d₁ on the internal variables and d₂ and the outputs, can be perfectly simulated with at most d₁ shares of each input



Strong Non-Interference (SNI)

- $d\text{-SNI} \Rightarrow d\text{-NI} \Rightarrow d\text{-probing secure}$
- a circuit is d-SNI iff any set of d intermediate variables, whose d₁ on the internal variables and d₂ and the outputs, can be perfectly simulated with at most d₁ shares of each input



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• Example: AES S-box on $GF(2^8)$

 $\begin{array}{c} \text{Constraint:}\\ d_0+d_1+d_2+d_3\leqslant d\\ \text{observations} \end{array} \left\{ \begin{array}{c} [x]\\ \hline [.2]\\ R \end{array} \right\} d_1+d_2\\ \text{observations}\\ \hline \\ d_3 \text{ output observations}\\ \hline \\ \hline \\ \hline \end{array} \right\} d_3 \text{ output observations}$

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GLP Features

- Introduced at CHES 2012 by Tim Güneysu, Vadim Lyubashevsky, and Thomas Pöppelmann
- No Gaussians, only uniform distributions
- Probabilistic algorithm
- Rejection sampling

GLP Key Derivation

$$\mathcal{R} = \frac{\mathbb{Z}_p[x]}{x^n + 1}$$

 \mathcal{R}_k : coefficients in the range [-k,k]

Algorithm 1 GLP key derivation

Ensure: Signing key sk, verification key pk1: $\mathbf{s_1}, \mathbf{s_2} \stackrel{\$}{\leftarrow} \mathcal{R}_1 / / \mathbf{s_1}$ and $\mathbf{s_2}$ have coefficients in $\{-1, 0, 1\}$ 2: $\mathbf{a} \stackrel{\$}{\leftarrow} \mathcal{R}$ 3: $\mathbf{t} \leftarrow \mathbf{as_1} + \mathbf{s_2}$ 4: $sk \leftarrow (\mathbf{s_1}, \mathbf{s_2})$ 5: $pk \leftarrow (\mathbf{a}, \mathbf{t})$

Based on the Decisional Compact Knapsack problem

GLP Signature

Fiat-Shamir with abort signature

Algorithm 2 GLP signature

Require: m, pk, skEnsure: Signature σ 1: $\mathbf{y}_1, \mathbf{y}_2 \stackrel{\$}{\leftarrow} \mathcal{R}_k$ 2: $\mathbf{c} \leftarrow H(\mathbf{r} = \mathbf{ay_1} + \mathbf{y_2}, \mathbf{m})$ 3: $\mathbf{z}_1 \leftarrow \mathbf{s}_1 \mathbf{c} + \mathbf{y}_1$ 4: $\mathbf{z}_2 \leftarrow \mathbf{s}_2 \mathbf{c} + \mathbf{y}_2$ 5: if \mathbf{z}_1 or $\mathbf{z}_2 \notin \mathcal{R}_{k-\alpha}$ then restart return $\sigma = (\mathbf{z}_1, \mathbf{z}_2, \mathbf{c})$

Random generation Commitment

Rejection Sampling

Verification \mathbf{z}_1 , $\mathbf{z}_2 \in \mathcal{R}_{k-\alpha}$ and $\mathbf{c} = H(\mathbf{a}\mathbf{z}_1 + \mathbf{z}_2 - \mathbf{t}\mathbf{c}, \mathbf{m})$

Algorithm 3 GLP key derivation



Algorithm 4 GLP key derivation



Algorithm 5 GLP key derivation



Algorithm 6 GLP key derivation



Algorithm 7 GLP key derivation



Security of Individual Gadgets

Each sensitive variable must be masked

- mod-p arithmetic masking
- w-bit Boolean masking
- Each block (or sub-block) which manipulates secret data is individually proven according to one of the three properties
 - ► Non-Interference (NI)
 - Strong Non-Interference (SNI)
 - Non-Interference with public Outputs (NIo)

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where $2^{w_0} > 2k + 1 \ge 2^{w_0 - 1}$

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2. $(\delta_i)_{0 \le i \le d} \leftarrow (\mathbf{x}_i)_{0 \le i \le d} - (\mathbf{k}_i)_{0 \le i \le d}$

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- $\bullet H^1: t \leftarrow as_1 + s_2$
- FullAdd: refresh then add

Algorithm 8 GLP key derivation



Algorithm 9 GLP signature

 $\begin{array}{lll} \mbox{Require: } m, pk, sk \\ \mbox{Ensure: Signature } \sigma \\ \mbox{1: } \mathbf{y_1, y_2} & \stackrel{\$}{\leftarrow} \mathcal{R}_k \\ \mbox{2: } \mathbf{c} \leftarrow H(\mathbf{r} = \mathbf{ay_1} + \mathbf{y_2}, \mathbf{m}) \\ \mbox{3: } \mathbf{z_1} \leftarrow \mathbf{s_1}\mathbf{c} + \mathbf{y_1} \\ \mbox{4: } \mathbf{z_2} \leftarrow \mathbf{s_2}\mathbf{c} + \mathbf{y_2} \\ \mbox{5: } \mbox{if } \mathbf{z_1 or z_2} \notin \mathcal{R}_{k-\alpha} \mbox{ then restart} \\ \mbox{return } \sigma = (\mathbf{z_1, z_2, c}) \end{array}$



Algorithm 10 GLP signature



Algorithm 11 GLP signature



Algorithm 12 GLP signature



Algorithm 13 GLP signature



Algorithm 14 GLP signature



- **DG:** generation of sharings for coefficients $x \in [-k, k]$
- H^1 : $\mathrm{ay_1} + \mathrm{y_2}$
- FullAdd: refresh then add
- Hash: unmasked
- RS: some details follow
- H²: linear function
- FullAdd: refresh then add

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- Rejection Sampling: are coefficients of z_1 and z_2 in $[-k + \alpha, k \alpha]$?
 - 1. convert mod-p arithmetic sharing into Boolean masking
 - 2. as in Data Generation, compute the masked difference with $k \alpha$ difference
 - 3. securely check the most significant bit

Algorithm 15 GLP signature





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Implementation

- unoptimized implementation
- based on a public domain implementation called GLYPH (n = 1024, p = 59393, k = 16383, and $\alpha = 16$)

Table: Implementation results. Timings are provided for 100 executions of the signing and verification algorithms, on one core of an Intel Core i7-3770 CPU-based desktop machine.

Number of shares $(d+1)$	Unprotected	2	3	4	5	6
Total CPU time (s)	0.540	8.15	16.4	39.5	62.1	111
Masking overhead	—	$\times 15$	$\times 30$	$\times 73$	$\times 115$	$\times 206$

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Conclusion

In a nutshell...

- Higher-order masking of GLP with proof in the probing model
- New security notions to mask lattice-based signatures

To continue...

- Extend these results to other lattice-based signatures
- Extend these results to other post-quantum schemes