Higher-Order Masking of Lattice-Based Signatures

Sonia BELAÏD

CryptoExperts

Outline

Introduction

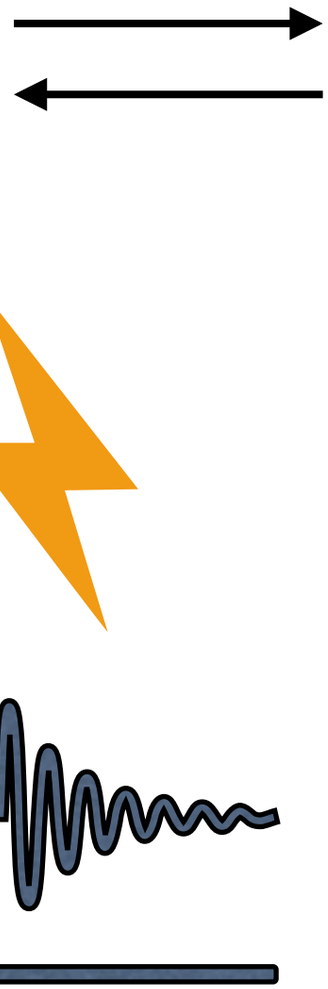
- Side-Channel Attacks
- Side-Channel Attacks Against Lattice-Based Signatures
- Masking

Masking Lattice-Based Signatures

- An Example: Masking GLP
- Other Signature Schemes

Introduction

Side-Channel Attacks

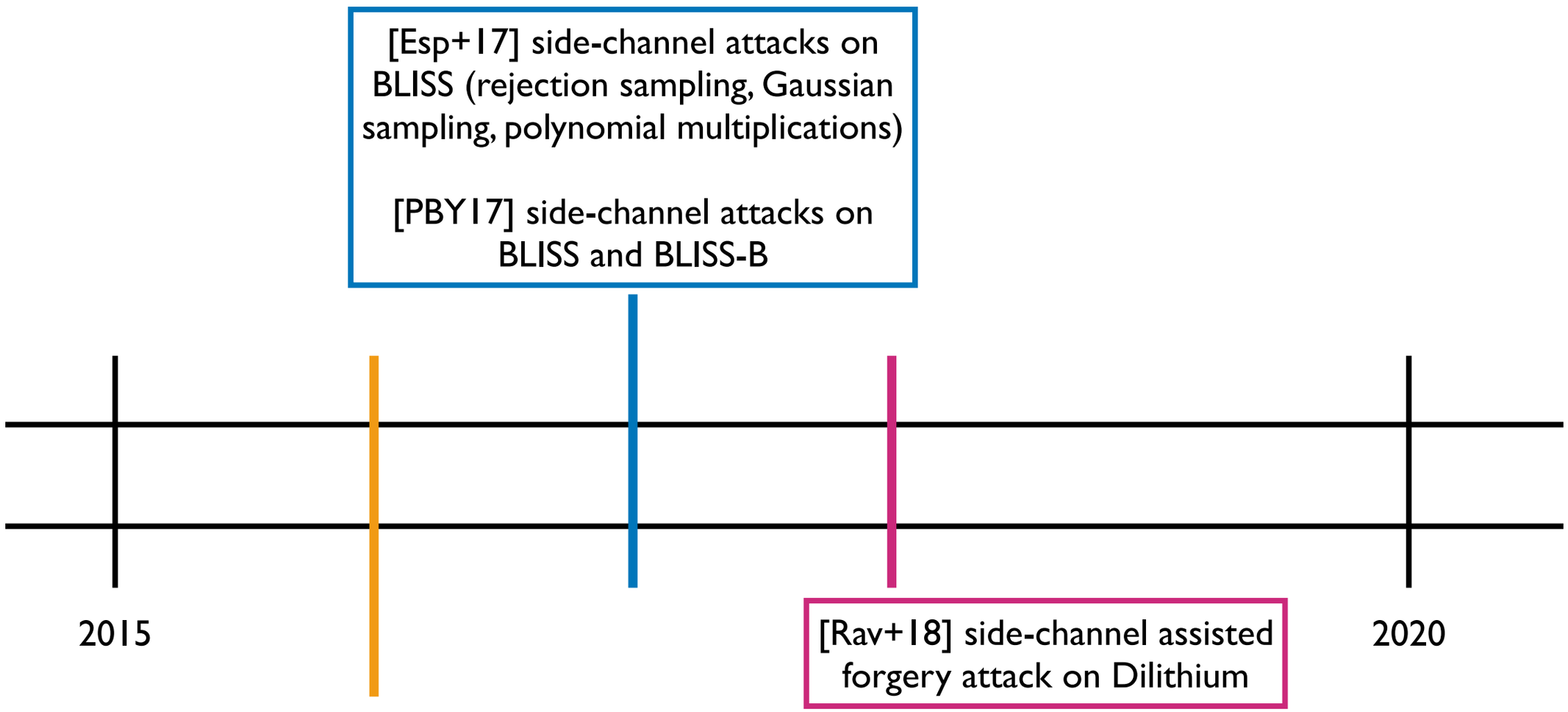


inputs / outputs

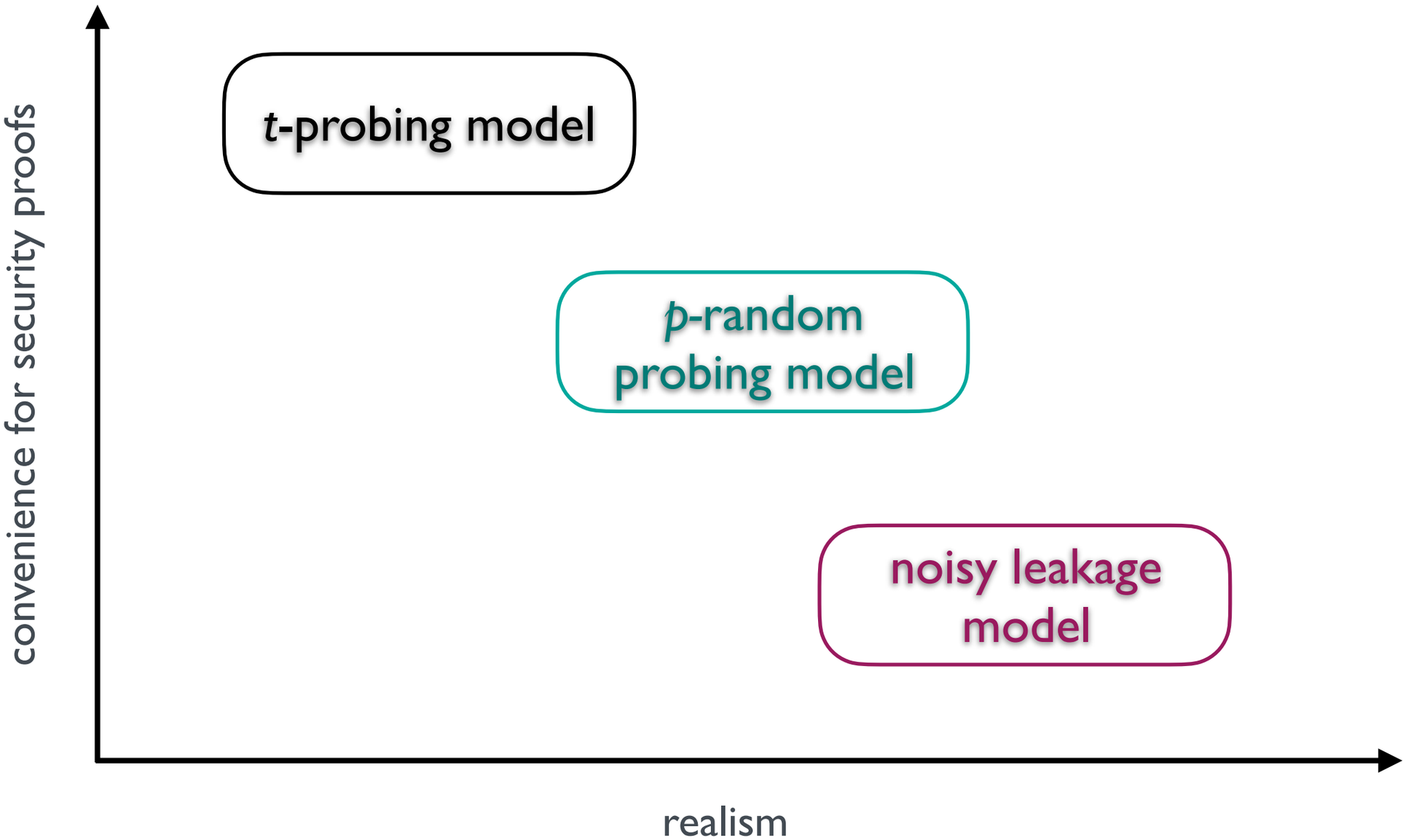
Power consumption

$$\approx \left|\left|_i \left(f(v_i) + \eta\right)\right.\right.$$

Side-Channel Attacks against Lattice-Based Signatures



Leakage Models



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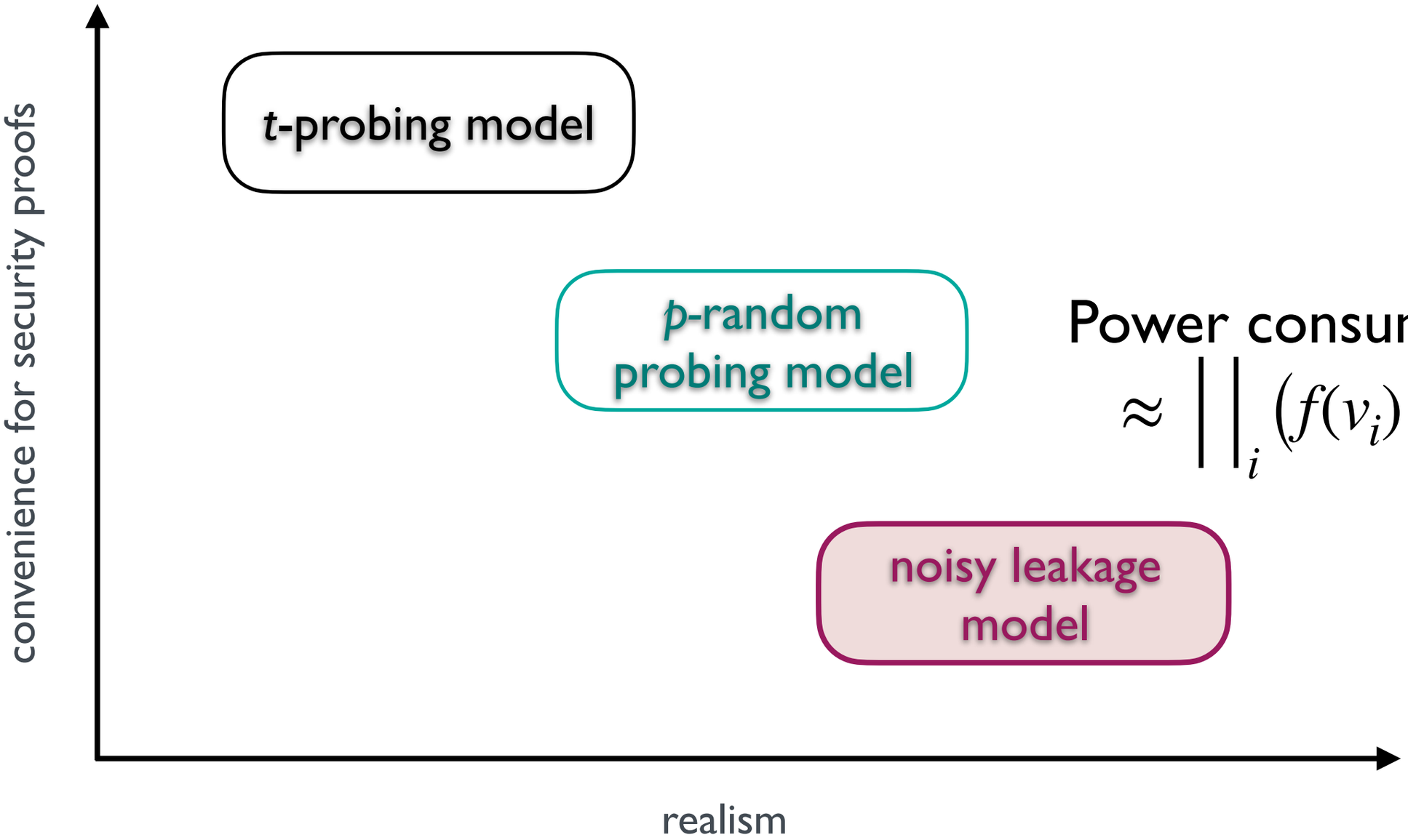

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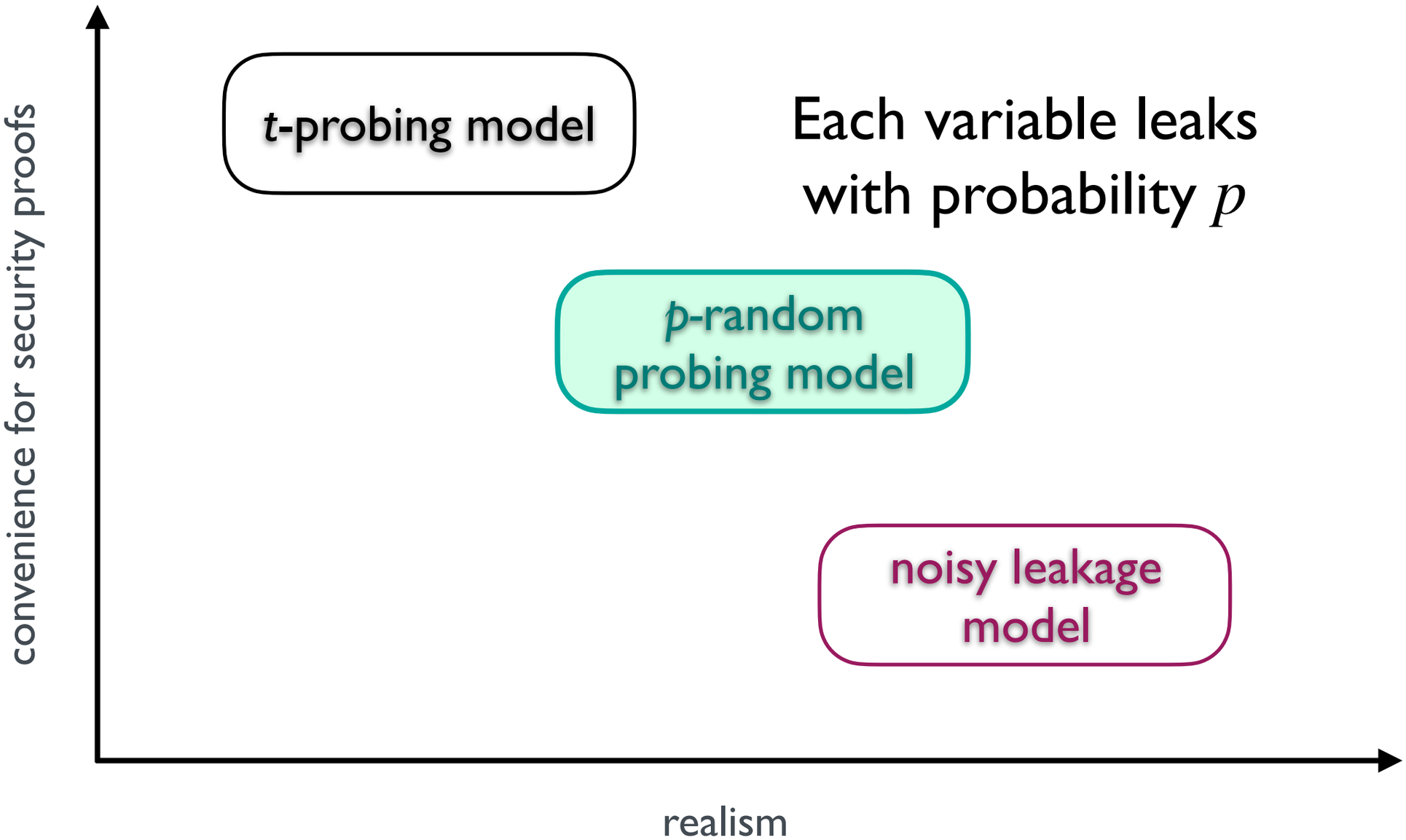

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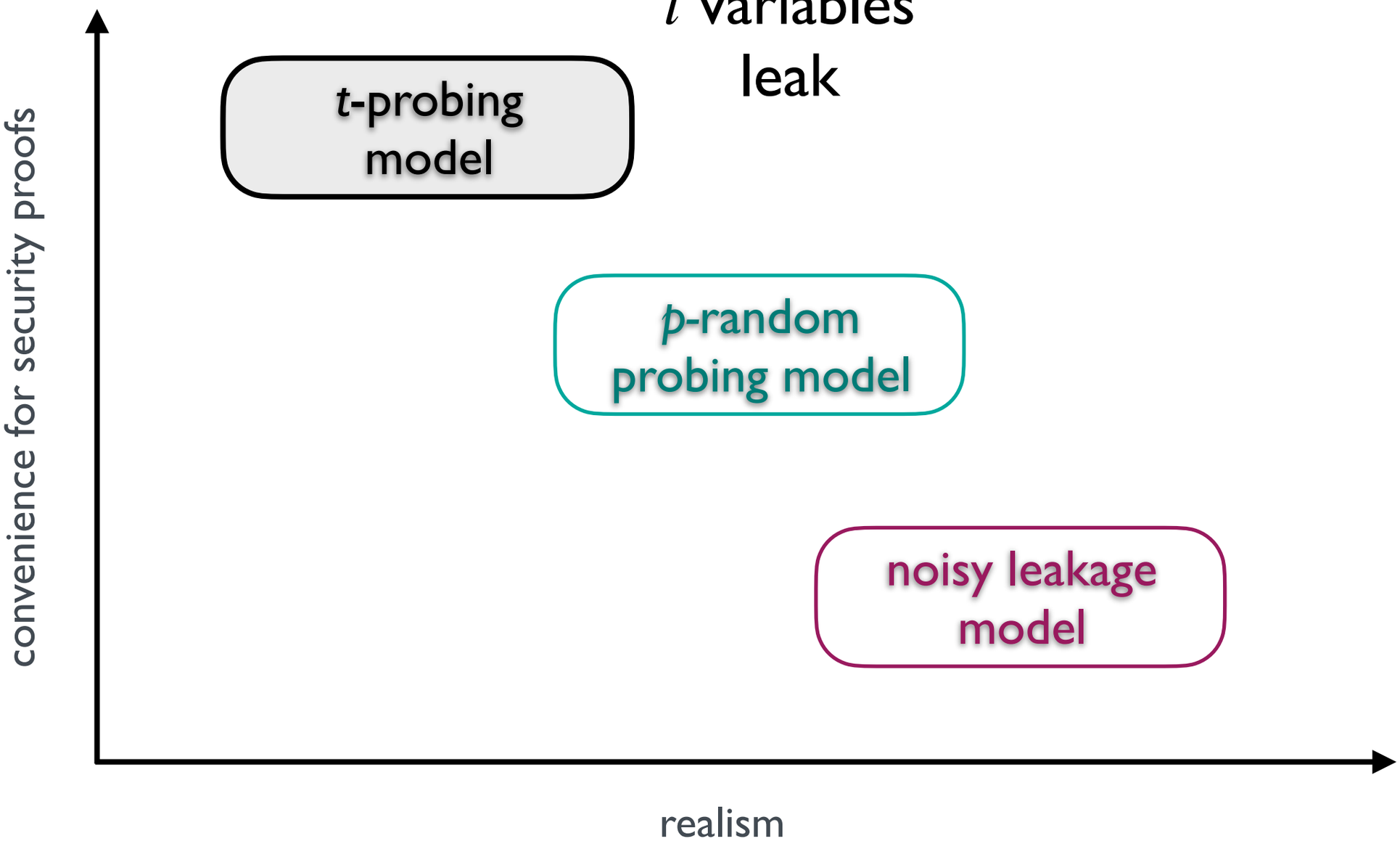

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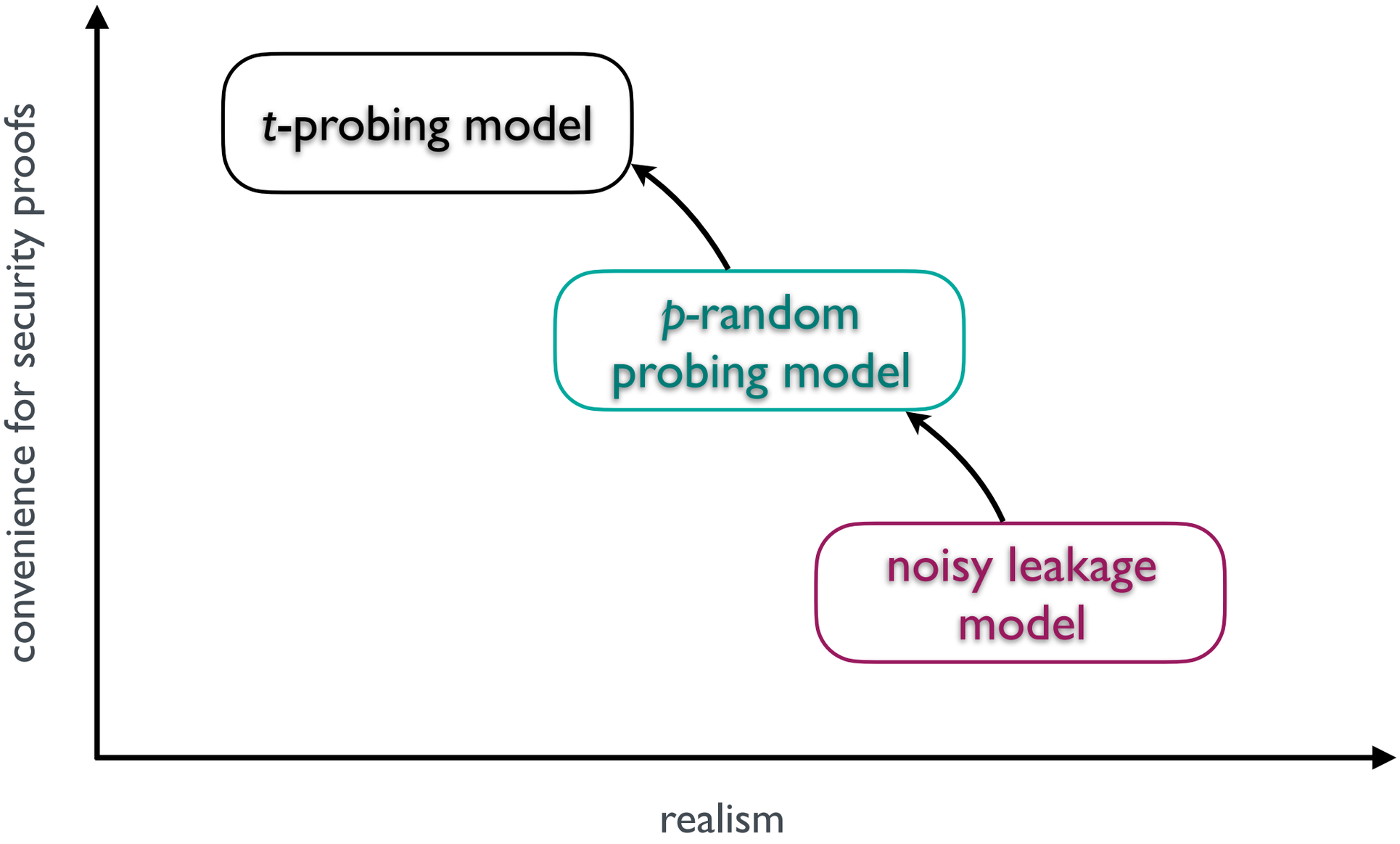


[EC:DDF14] *Unifying Leakage Models: From Probing Attacks to Noisy Leakage*. Alexandre Duc, Stefan Dziembowski, Sebastian Faust. Eurocrypt 2014

Probing Security [ISW03]

Attacker model: adversary gets the exact values of t intermediate variables

Security: any set of *at most* t intermediate variables must be independent of the secrets

Masking

Sensitive variable v

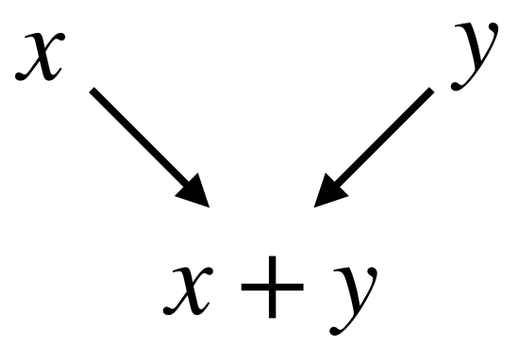
$$v_1, \ldots, v_{n-1} \leftarrow \$ \text{ } \textit{(uniform distribution)}$$

$$v_n \leftarrow v + v_1 + \ldots + v_{n-1}$$

Each strict subset of $(v_i)_{1 \leq i \leq n}$ is independent from v

Masking

Additions:



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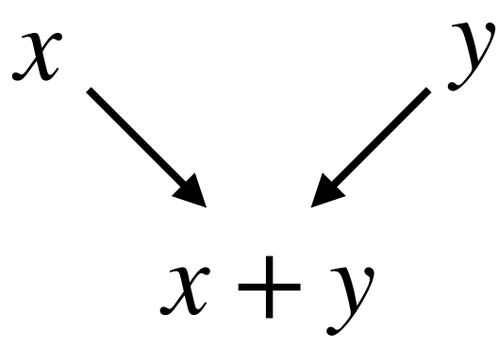

$(x_1, \ldots, x_n)$ $(y_1, \ldots, y_n)$

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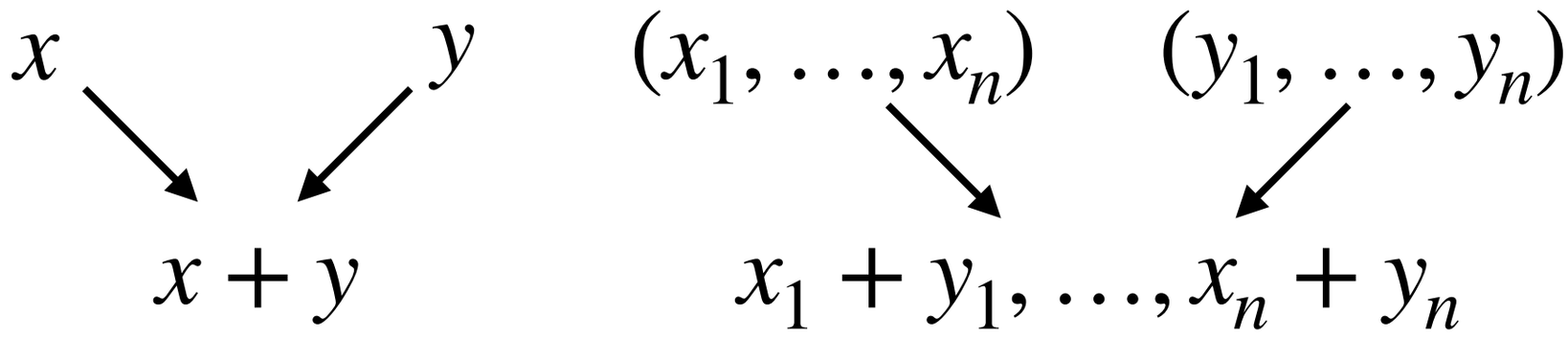

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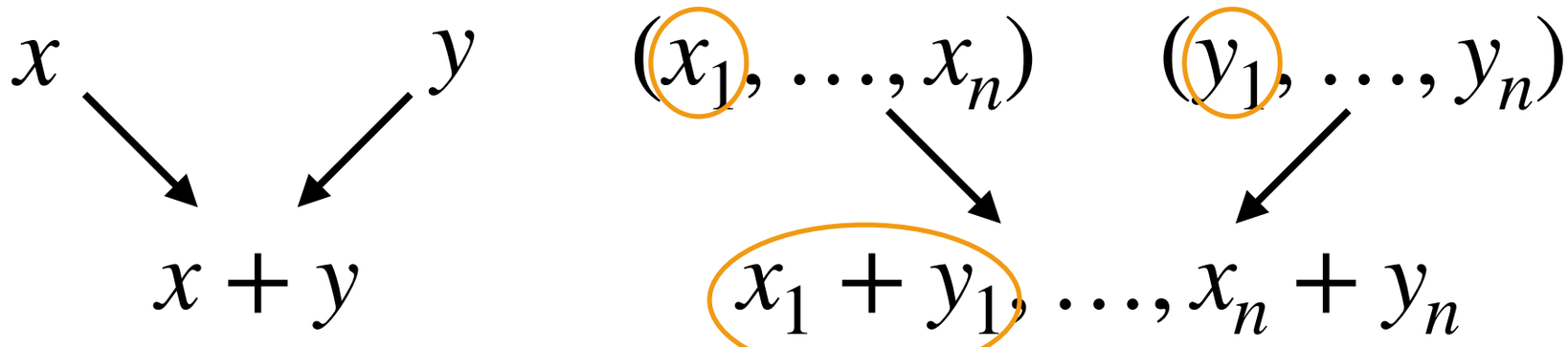

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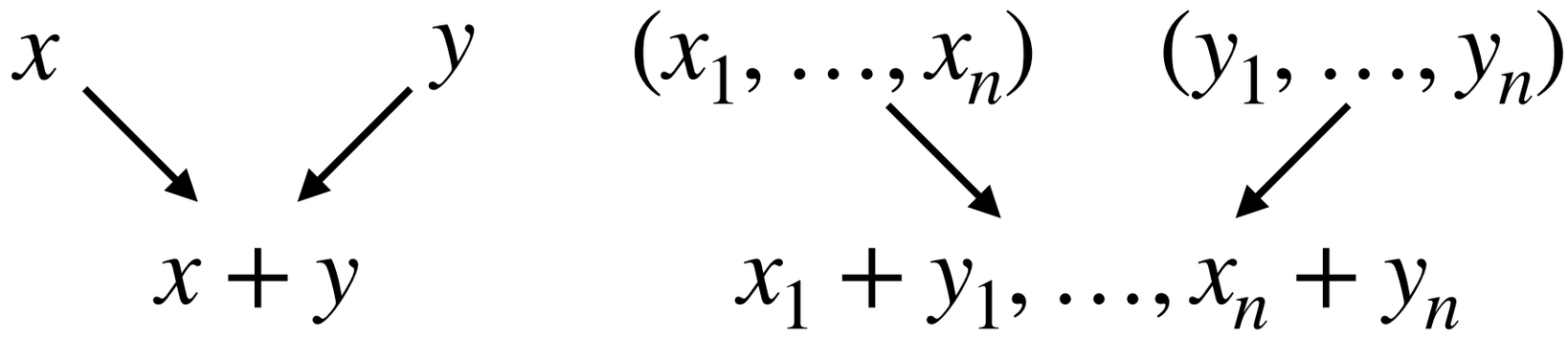

Multiplications:

$$x \cdot y \rightarrow ?$$

Need for extra randomness to mix shares without introducing a bias

Masking

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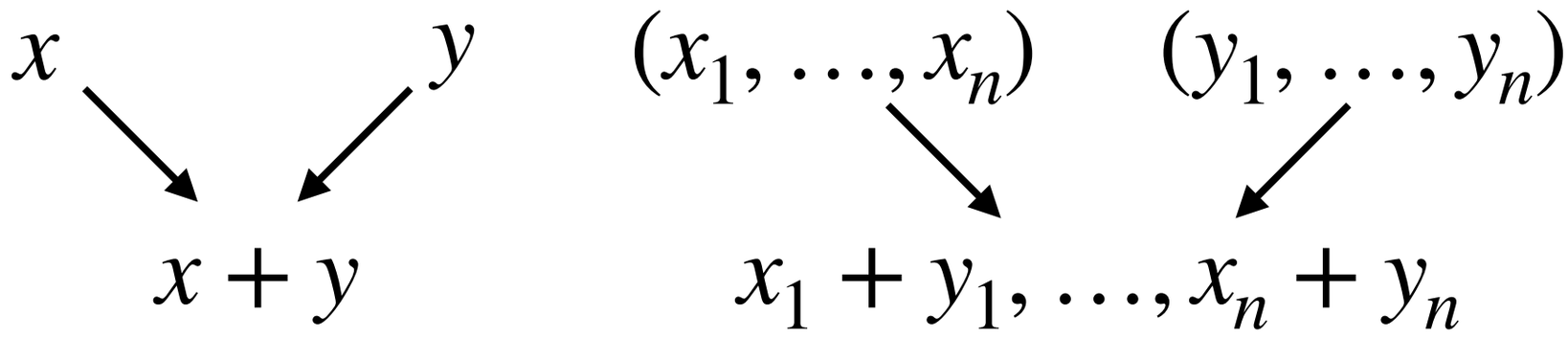

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Small example for $n = 2$

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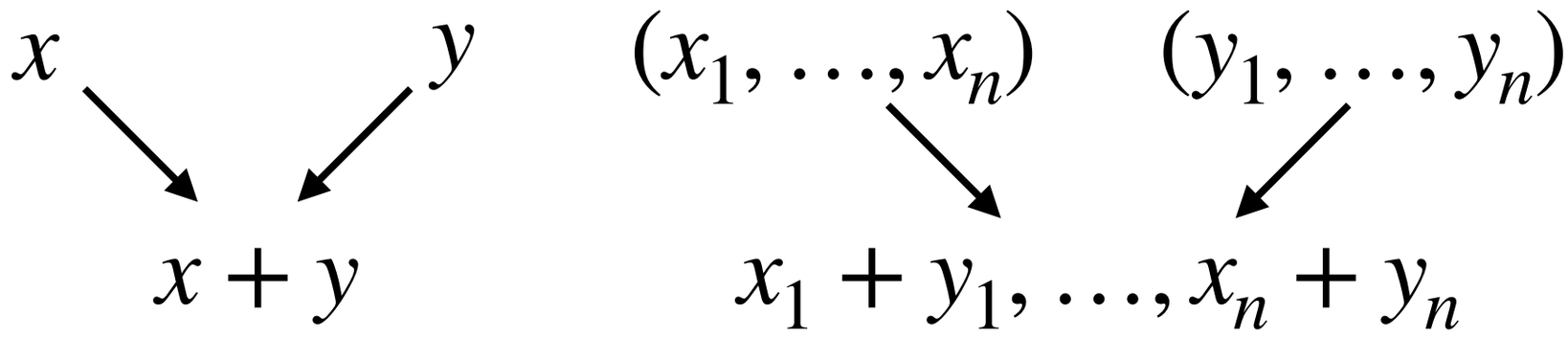

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Small example for $n = 2$

$$x = x_0 + x_1$$

$$y = y_0 + y_1$$

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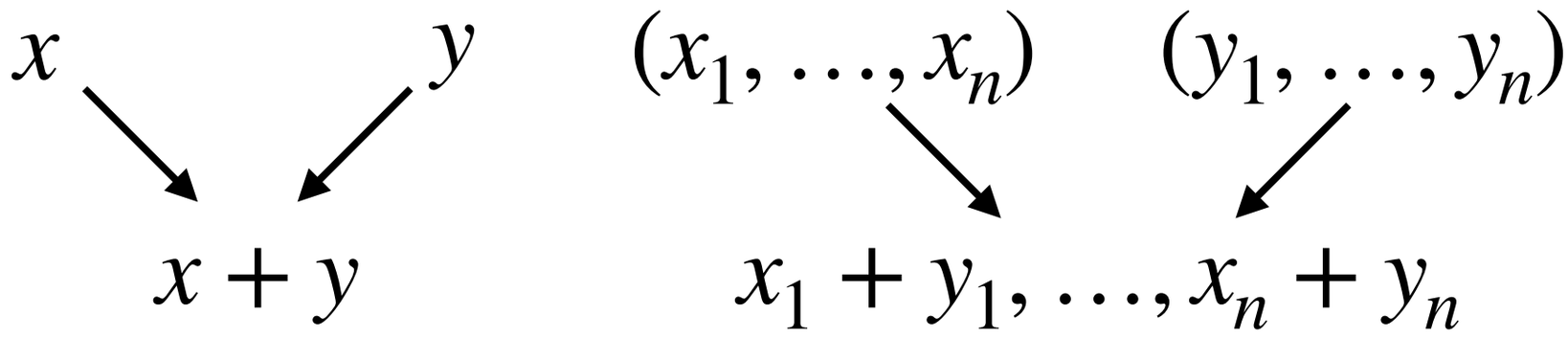

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$$x = x_0 + x_1 \qquad z_0 \leftarrow x_0 y_0 + x_0 y_1$$

$$y = y_0 + y_1 \qquad z_1 \leftarrow x_1 y_1 + x_1 y_0$$

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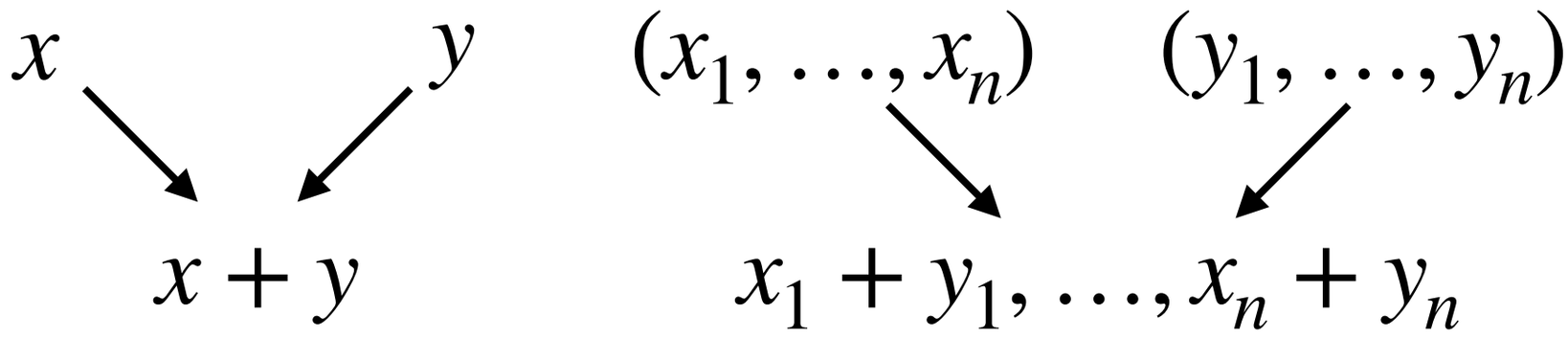

Multiplications:

$$x \cdot y \rightarrow ?$$

Small example for $n = 2$

$$x = x_0 + x_1 \qquad z_0 \leftarrow x_0 y_0 + r + x_0 y_1$$

$$y = y_0 + y_1 \qquad z_1 \leftarrow x_1 y_1 + r + x_1 y_0$$

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Masking Lattice-Based Signatures

Main Challenges

Symmetric implementations

Linear operations

- Boolean masking

Non-linear operations

- Boolean masking
- Generic (non-linear) multiplications

Lattice-Based Signatures

Linear operations

- Boolean & Arithmetic masking
- Conversions

Non-linear operations

- Boolean & Arithmetic masking
- Conversions
- Secret-dependent branches
- New generic algorithms to exhibit

Example: GLP Signature Scheme

Only uniform distributions

Inputs: $m, pk = (a, t), sk = (s_1, s_2)$
Outputs: signature sig

1. $y_1, y_2 \leftarrow R_{q,\kappa}$
2. $r \leftarrow a \cdot y_1 + y_2$
3. $c \leftarrow \text{hash}(r,m)$
4. $z_1 \leftarrow s_1 \cdot c + y_1$
5. $z_2 \leftarrow s_2 \cdot c + y_2$
6. If z_1 or $z_2 \notin R_{q,\kappa-\alpha}$ then
7. Restart
8. End
9. Return $sig \leftarrow (z_1, z_2, c)$

Steps

1. Identify sensitive variables
2. Identify operations to mask
3. Mask atomic operations & convert Boolean / arithmetic masking
4. Safely compose operations

[CHES:GLP12] *Practical lattice-based cryptography: A signature scheme for embedded systems.* T. Güneysu, V. Lyubashevsky, and T. Pöppelmann. CHES 2012.

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[EC:BBEFGRT18] *Masking the GLP Lattice-Based Signature Scheme at Any Order.* Gilles Barthe, Sonia Belaïd, Thomas Espitau, Pierre-Alain Fouque, Benjamin Grégoire, Mélissa Rossi, and Mehdi Tibouchi. Eurocrypt 2018

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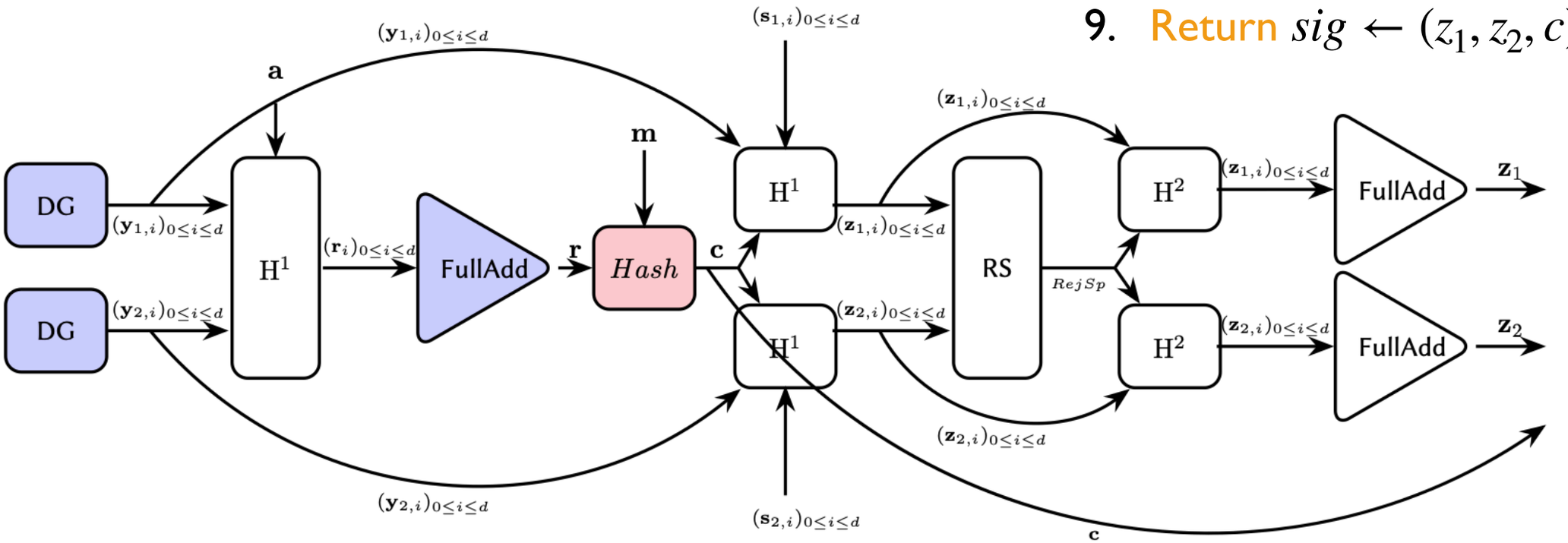

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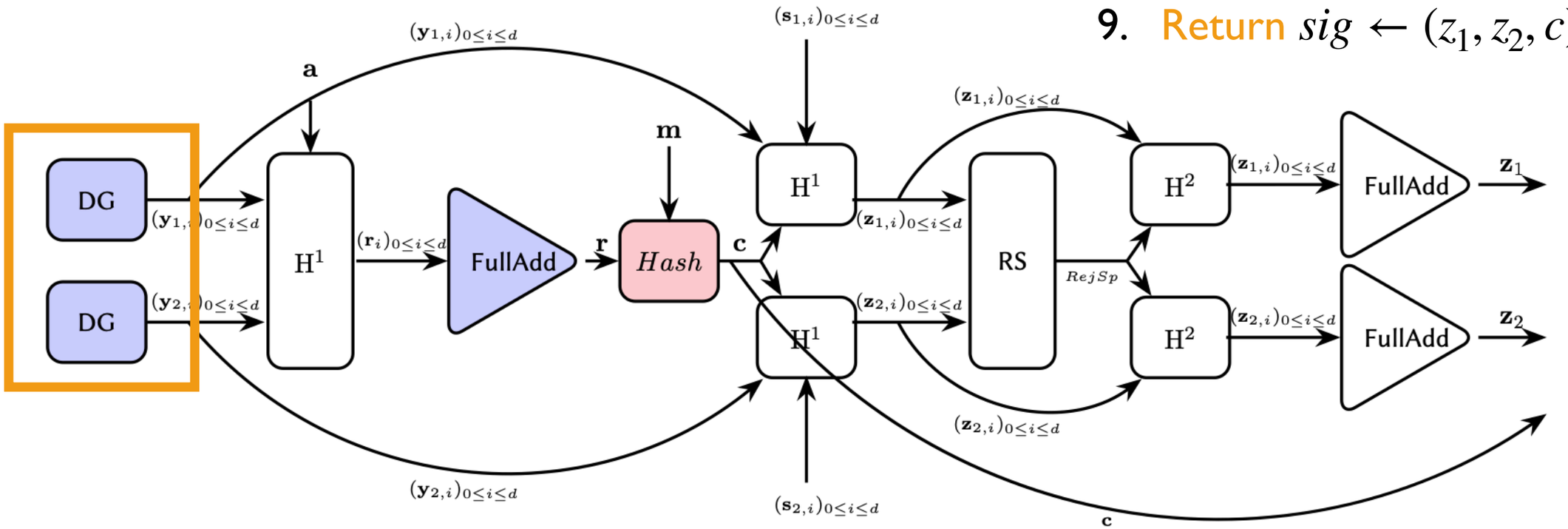

CryptoExperts

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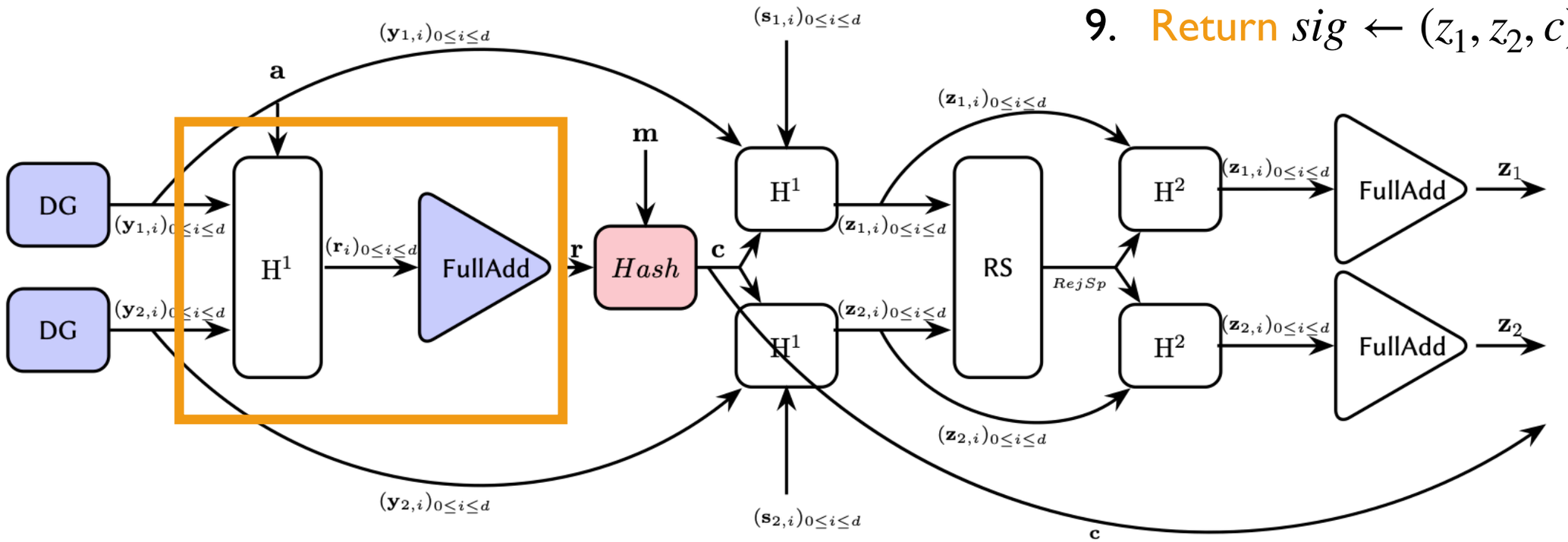

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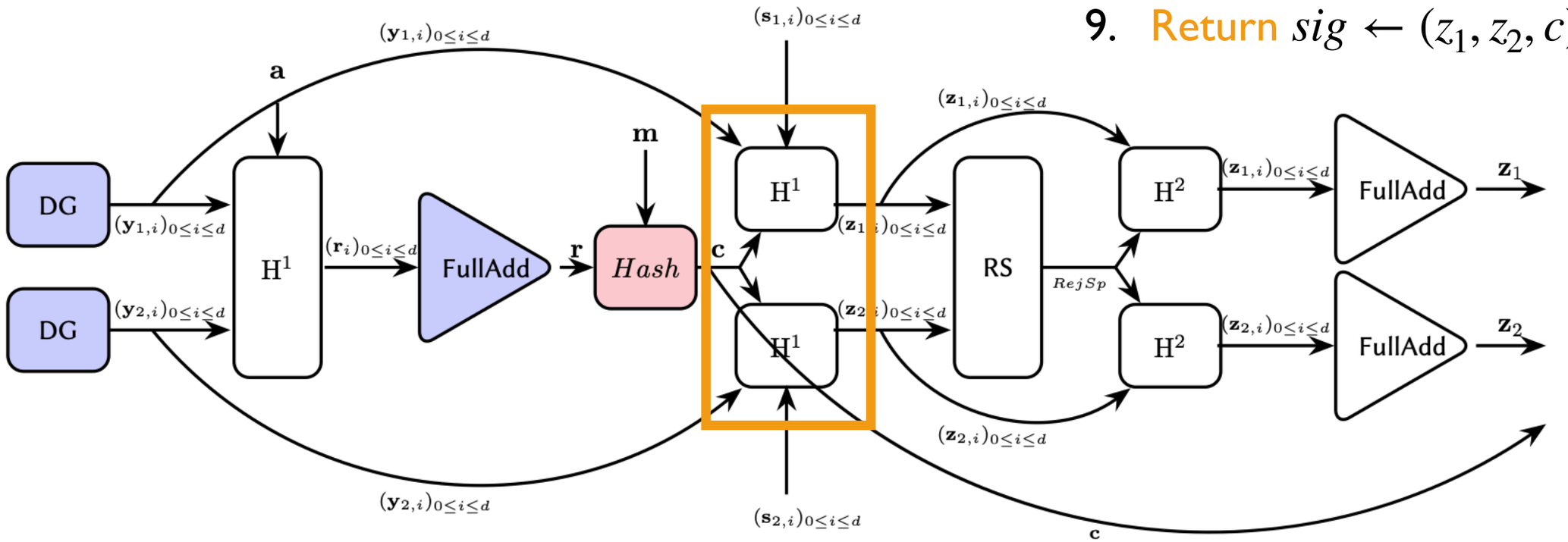

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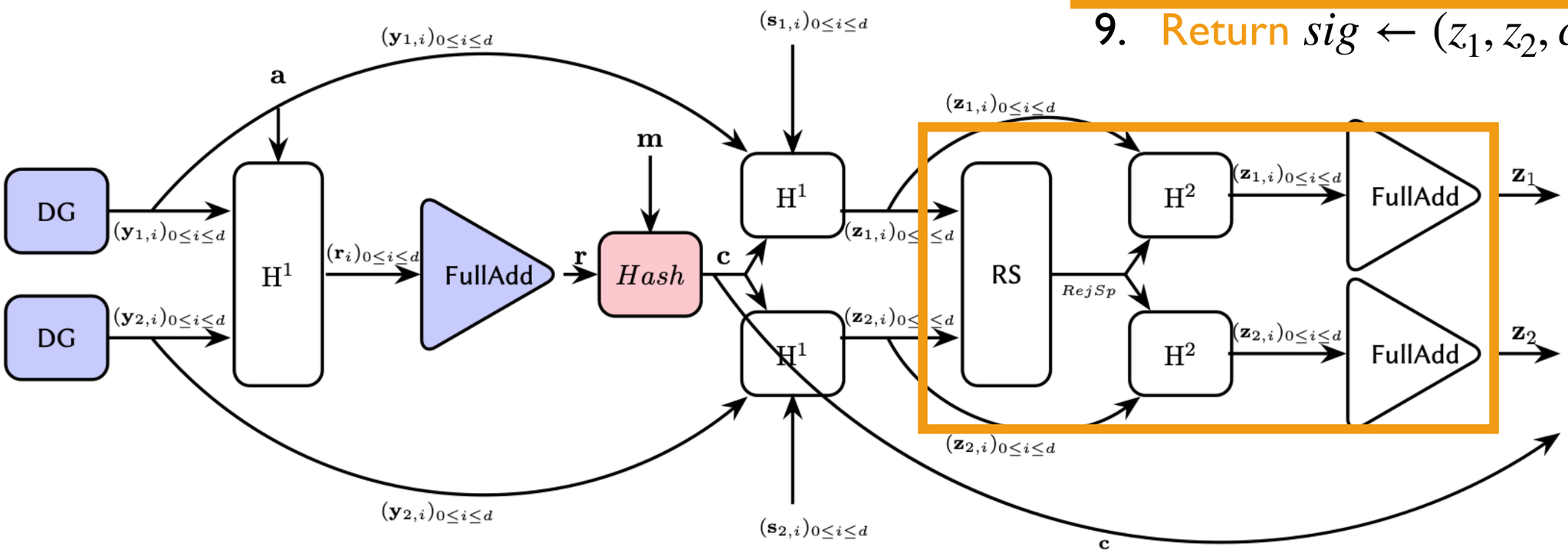

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Generate each coefficient in $[-k, k]$:

- generate Boolean masking of a uniformly random value
  - with rejection
- convert the Boolean masking into an arithmetic masking
  - variant of existing conversions but with arbitrary modulus

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- Convert both arithmetic maskings into Boolean maskings
- Compare the maskings by safely checking the most significant bit of the difference
- Unmask the final result

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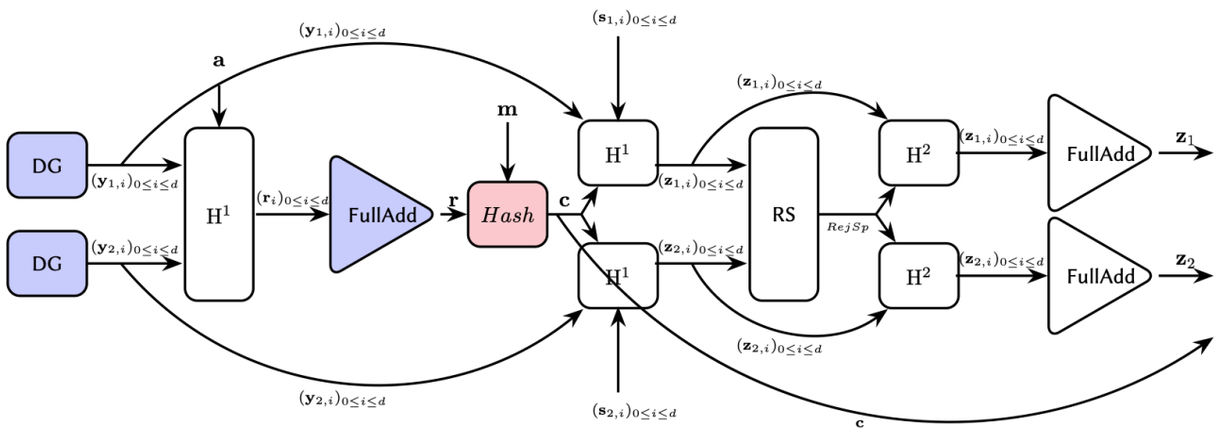

Steps

1. Identify sensitive variables
2. Identify operations to mask
3. Mask atomic operations & convert Boolean / arithmetic masking
4. Safely compose operations
   - Add randomness at careful locations

Other Signature Schemes

With Gaussian distributions

- Gaussian distributions
  - Pros: better parameters and security reductions
  - Cons: secure implementations are difficult to build for discrete Gaussians (timing attacks, power analysis attacks)

Almost all (PQC NIST) candidates chose to stay away from Gaussian distributions

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- Example
  - **BLISS**: Léo Ducas, Alain Durmus, Tancrède Lepoint, and Vadim Lyubashevsky. *Lattice Signatures and Bimodal Gaussians*. In CRYPTO 2013

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- Main Challenges
  - Gaussian sampling $\mathscr{D}_\sigma$
  - Rejection Sampling

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[CCS:BBEFRT] G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, M. Rossi, and M. Tibouchi. *GALACTICS: Gaussian Sampling for Lattice-Based Constant-Time Implementation of Cryptographic Signatures, Revisited.* In CCS 2019.

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With Gaussian distributions

- Main Challenges
  - Gaussian sampling $\mathscr{D}_\sigma$
    - Naive method: cumulative distribution table (CDT) sampling
      - Generate a random value α in [0,1] with high precision
      - Return the index of the first entry greater than α

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With Gaussian distributions

- Main Challenges
  - Gaussian sampling $\mathscr{D}_\sigma$
    - Naive method: cumulative distribution table (CDT) sampling
      - Generate a random value α in [0,1] with high precision
      - Return the index of the first entry greater than α
    - Better strategy: construct a distribution somehow close to $\mathscr{D}_\sigma^+$ (with smaller standard deviation) then use rejection sampling
      - Less entries for the CDT
      - Relies on uniform generation + rejection sampling
      - Masking of the comparisons in the CDT

Other Signature Schemes

With Gaussian distributions

- Main Challenges
  - Rejection Sampling
    - *Main idea: compare uniform values in [0,1] to some probability values*

Other Signature Schemes

With Gaussian distributions

- Main Challenges
  - Rejection Sampling
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    - Naive method: repeated Bernoulli trials with known constant probabilities
      - Not in constant time

Other Signature Schemes

With Gaussian distributions

- Main Challenges
  - Rejection Sampling
    - *Main idea: compare uniform values in [0,1] to some probability values*
    - Naive method: repeated Bernoulli trials with known constant probabilities
      - Not in constant time
    - Better method: approximate the probabilities of rejection with a sufficiently close polynomial
      - Integer arithmetic only
      - High precision with the methodology [AC:Prest17] based on the Rényi divergence
      - Masking of the comparison using Boolean sharing

[AC:Prest17] Thomas Prest. *Sharper Bounds in Lattice-Based Cryptography Using the Rényi Divergence*. In ASIACRYPT 2017.

Conclusion

Conclusion

- Operations in lattice-based signatures require new masking techniques
  - Conversion between Boolean and arithmetic masking (with different modulus)
  - Sampling from different distributions
    - Uniform distributions
    - More challenging Gaussian distributions
    - etc.

- Remaining Challenges
  - Many schemes are not investigated yet
  - Maybe different operations to mask
  - Design masking-friendly signature schemes (e.g., Mitaka, AsiaCCS 2021)

Related Works

[EC:BBEFGRT18] *Masking the GLP Lattice-Based Signature Scheme at Any Order.* Gilles Barthe, Sonia Belaïd, Thomas Espitau, Pierre-Alain Fouque, Benjamin Grégoire, Mélissa Rossi, and Mehdi Tibouchi. Eurocrypt 2018

[CCS:BBEFRT19] *GALACTICS: Gaussian Sampling for Lattice-Based Constant-Time Implementation of Cryptographic Signatures, Revisited.* Gilles Barthe, Sonia Belaïd, Thomas Espitau, Pierre-Alain Fouque, Mélissa Rossi, and Mehdi Tibouchi. CCS 2019

[CARDIS:GR19] *An Efficient and Provable Masked Implementation of qTESLA.* François Gérard, Mélissa Rossi. CARDIS 2019

[ACNS:MGTF19] *Masking Dilithium - Efficient Implementation and Side-Channel Evaluation.* Vincent Migliore, Benoît Gérard, Mehdi Tibouchi, Pierre-Alain Fouque. ACNS 2019

[AsiaCCS:E21] *Mitaka: Faster, Simpler, Parallelizable and Maskable Hash-and-Sign Signatures on NTRU Lattices.* Thomas Espitau. AsiaCCS 2021