Improved Side-Channel Analysis of Finite-Field Multiplication

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Outline

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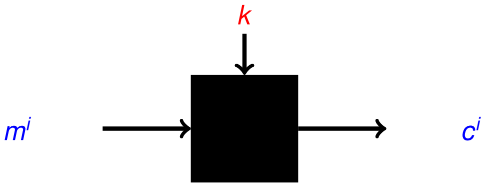
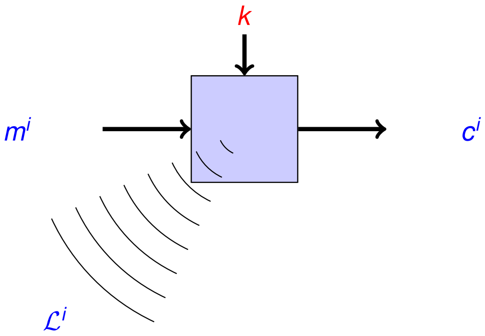
- Black-box cryptanalysis
- Side-channel analysis

- Black-box cryptanalysis: $\mathcal{A} \leftarrow (m^i, c^i)$
- Side-Channel Analysis



- ➜ Black-box cryptanalysis
- ➜ Side-Channel Analysis: $\mathcal{A} \leftarrow (m^i, c^i, \mathcal{L}^i)$



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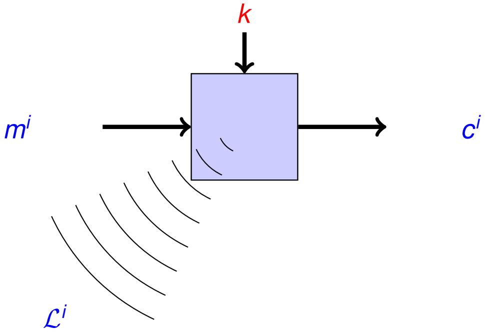

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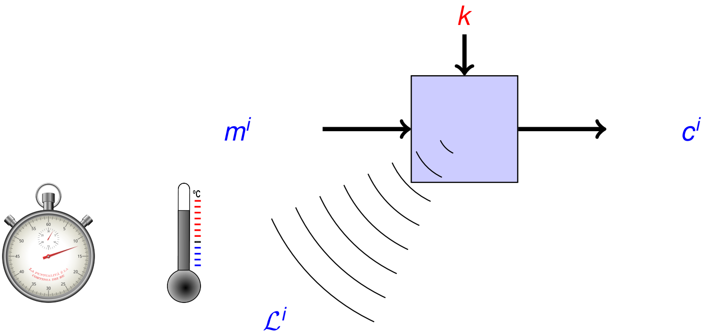

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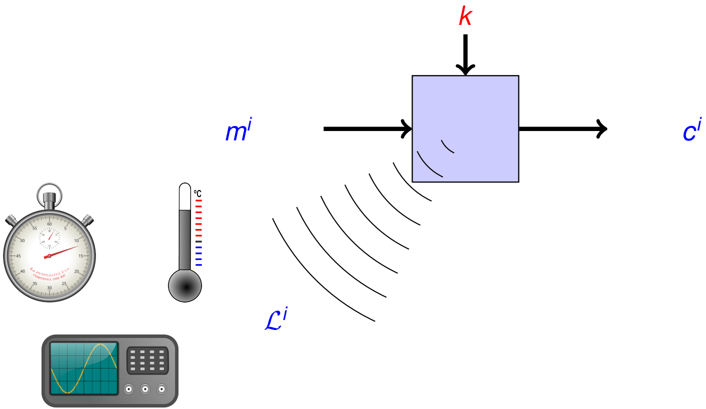

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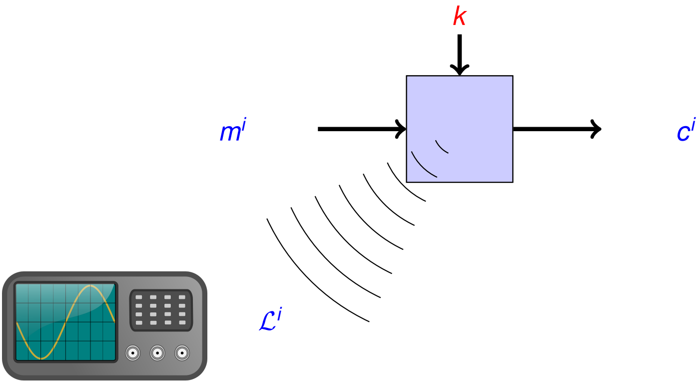
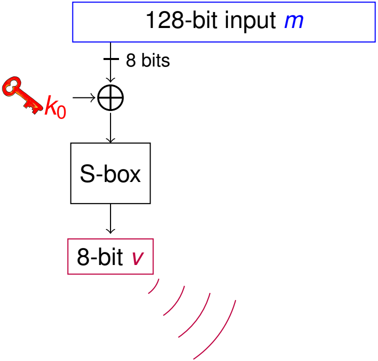


Classical Power-Analysis Attack against AES



Attack on 8 bits

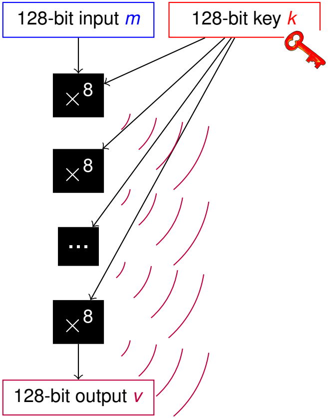
- prediction of the outputs for the 256 possible 8-bit secret
- correlation between predictions and leakage
- selection of the best correlation to find the correct 8-bit secret

Attack on 128 bits

- repetition of the attack on 8 bits on each S-box

Power-Analysis Attack against AES-GCM authentication, multiplication-based fresh re-keying, ...

➜ *k* is only manipulated in multiplications



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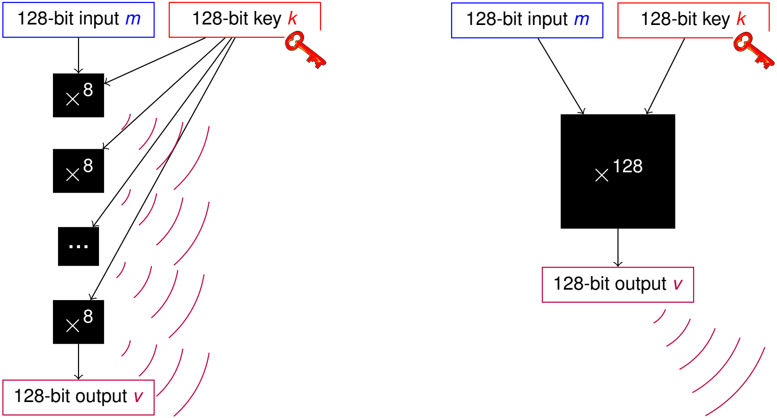


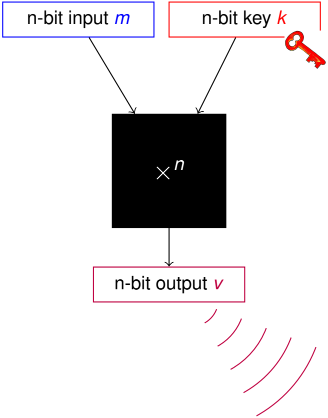
Hidden Multiplier Problem

Definition

Let $k \leftarrow \mathrm{GF}(2^n)$. Let $\ell \in \mathbb{N}$.
Given a sequence $\{m^i, \mathcal{L}^i\}_{1 \leq i \leq \ell}$ where

- $m^i \leftarrow \mathrm{GF}(2^n)$
- $\mathcal{L}^i = \mathrm{HW}(v^i) + \varepsilon^i$, $\varepsilon^i \sim \mathcal{N}(0, \sigma^2)$

recover k.



State of The Art

Sonia Belaïd, Pierre-Alain Fouque, and Benoît Gérard.
Side-channel analysis of multiplications in GF(2^{128}) - application to AES-GCM.
In *Asiacrypt 2014, Proceedings, Part II*, pages 306–325.

- ➜ use Hamming Weights' LSB
- ➜ solve a system with errors

	Signal-to-Noise Ratio $= \frac{\text{signal variance}}{\text{noise variance}} = 32/\sigma^2$			
Method	3.200	800	200	128
Naive method $(\mathcal{C}_s, \mathcal{C}_t)$	$(2^8, 2^{21})$	$(2^8, 2^{21})$	$(2^8, 2^{65})$	$(2^8, 2^{107})$
LPN (LF Algo) $(\mathcal{C}_s, \mathcal{C}_t)$	$(2^{11}, 2^{14})$	$(2^{20}, 2^{22})$	$(2^{32}, 2^{34})$	$(2^{48}, 2^{50})$
Linear decoding $(\mathcal{C}_s, \mathcal{C}_t)$	$(2^6, 2^6)$	$(2^6, 2^7)$	$(2^8, 2^{25})$	$(2^9, 2^{62})$

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- ✘ specific to multiplication in GF(2^{128})
- ✘ highly impacted by noise

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Contributions

New Attack:

- ➜ filter the multiplication's outputs leakage to extract high and low Hamming weights
- ➜ solve a system with errors

Contributions

New Attack:

- ➜ filter the multiplication's outputs leakage to extract high and low Hamming weights
- ➜ solve a system with errors
- ✔ less impacted by noise
- ✔ more generic

Main Idea of The Attack

Reminder:

$$\mathcal{L}(v) = \mathrm{HW}(v) + \varepsilon = \mathrm{HW}(m \cdot k) + \varepsilon$$

Extreme cases:

$\mathrm{HW}(v) = 0 \rightarrow v = 0$

$$\begin{cases} v_0 & = \bigoplus_{0 \leqslant j < n} \left(\bigoplus_{i \in I^{(0,j)}} m_i \right) k_j = 0 \\ v_1 & = \bigoplus_{0 \leqslant j < n} \left(\bigoplus_{i \in I^{(1,j)}} m_i \right) k_j = 0 \\ \vdots & \quad \vdots \qquad \vdots \\ v_{n-1} & = \bigoplus_{0 \leqslant j < n} \left(\bigoplus_{i \in I^{(n-1,j)}} m_i \right) k_j = 0 \end{cases}$$

$\mathrm{HW}(v) = n \rightarrow v = 2^n - 1$

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Reminder:

$$\mathcal{L}(v) = \mathrm{HW}(v) + \varepsilon = \mathrm{HW}(m \cdot k) + \varepsilon$$

Usual cases:

$\mathcal{L}(v)$ low ➜ $v \approx 0$

$$\begin{cases} v_0 &= \bigoplus_{0 \leqslant j < n} \left(\bigoplus_{i \in I^{(0,j)}} m_i \right) k_j = 0 \\ v_1 &= \bigoplus_{0 \leqslant j < n} \left(\bigoplus_{i \in I^{(1,j)}} m_i \right) k_j = 0 \\ \vdots & \quad \vdots \qquad \qquad \vdots \\ v_{n-1} &= \bigoplus_{0 \leqslant j < n} \left(\bigoplus_{i \in I^{(n-1,j)}} m_i \right) k_j = 0 \end{cases}$$

$\mathcal{L}(v)$ high ➜ $v \approx 2^n - 1$

$$\begin{cases} v_0 &= \bigoplus_{0 \leqslant j < n} \left(\bigoplus_{i \in I^{(0,j)}} m_i \right) k_j = 1 \\ v_1 &= \bigoplus_{0 \leqslant j < n} \left(\bigoplus_{i \in I^{(1,j)}} m_i \right) k_j = 1 \\ \vdots & \quad \vdots \qquad \qquad \vdots \\ v_{n-1} &= \bigoplus_{0 \leqslant j < n} \left(\bigoplus_{i \in I^{(n-1,j)}} m_i \right) k_j = 1 \end{cases}$$

with an error probability p

Two Steps

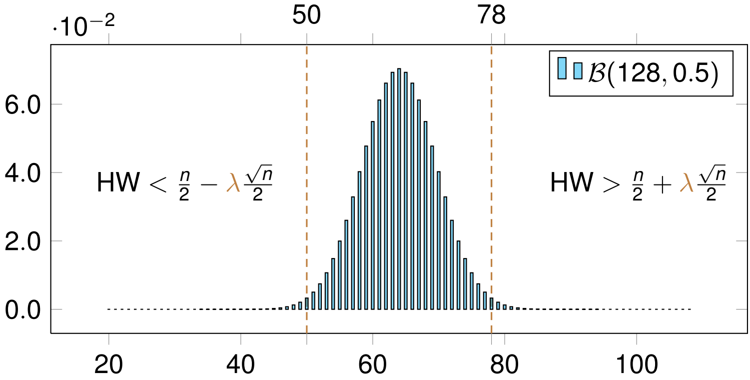
1. filter the lowest and highest Hamming weights with a limited number of consumption traces to limit the error probability p

   ➜ obtain a linear system with errors

2. solve the system with the error probability p

   ➜ recover the secret key k

Step 1: Filtering



SNR $= 128$

$n = 128$

$\lambda \approx 2.5$

filtering: 1 trace over 2^5

error probability: $p \approx 0.38$

Step 1: Filtering

Proportion of filtered acquisitions:

$$F(\lambda, \sigma) = 1 - 2^{-n} \sum_{y=0}^{n} \binom{n}{y} \int_{n/2-\lambda s}^{n/2+\lambda s} \phi_{y,\sigma}(t)dt, \quad \text{with } \; s = \sqrt{n}/2$$

Error probability:

$$p(\lambda, \sigma) = \frac{1}{F(\lambda, \sigma)} \sum_{y=0}^{n} \frac{\binom{n}{y}}{2^n} \left(\underbrace{\frac{y}{n} \int_{-\infty}^{n/2-\lambda s} \phi_{y,\sigma}(t)dt}_{\text{low Hamming weights}} + \underbrace{\left(1 - \frac{y}{n}\right) \int_{n/2+\lambda s}^{+\infty} \phi_{y,\sigma}(t)dt}_{\text{high Hamming weights}} \right)$$

Step 1: Filtering

$\log_2(1/F(\lambda))$	30	25	20	15	10	5
SNR $= 128$, $\sigma = 0.5$						
λ	6.00	5.46	4.85	4.15	3.29	2.16
p	0.23	0.25	0.28	0.31	0.34	0.39
p [BFG14]	0.31					
SNR $= 8$, $\sigma = 2$						
λ	6.37	5.79	5.14	4.39	3.48	2.28
p	0.25	0.27	0.29	0.32	0.35	0.40
p [BFG14]	> 0.49					
SNR $= 2$, $\sigma = 4$						
λ	7.42	6.73	5.97	5.09	4.03	2.64
p	0.28	0.30	0.32	0.34	0.37	0.41
p [BFG14]	> 0.49					
SNR $= 0.5$, $\sigma = 8$						
λ	10.57	9.58	8.48	7.21	5.71	3.73
p	0.34	0.36	0.37	0.39	0.41	0.44
p [BFG14]	> 0.49					

Step 2: Solving the System with Errors

Classical LPN problem: recover the secret key from a noisy system

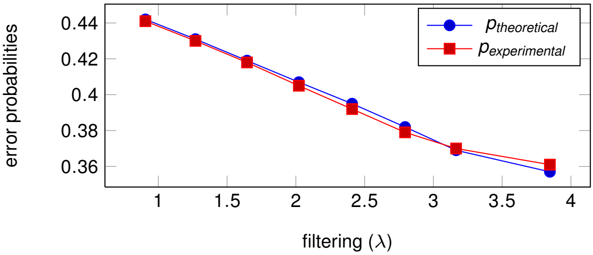
- limited memory
- limited computational power

Specific constraints:

- limited number of equations/consumption traces
- key size n (*e.g.*, 128)
- probability of errors dependent on the filtering and on the noise

Experiments

- Filtering on a Virtex 5 (128 bits) : SNR $= 8.21$, $\sigma = 7.11$



- Expected complexities to recover k with 2^{20} traces ($p \approx 0.29$)

	$(2^{59.31}, 2^{27.00})$
(time , memory)	$(2^{51.68}, 2^{36.00})$
	$(2^{50.00}, 2^{44.00})$

Extension: Chosen Inputs in GF(2^{128})

1. Exhibit the noisy system:

- $\mathrm{MSB}(m \cdot k) = 0$ ➜ $\mathrm{HW}((2 \cdot m) \cdot k) = \mathrm{HW}(m \cdot k)$
- $\mathrm{MSB}(m \cdot k) = 1$ ➜

$$|\mathrm{HW}((2 \cdot m) \cdot k) - \mathrm{HW}(m \cdot k)| = \begin{cases} 1 & \text{with probability} = 3/4 \\ 3 & \text{with probability} = 1/4 \end{cases}$$

SNR (σ)	128 (0.5)	8 (2)	2 (4)	0.5 (8)
p	0.003	0.27	0.39	0.46

Extension: Chosen Inputs in $GF(2^{128})$

1. Exhibit the noisy system:

- $\mathrm{MSB}(m \cdot k) = 0$ ➜ $HW((2 \cdot m) \cdot k) = HW(m \cdot k)$
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SNR (σ)	128 (0.5)	8 (2)	2 (4)	0.5 (8)
p	0.003	0.27	0.39	0.46

2. Solve the noisy system:

- only 128 equations
- repetitions to obtain a system with almost no error

  Example:

  ➜ SNR of 128 can be achieved from an SNR of 2 and 64 repetitions

  ➜ $128 \times 0.003 = 0.384$ errors

  ➜ solving the system with a single error: 2^7 key verifications

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Conclusion

Summary

- attack on multiplications without looking inside the multiplication
- less noise sensitive than [BFG14]
- practical for $n = 128$

Further Work

- application of similar attacks to other primitives
- deeper analysis of LPN techniques to improve solving in side-channel contexts

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Thank you for your attention.