



Security of Cryptosystems Against Power-Analysis Attacks PhD Defense

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Presented by Sonia Belaïd

Cryptology



Cryptology



Cryptology



- → Asymmetric cryptography
- → Symmetric cryptography

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- → Symmetric cryptography



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- → Asymmetric cryptography
- Symmetric cryptography

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Example: confidentiality with encryption

- ➔ Black-box cryptanalysis
- → Side-channel analysis



→ Black-box cryptanalysis: $\mathscr{A} \leftarrow (m_i, c_i)$

➔ Side-Channel Analysis



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- → Side-Channel Analysis: $\mathscr{A} \leftarrow (m_i, c_i, \mathscr{L}_i)$



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Cryptography: countermeasures against Power-Analysis Attacks







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- 1. Successful attack with low noise
- 2. Improved attack for higher noise

S. Belaïd, J-S. Coron, B. Gérard, P-A. Fouque, J-G. Kammerer, and E. Prouff CHES 2015

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Classical Power-Analysis Attack against AES-128



Attack on 8 bits

- prediction of the outputs for the 256 possible 8-bit secret
- correlation between predictions and leakage
- selection of the best correlation to find the correct 8-bit secret

Attack on 128 bits

 repetition of the attack on 8 bits on each S-box

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Power-Analysis Attack against AES-GCM authentication, multiplication-based fresh re-keying, ...

 \rightarrow k is only manipulated in multiplications

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Hidden Multiplier Problem

Let $k \leftarrow GF(2^n)$. Let $\ell \in \mathbb{N}$. Given a sequence $\{m^i, \mathscr{L}^i\}_{1 \le i \le \ell}$ where

►
$$m^i \leftarrow GF(2^n)$$

•
$$\mathscr{L}^{i} = HW(v^{i}) + \varepsilon^{i}, \ \varepsilon^{i} \sim \mathcal{N}(0, \sigma^{2})$$

recover k .



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Main Observation

Current Issue: each bit of the 128-bit multiplication's result depends on all the key bits

➔ no divide-and-conquer strategy

Hypotheses:

- ► leakage of multiplication's outputs $HW(v) + \varepsilon$
- multiplication in GF(2¹²⁸)



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Main Observation

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Main observation:

the LSB of a variable's Hamming weight is a linear function of its bits:

$$\operatorname{Isb}_{0}(\operatorname{HW}(\boldsymbol{v})) = \bigoplus_{0 \le i \le 127} \boldsymbol{v}_{i} = \bigoplus_{0 \le i \le 127} \left(\bigoplus_{0 \le j \le 127 - i} m_{j} \right) \boldsymbol{k}_{i}$$

With ℓ Hamming weight values $\{HW(v^{(i)})\}_{0 \le i < \ell}$, we recover *k* by solving \mathscr{S} :

$$\mathscr{S} = \begin{cases} \bigoplus_{0 \le i \le 127} \left(\bigoplus_{0 \le j \le 127 - i} m_j^{(0)} \right) k_i = \operatorname{lsb}_0 (\operatorname{HW}(v))^{(0)} \\ \bigoplus_{0 \le i \le 127} \left(\bigoplus_{0 \le j \le 127 - i} m_j^{(1)} \right) k_i = \operatorname{lsb}_0 (\operatorname{HW}(v))^{(1)} \\ \dots \\ \bigoplus_{0 \le i \le 127} \left(\bigoplus_{0 \le j \le 127 - i} m_j^{(\ell-1)} \right) k_i = \operatorname{lsb}_0 (\operatorname{HW}(v))^{(\ell-1)} \end{cases}$$

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But in practice, the leakage comes with noise: $\mathcal{L} = HW(v) + \varepsilon$

$$\mathsf{Isb}_0([\mathscr{L}]) = \mathsf{Isb}_0(\mathsf{HW}(\mathbf{v})) \oplus \mathbf{b}_{\varepsilon}$$

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Complexities

	Signal-to-Noise Ratio			
Method	3.200	800	200	128
Naive method ($\mathscr{C}_{s}, \mathscr{C}_{t}$)	(<mark>2⁹,2²¹)</mark>	(<mark>2⁹,2²¹)</mark>	(<mark>2⁹,2⁶⁵)</mark>	(<mark>2⁹,2¹⁰⁷)</mark>
LPN (LF Algo) ($\mathscr{C}_{s}, \mathscr{C}_{t}$)	(<mark>2¹²,2¹⁴)</mark>	(<mark>2²¹,2²²)</mark>	(<mark>2³³,2³⁴)</mark>	(2 ⁴⁹ , 2 ⁵⁰)
Linear decoding $(\mathscr{C}_{s}, \mathscr{C}_{t})$	(<mark>2⁷,2⁶)</mark>	(<mark>2⁷,2⁷)</mark>	(2 ⁹ ,2 ²⁵)	(2 ¹⁰ , 2 ⁶²)

Signal-to-noise ratio =	signal variance _	32
	noise variance	$\overline{\sigma^2}$

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Main Observation

New Attack:

- → filter the multiplication's outputs leakage to extract high and low Hamming weights
- → solve a system with errors

Improvements:

- ✓ more generic
- less impacted by noise



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Reminder:

$$\mathscr{L}(\mathbf{v}) = HW(\mathbf{v}) + \varepsilon = HW(\mathbf{m} \odot \mathbf{k}) + \varepsilon$$

Extreme cases:

 $HW(\mathbf{v}) = 0 \Rightarrow \mathbf{v} = 0$ $HW(\mathbf{v}) = n \Rightarrow \mathbf{v} = 2^{n} - 1$ $\begin{cases} \mathbf{v}_{0} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(0,j)}} m_{i} \right) \mathbf{k}_{j} = 0$ $\begin{cases} \mathbf{v}_{0} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(0,j)}} m_{i} \right) \mathbf{k}_{j} = 1$ $\vdots \vdots \vdots$ $\mathbf{v}_{n-1} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(n-1,j)}} m_{i} \right) \mathbf{k}_{j} = 0$ $\begin{cases} \mathbf{v}_{0} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(n-1,j)}} m_{i} \right) \mathbf{k}_{j} = 1$ $\vdots \vdots$ $\mathbf{v}_{n-1} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(n-1,j)}} m_{i} \right) \mathbf{k}_{j} = 0$

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Reminder:

$$\mathscr{L}(\mathbf{v}) = HW(\mathbf{v}) + \varepsilon = HW(\mathbf{m} \odot \mathbf{k}) + \varepsilon$$

Usual cases:

 $\mathscr{L}(\mathbf{v}) \text{ low } \rightarrow \mathbf{v} \approx 0 \qquad \qquad \mathscr{L}(\mathbf{v}) \text{ high } \rightarrow \mathbf{v} \approx 2^{n} - 1$ $\begin{cases} \mathbf{v}_{0} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(0,j)}} m_{i} \right) \mathbf{k}_{j} = 0 \\ \mathbf{v}_{1} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(1,j)}} m_{i} \right) \mathbf{k}_{j} = 0 \\ \vdots \quad \vdots \quad \vdots \\ \mathbf{v}_{n-1} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(n-1,j)}} m_{i} \right) \mathbf{k}_{j} = 0 \end{cases} \qquad \qquad \begin{cases} \mathbf{v}_{0} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(n-1,j)}} m_{i} \right) \mathbf{k}_{j} = 1 \\ \vdots \quad \vdots \\ \mathbf{v}_{n-1} = \bigoplus_{0 \leq j < n} \left(\bigoplus_{i \in I^{(n-1,j)}} m_{i} \right) \mathbf{k}_{j} = 0 \end{cases}$

with an error probability p
Filtering



Error Probabilities

$\log_2(1/F(\lambda))$	30	25	20	15	10	5
SNR = 128, <i>σ</i> = 0.5						
λ	6.00	5.46	4.85	4.15	3.29	2.16
р	0.23	0.25	0.28	0.31	0.34	0.39
p [BFG14]	0.31					
SNR = 8, <i>σ</i> = 2						
λ	6.37	5.79	5.14	4.39	3.48	2.28
р	0.25	0.27	0.29	0.32	0.35	0.40
p [BFG14]	> 0.49					
$SNR = 2, \sigma = 4$						
λ	7.42	6.73	5.97	5.09	4.03	2.64
р	0.28	0.30	0.32	0.34	0.37	0.41
p [BFG14]	> 0.49					
SNR = 0.5, <i>σ</i> = 8						
λ	10.57	9.58	8.48	7.21	5.71	3.73
р	0.34	0.36	0.37	0.39	0.41	0.44
p [BFG14]	> 0.49					

Signal-to-noise ratio = $\frac{\text{signal variance}}{\text{noise variance}} = \frac{32}{\sigma^2}$

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Experiments for n = 128

Filtering on a Virtex 5 - 128 bits (n = 128) : SNR = 8.21, $\sigma = 7.11$



• Expected complexities to recover k with 2^{20} consumption traces

		(2 ^{59.31} ,2 ^{27.00})
trade-offs	(time , memory)	(2 ^{51.68} , 2 ^{36.00})
		(2 ^{50.00} , 2 ^{44.00})

Conclusion on the Multiplication Cryptanalysis



Summary

✓ successful attacks on multiplications from the output's leakage

✓ practical for n = 128 (use cases: AES-GCM, re-keying)

Further Work

- → application of similar attacks on other primitives
- deeper analysis of LPN techniques in the context of side-channel analysis

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Cryptography: countermeasures against Power-Analysis Attacks











Countermeasures against Power-Analysis Attacks



Problem: leakage \mathscr{L} is key-dependent

Fresh Re-keying

Idea: regularly change k



Masking

Idea: make leakage ℒ random



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→ each *t*-uple of *v_i* is independent from *v*

Countermeasures against Power-Analysis Attacks



Problem: leakage ℒ is key-dependent

Masking

Idea: make leakage ℒ random



→ each *t*-uple of *v_i* is independent from *v*



Security of Masked Programs: Leakage Model



realism

Security of Masked Programs: Leakage Model



realism









- 1. show that a *t*-uple is independent from the secret
- 2. test all the possible t-uples

function Ex-t3(x_1, x_2, x_3, x_4, c): $(* X_1, X_2, X_3 =$ *) $(* X_{4} = X + X_{1} + X_{2} + X_{3})$ *r*₁ ← \$ $r_2 \leftarrow \$$ $V_1 \leftarrow X_1 + I_1$ $V_2 \leftarrow (X + X_1 + X_2 + X_3) + I_2$ $t_1 \leftarrow X_2 + I_1$ $t_2 \leftarrow (\chi_2 + r_1) + \chi_3$ $y_3 \leftarrow (x_2 + r_1 + x_3) + r_2$ $V_4 \leftarrow C + \frac{r_2}{r_2}$ return (V_1, V_2, V_3, V_4)

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function Ex-t3(x_1, x_2, x_3, x_4, c): $(* X_1, X_2, X_3 =$ *) $(* X_{4} = X + X_{1} + X_{2} + X_{3})$ *r*₁ ← \$ $r_2 \leftarrow \$$ 1. independent $V_1 \leftarrow X_1 + I_1$ from the secret? $V_2 \leftarrow (X + X_1 + X_2 + X_3) + I_2$ × many mistakes $t_1 \leftarrow X_2 + I_1$ $t_2 \leftarrow (\chi_2 + r_1) + \chi_3$ $y_3 \leftarrow (x_2 + r_1 + x_3) + r_2$ $V_4 \leftarrow C + \frac{r_2}{r_2}$ return (V_1, V_2, V_3, V_4)

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2. test 286 3-uplesX missing casesX inefficient

Inputs: t intermediate variables, $b \leftarrow true$ function Ex-t3(x_1, x_2, x_3, x_4, c): (Rule 1) secret variables? **/**1 ← \$ yes \rightarrow (Rule 2) <u>r</u>₂ ← \$ no 🔿 🖌 $V_1 \leftarrow X_1 + I_1$ (Rule 2) an expression v is invertible in the $V_2 \leftarrow (X + X_1 + X_2 + X_3) + I_2$ only occurrence of a random r? $t_1 \leftarrow x_2 + r_1$ yes $\rightarrow v \leftarrow r$; (Rule 1) $t_2 \leftarrow (x_2 + r_1) + x_3$ no \rightarrow (Rule 3) $V_3 \leftarrow (X_2 + I_1 + X_3) + I_2$ (Rule 3) is flag b = true? $V_A \leftarrow C + r_0$ ves \rightarrow simplify; $b \leftarrow$ false; (Rule 1) return (y_1, y_2, y_3, y_4) no 🔿 🗙 ✓ → distribution independent from the secret

✗ → might be used for an attack

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Problem: *n* intermediate variables $\rightarrow \binom{n}{t}$ proofs

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New Idea: proofs for sets of more than t variables

 find larger sets which cover all the intermediate variables is a hard problem

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two algorithms efficient in practice

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Algorithm 1:

1. select X = (t variables) and prove its independence

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Algorithm 1:

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- 2. extend X to \hat{X} with more observations but still independence

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3. recursively descend in set $\mathscr{C}(\widehat{X})$

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Algorithm 1:

- 1. select X = (t variables) and prove its independence
- 2. extend X to \hat{X} with more observations but still independence
- 3. recursively descend in set $\mathscr{C}(\widehat{X})$
- 4. merge \hat{X} and $\mathscr{C}(\hat{X})$ once they are processed separately.

function Ex-t3(x_1, x_2, x_3, x_4, c): *r*₁ ← \$ *r*₂ ← \$ $V_1 \leftarrow X_1 + I_1$ $y_2 \leftarrow (x + x_1 + x_2 + x_3) + r_2$ $t_1 \leftarrow X_2 + I_1$ $t_2 \leftarrow (X_2 + I_1) + X_3$ $V_3 \leftarrow (X_2 + I_1 + X_3) + I_2$ $y_4 \leftarrow C + r_2$ return $(\gamma_1, \gamma_2, \gamma_3, \gamma_4)$

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function Ex-t3(
$$x_1, x_2, x_3, x_4, c$$
):
(r₁) $\leftarrow \$$
(r₂) $\leftarrow \$$
(y₁) $\leftarrow x_1 + r_1$
(y₂) $\leftarrow (x + x_1 + x_2 + x_3) + r_2$
(t₁) $\leftarrow x_2 + r_1$
(t₂) $\leftarrow (x_2 + r_1) + x_3$
(y₃) $\leftarrow (x_2 + r_1 + x_3) + r_2$
(y₄) $\leftarrow c + r_2$
return(y₁, y₂, y₃, y₄)

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→ 207 proofs instead of 286
Application to the Sbox [CPRR13, Algorithm 4]

Method	# tuples	Security	Complexity					
			# sets	time*				
First-Order Masking								
naive		~	63	0.001s				
Alg. 1	63		17	0.001s				
Alg. 2			17	0.001s				
Second-Order Masking								
naive			12,561	0.180s				
Alg. 1	12,561	~	851	0.046s				
Alg. 2			619	0.029s				
Third-Order Masking								
naive			4,499,950	140.642s				
Alg. 1	4,499,950	~	68,492	9.923s				
Alg. 2			33,075	3.894s				
Fourth-Order Masking								
naive			-	unpractical				
Alg. 1	2,277,036,685	l 🖌	8,852,144	2959.770s				
Alg. 2			3,343,587	879.235s				

*run on a headless VM with a dual core (only one core is used in the computation) 64-bit processor clocked at 2GHz イロトイロトイラト モミト ミークへで 28/40

Benchmarks

Poforonco	Torget	# tuploc	Socurity	Complexity					
nelerence	laigei	# tupies	Security	# sets	time (s)				
First-Order Masking									
FSE13	full AES	17,206	 ✓ 	3,342	128				
MAC-SHA3	full Keccak-f	13,466	 ✓ 	5,421	405				
Second-Order Masking									
RSA06	Sbox	1,188,111	 ✓ 	4,104	1.649				
CHES10	Sbox	7,140	1 st -order flaws (2)	866	0.045				
CHES10	AES KS	23,041,866	 ✓ 	771,263	340,745				
FSE13	2 rnds AES	25,429,146	/ /	511,865	1,295				
FSE13	4 rnds AES	109,571,806	V V	2,317,593	40,169				
Third-Order Masking									
RSA06	Sbox	2,057,067,320	3 rd -order flaws (98,176)	2,013,070	695				
FSE13	Sbox(4)	4,499,950	 ✓ 	33,075	3.894				
FSE13	Sbox(5)	4,499,950	V V	39,613	5.036				
Fourth-Order Masking									
FSE13	Sbox (4)	2,277,036,685	✓	3,343,587	879				
Fifth-Order Masking									
CHES10	•	216,071,394		856,147	45				

Chosen Contributions





Chosen Contributions



Cryptography: countermeasures against Power-Analysis Attacks









C

A refresh algorithm takes as input a sharing $(x_i)_{i\geq 0}$ of x and returns a new sharing $(x'_i)_{i\geq 0}$ of x such that $(x_i)_{i\geq 1}$ and $(x^r_i)_{i\geq 1}$ are mutually independent.



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if *t* is fixed: show that any set of *t* intermediate variables is independent from the secret

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if *t* is not fixed: show that any set of *t* intermediate variables can be simulated with at most *t* shares of each input

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if *t* is fixed: show that any set of *t* intermediate variables is independent from the secret

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function Linear-function-t($a_0, ..., a_i, ..., a_t$):

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for i = 0 to t $c_i \leftarrow f(a_i)$ return $(c_0, ..., c_i, ..., c_t)$

→ straightforward for linear functions

if *t* is fixed: show that any set of *t* intermediate variables is independent from the secret

if *t* is not fixed: show that any set of *t* intermediate variables can be simulated with at most *t* shares of each input





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- → straightforward for linear functions
- → formal proofs with EasyCrypt and pen-and paper proofs for small non-linear functions











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Stronger security property for Refresh

Strong Non-Interference in the *t*-probing model:

if t is not fixed: show that any set of t intermediate variables with

- t1 on internal variables
- $t_2 = t t_1$ on the outputs

can be simulated with at most t_1 shares of each input



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Automatic tool for C-based algorithms

► unprotected algorithm → higher-order masked algorithm

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example for AES S-box


Secure Composition

Automatic tool for C-based algorithms

- ► unprotected algorithm → higher-order masked algorithm
- example for AES S-box



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Some Results

Resource usage statistics for generating masked algorithms (at any order) from some unmasked implementations¹

Scheme	# Refresh	Time	Memory
AES (⊙)	2	0.09s	4Mo
AES $(x \odot g(x))$	0	0.05s	4Mo
Keccak with Refresh	0	121.20	456Mo
Keccak	600	2728.00s	22870Mo
Simon	67	0.38s	15Mo
Speck	61	6.22s	38Mo

¹On a Intel(R) Xeon(R) CPU E5-2667 0 @ 2.90GHz with 64Go of memory running Linux (Fedora)

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Conclusion on Higher-Order Masking

Summary

- ✓ verification of higher-order masking schemes
- efficient and proven composition
- ✓ two automatic tools

Further Work

- → extend the verification to higher orders using composition
- → integrate transition/glitch-based model
- build practical experiments for both attacks and new countermeasures



Conclusion



- investigate the LPN algorithms in the context of power-analysis attacks
- → analyze the operation modes

Cryptography: countermeasures against Power-Analysis Attacks

- implement and evaluate our countermeasures on real devices (software and hardware)
- ➔ make verifications and compositions as practical as possible
- → use the characterization of a device as a leakage model

Publications



Sonia Belaïd, Luk Bettale, Emmanuelle Dottax, Laurie Genelle, and Franck Rondepierre. Differential Power Analysis of HMAC SHA-2 in the Hamming weight model. SECRYPT 2013.



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Masking and leakage-resilient primitives: One, the other(s) or both? Cryptography and Communications 2015.



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