Dynamic Random Probing Expansion with Quasi Linear Asymptotic Complexity

Sonia Belaïd [1], Matthieu Rivain [1],
Abdul Rahman Taleb [1,2] and Damien Vergnaud [2,3]

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December 7, 2021

Side-Channel Attacks & Masking

Security against **side-channel attacks**

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Masking countermeasure (sensitive variable x over field $\mathbb{K}$)

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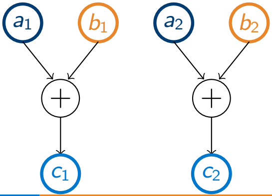

Leakage Models

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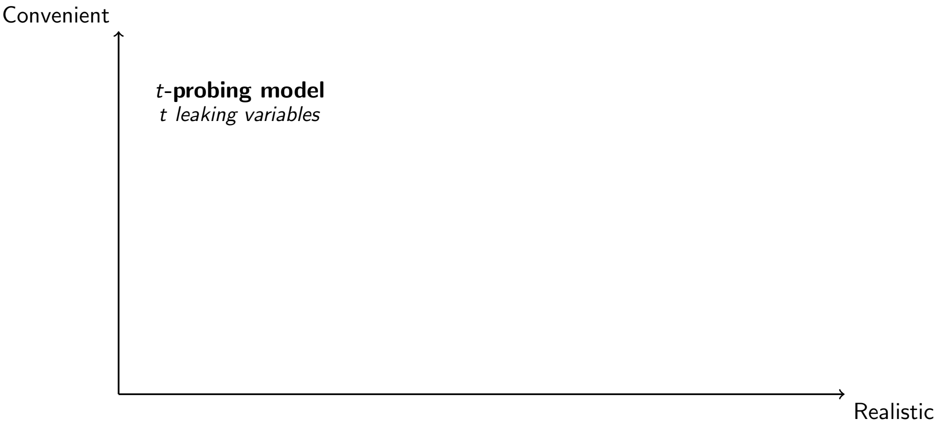

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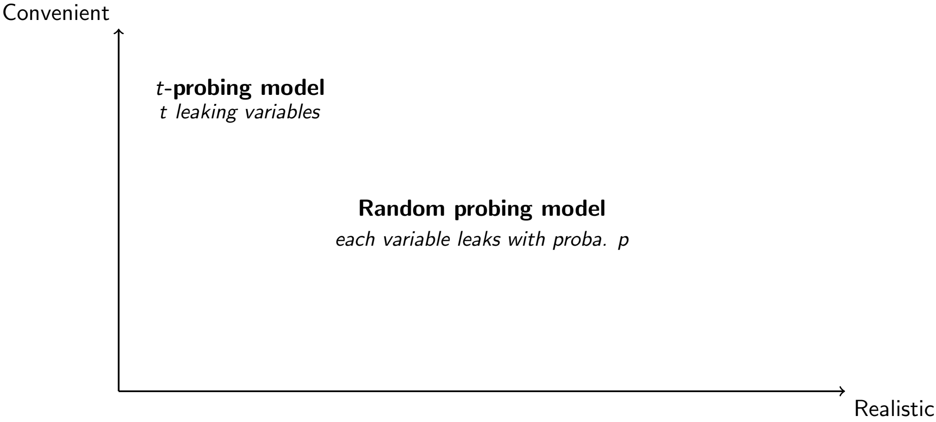

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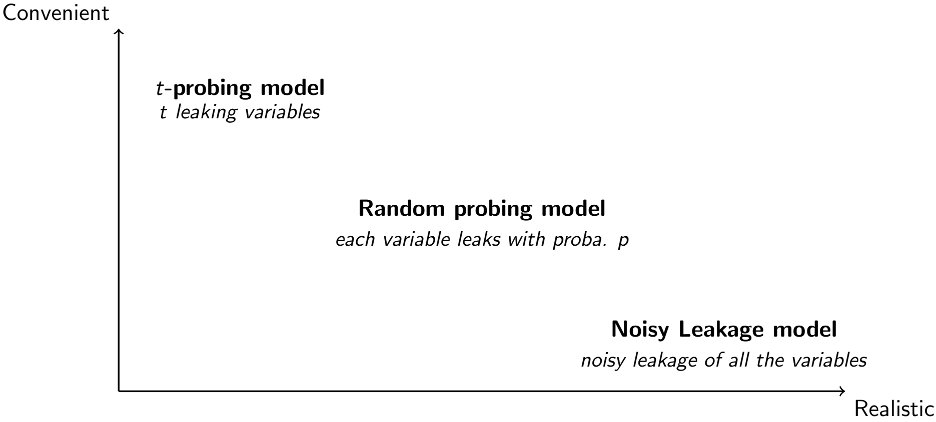

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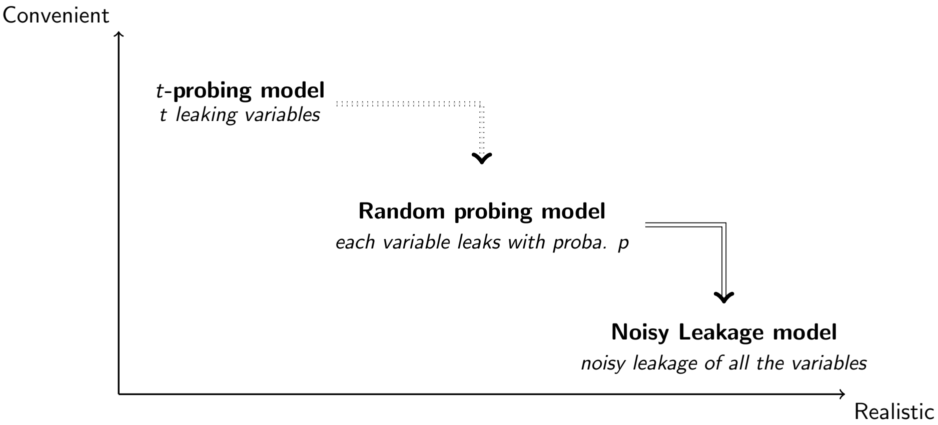

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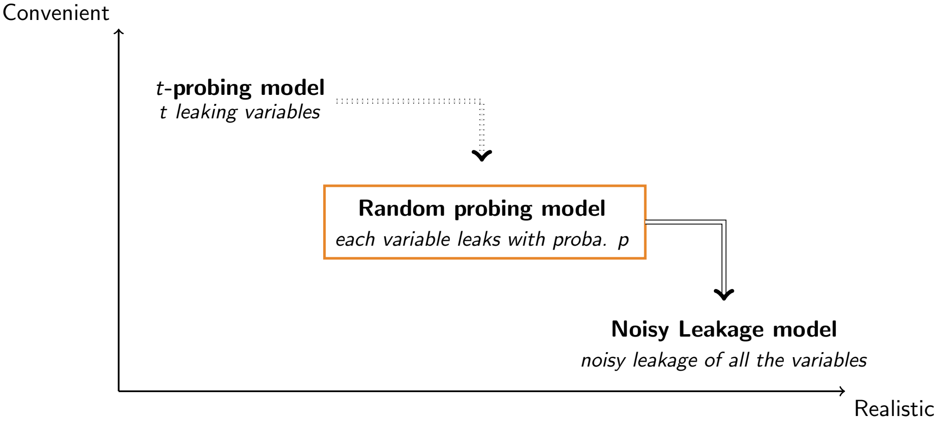

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- In-depth analysis of RP expansion
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- Concrete instantiations for RP expansion tolerating a leakage rate of $p \approx 2^{-7.5}$

Contributions

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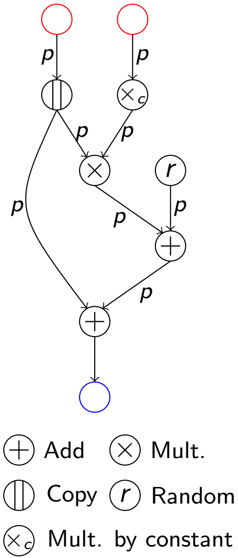
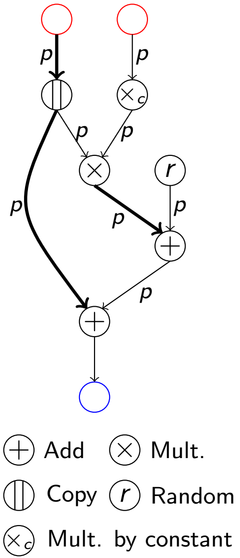
RP Security



(p, ε)-RP Security

RP Security



Add Mult.

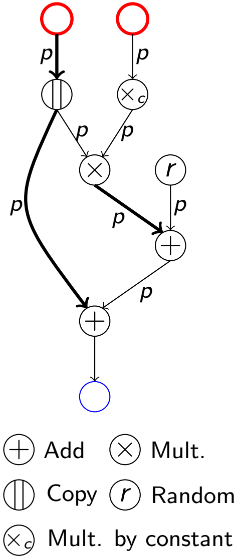
Copy Random

Mult. by constant

(p, ε)-RP Security

W set of wires

RP Security



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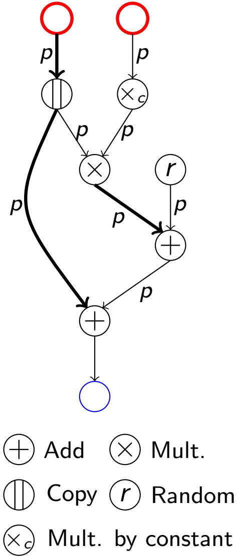
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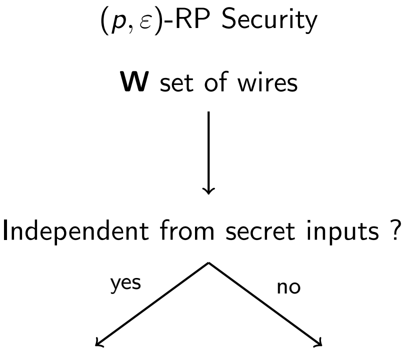
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Independent from secret inputs ?
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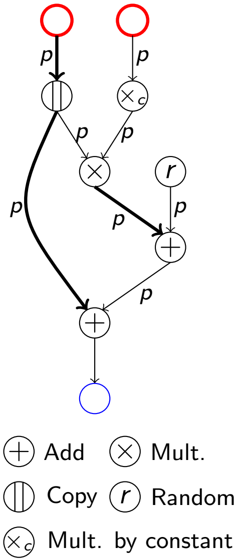
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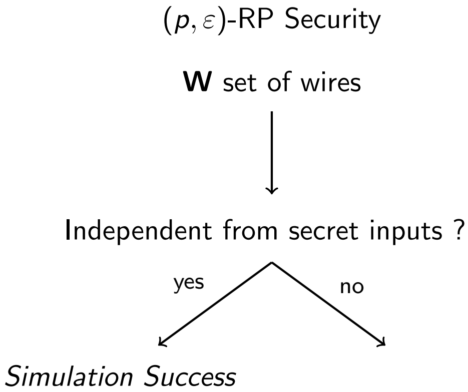
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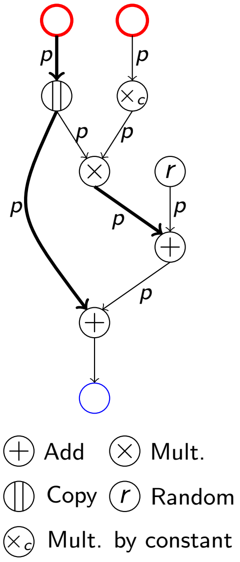
Simulation Success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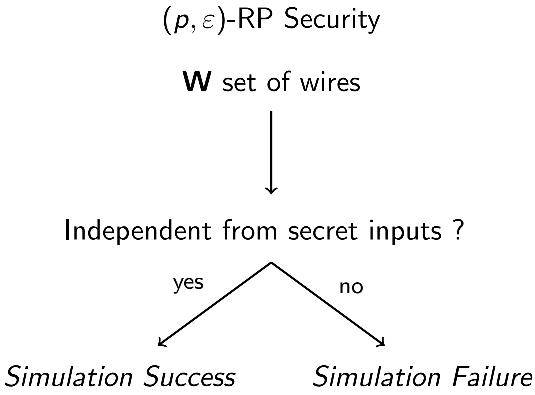
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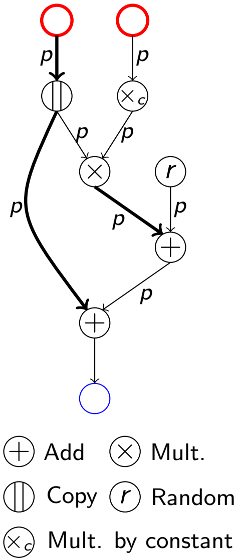

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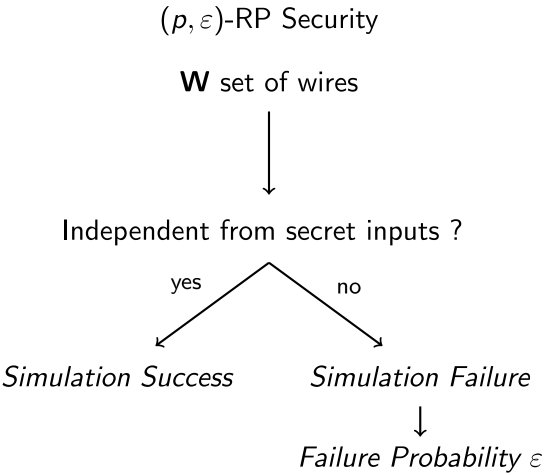

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Using n-share gadgets $G_1, \ldots, G_\beta$

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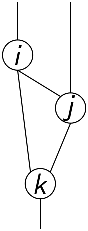

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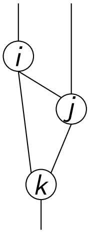

Leakage probability
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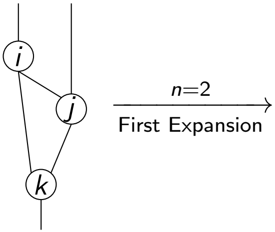

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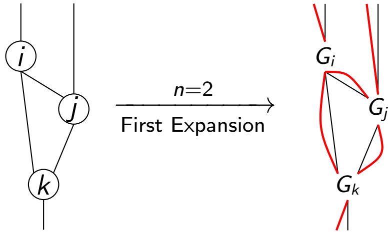

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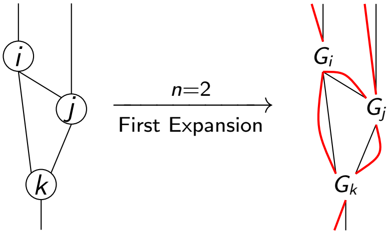

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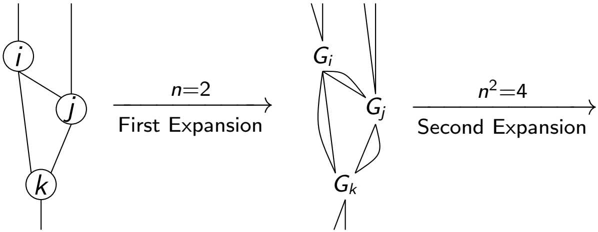

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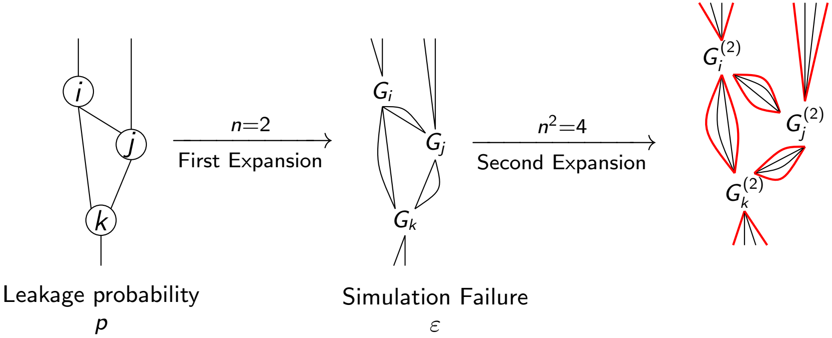

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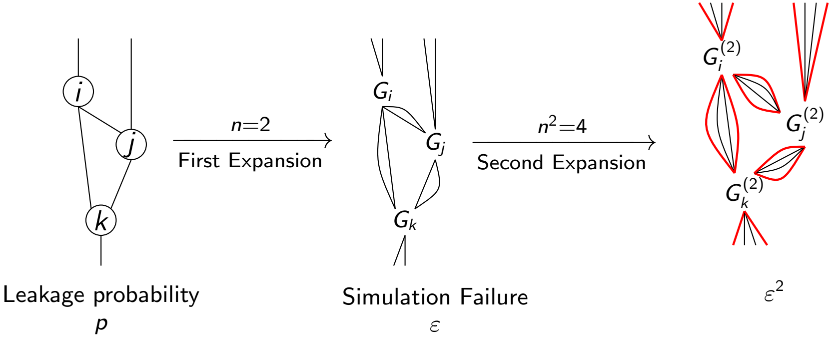

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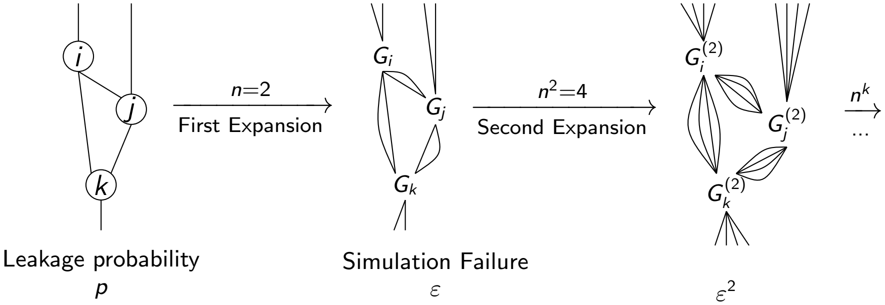

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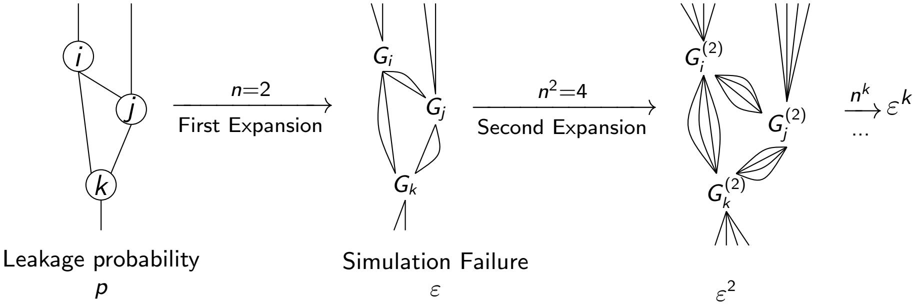

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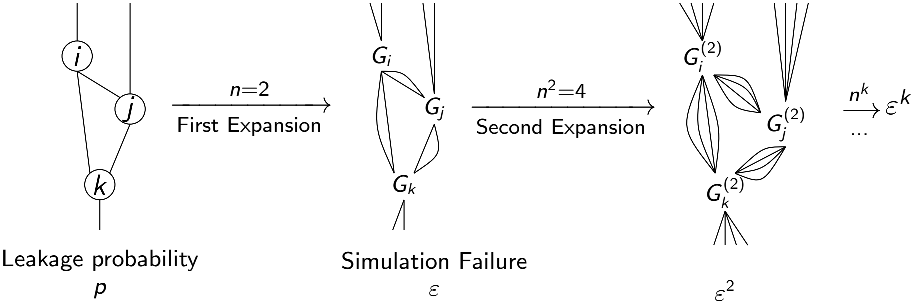

Condition : $\varepsilon < p$ (tolerated leakage rate)

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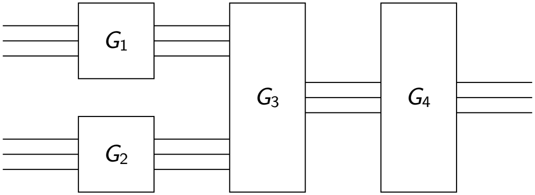

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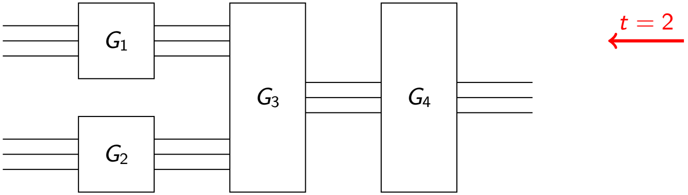

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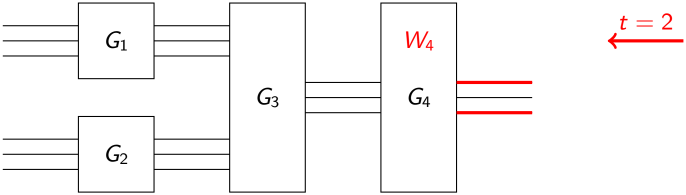

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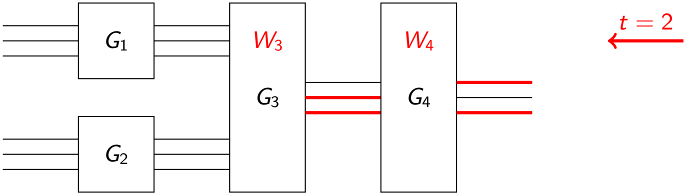

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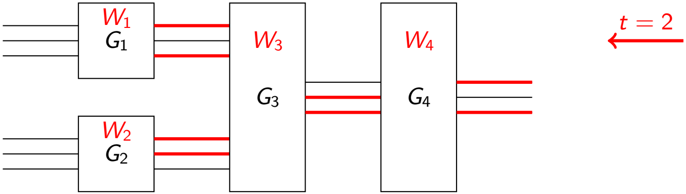

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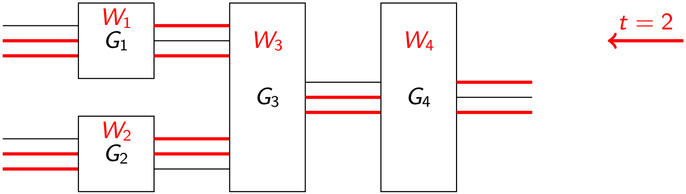

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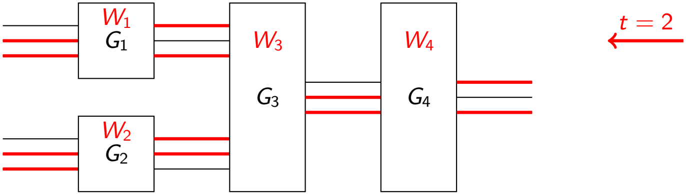

- Independent failure probability on each input sharing

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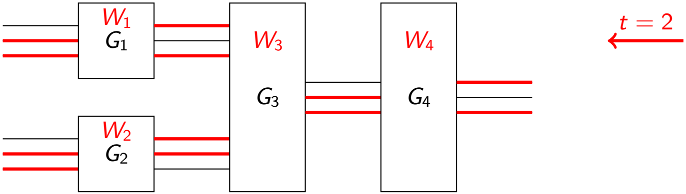

- Independent failure probability on each input sharing
- $G_1, \ldots, G_\beta$ are (t, p, ε)-RPE $\implies$ compiled circuit C is $(p, 2.|C|.\varepsilon^k)$-RP Secure

RP Expansion

Parameters

Complexity of expanded circuit C of security parameter κ:

RP Expansion

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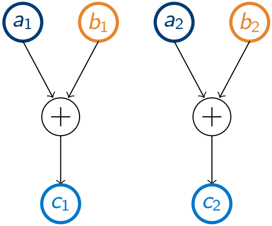

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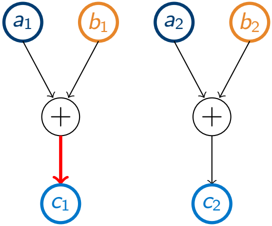

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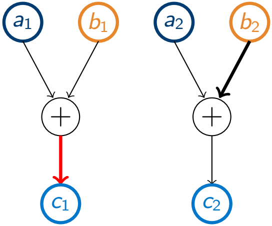

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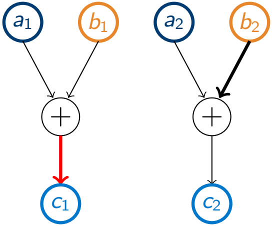

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Simulation needs a_1 ($\leq t$) and b_1, b_2 ($> t$)

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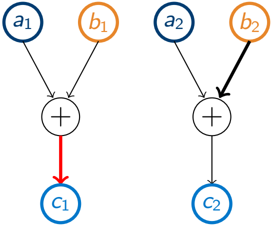

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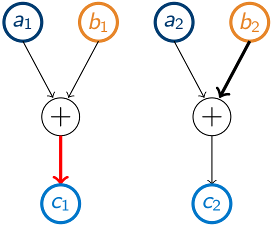

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Failure on $b \implies \mathbf{d} = |W| = \mathbf{1}$

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- higher **d** $\implies$ faster decrease in failure probability ($d_{\max} = \dfrac{n+1}{2}$)

Dynamic RP Expansion
Idea

Using RPE compilers $CC_1, \ldots, CC_\ell$ with numbers of shares $n_1, \ldots, n_\ell$

$$C \quad \xrightarrow[k_1 \text{ times}]{CC_1}$$

Leakage
rate p

Dynamic RP Expansion

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$$C \xrightarrow[k_1 \text{ times}]{CC_1} \hat{C}_1$$

Leakage rate p

$n_1^{k_1}$ shares

$\varepsilon_1^{k_1} = f_1^{(k_1)}(p)$

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Conditions: $\varepsilon_1 < p, \quad \varepsilon_2 < \varepsilon_1^{k_1}, \quad \ldots \quad , \quad \varepsilon_\ell < \varepsilon_{\ell-1}^{k_{\ell-1}}$

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Why?

n-share RPE compilers:

Dynamic RP Expansion

Motivation

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- **small** n: fewer sets of probes that reveal the secret $\implies$ tolerate better leakage rate p

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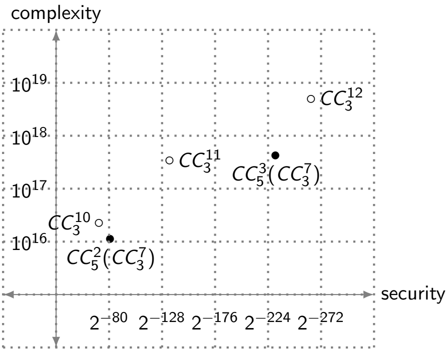

Complexity and security level of RP AES starting from tolerated leakage of $p = 2^{-7.6}$ using 3-share CC_3 and 5-share CC_5 by *Belaïd et al. - EuroCrypt 2021*

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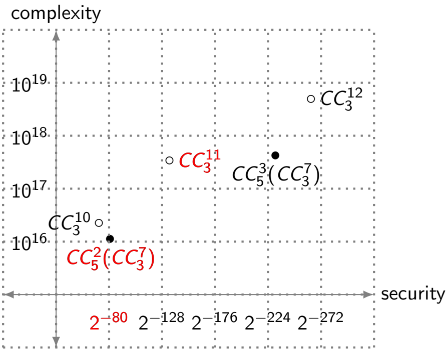

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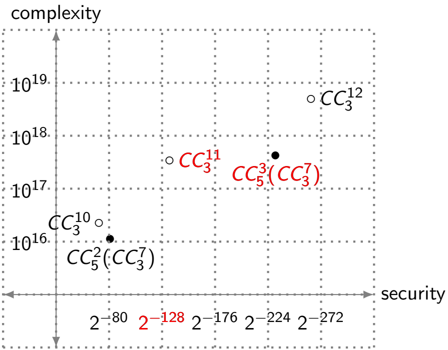

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In this work:

Dynamic RP Expansion
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In this work:

- construction of n-share linear $G_{\mathrm{add}}, G_{\mathrm{copy}}, G_{\mathrm{cmult}}$ with $\mathcal{O}(n \log n)$ asymptotic complexity and maximal amp. order

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In this work:

- construction of n-share linear $G_{\mathsf{add}}, G_{\mathsf{copy}}, G_{\mathsf{cmult}}$ with $\mathcal{O}(n \log n)$ asymptotic complexity and maximal amp. order
- construction of n-share G_{mult} with $\mathcal{O}(n \log n)$ **randomness** and $\mathcal{O}(n)$ **multiplications** between variables

$\mathcal{O}(n \log n)$ refresh gadget G_{refresh} by *Battistello et al. - CHES 2016*:

$$\underset{\longrightarrow}{\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}}$$

Linear Gadgets
Building Block

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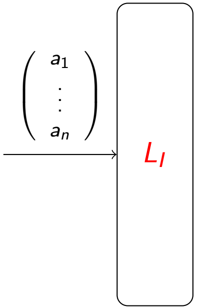

$n/2$ randoms

$b_i \leftarrow a_i + r_i$

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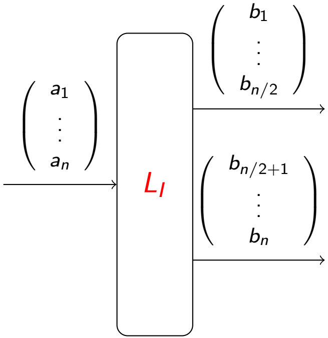

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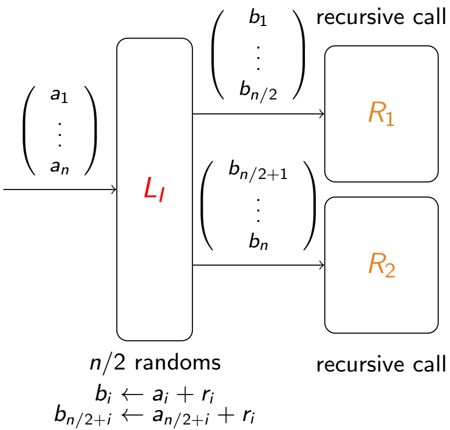

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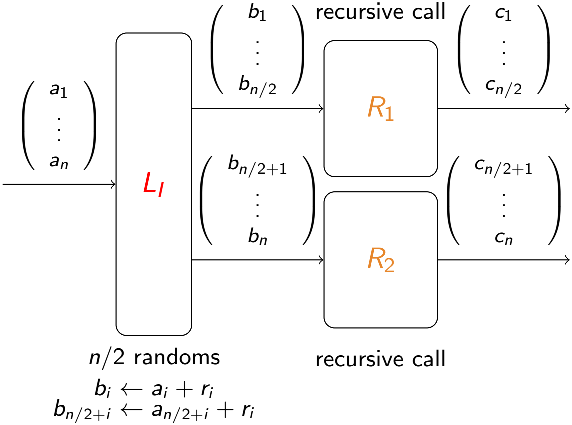

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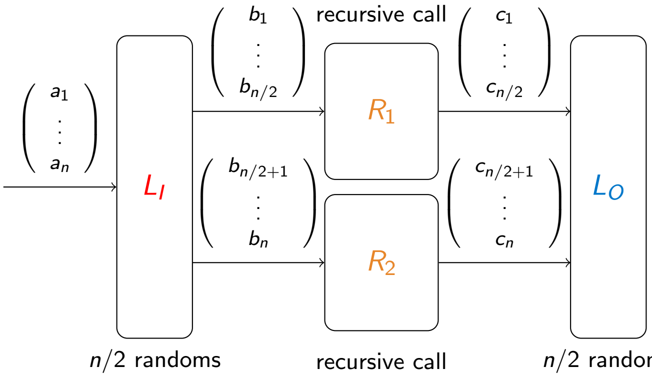

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$n/2$ randoms

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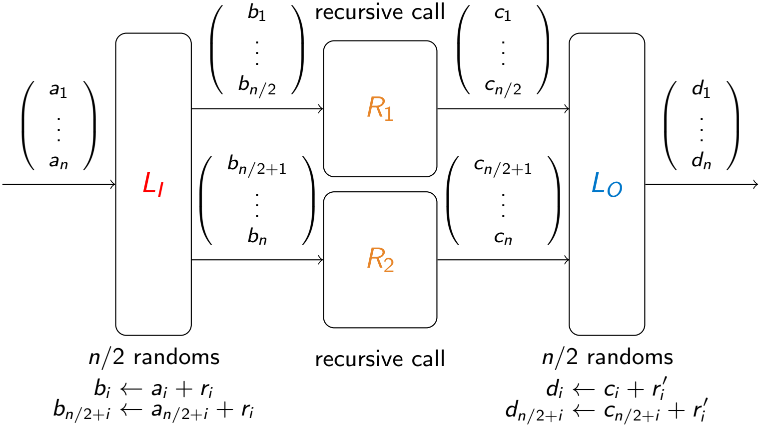

Linear Gadgets

Building Block

Example (4 shares):

$$d_1 \leftarrow (a_1 + r_1) + r_3 + r_5$$
$$d_2 \leftarrow (a_2 + r_2) + r_3 + r_6$$
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Linear Gadgets
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- proven by *Battistello et al.* to be $(n-1)$-SNI in the probing model

Linear Gadgets
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$$

- proven by *Battistello et al.* to be $(n-1)$-SNI in the probing model
- proven in **our work** to satisfy stronger requirements to be used as a building block for RPE secure constructions (extension of requirements proposed by *Belaïd et al. - EuroCrypt 2021*)

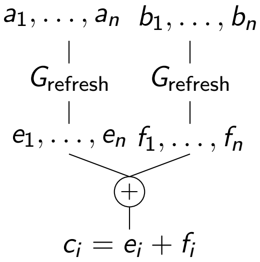
Linear Gadgets

Constructions

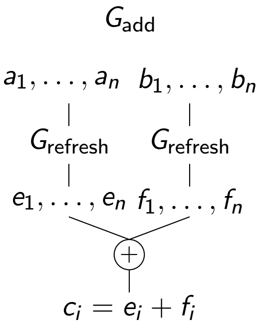
Using $\mathcal{O}(n \log n)$ G_{refresh}

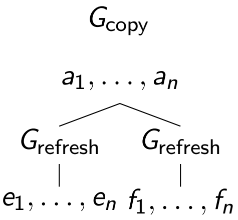
Using $\mathcal{O}(n \log n)$ G_{refresh}

G_{add}

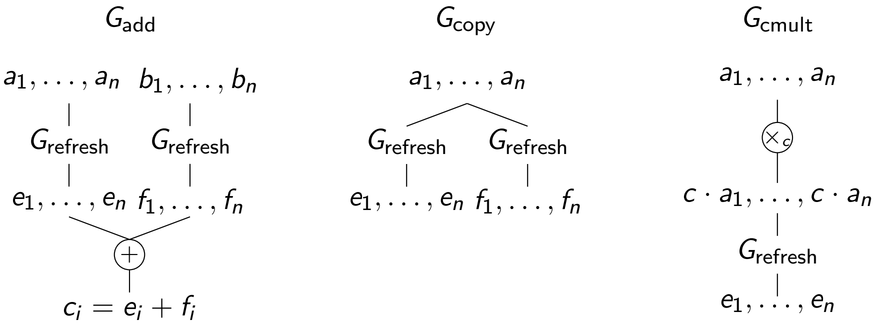


Using $\mathcal{O}(n \log n)$ G_{refresh}





Using $\mathcal{O}(n \log n)$ G_{refresh}



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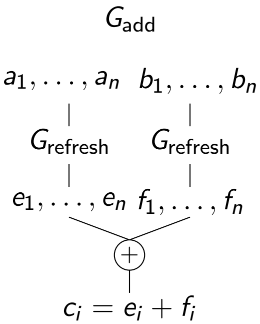

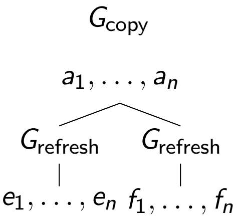

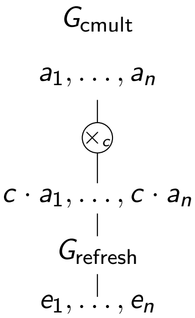

- Complexity in $\mathcal{O}(n \log n)$

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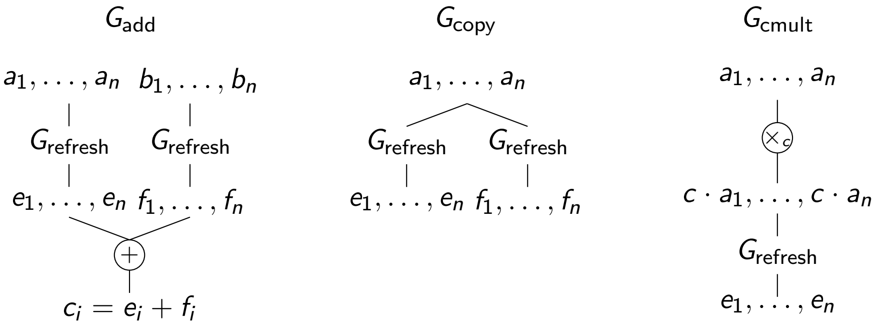

- Complexity in $\mathcal{O}(n \log n)$
- RPE secure with $d = d_{\max} = \dfrac{n+1}{2}$

Multiplication Gadget

Construction from $G_{\text{submult}}, G_{\text{compress}}$

G_{mult} (over $\mathbb{K}$) construction from 2 subgadgets

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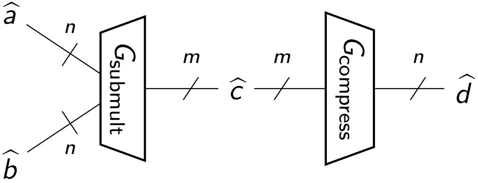

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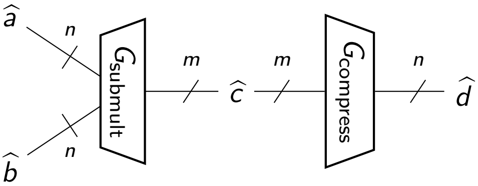

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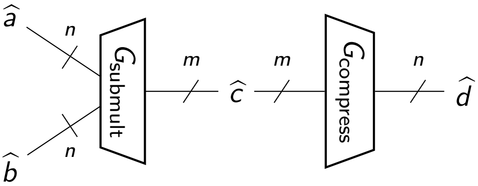

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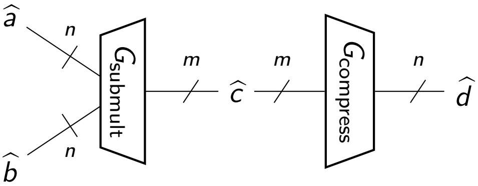

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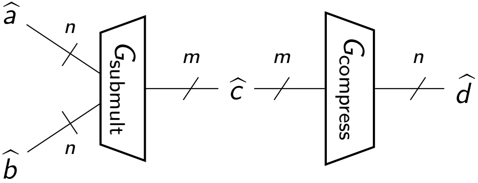

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Multiplication Gadget

Extension of G_{submult} by *Belaïd et al. - Crypto 2017*

Inputs a,b (illustration with 3 shares), field $\mathbb{K}$

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$$\gamma = \begin{pmatrix} \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} \qquad \delta = \begin{pmatrix} 1-\gamma_{1,1} & 1-\gamma_{2,1} & 1-\gamma_{3,1} \\ 1-\gamma_{1,2} & 1-\gamma_{2,2} & 1-\gamma_{3,2} \\ 1-\gamma_{1,3} & 1-\gamma_{2,3} & 1-\gamma_{3,3} \end{pmatrix}$$

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$$c_1 \leftarrow \big((r_1 + a_1) + (r_2 + a_2) + (r_3 + a_3)\big) \cdot \big((s_1 + b_1) + (s_2 + b_2) + (s_3 + b_3)\big)$$

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$$c_3 \leftarrow -r_2 \cdot ((\delta_{2,1} \cdot s_1 + b_1) + (\delta_{2,2} \cdot s_2 + b_2) + (\delta_{2,3} \cdot s_3 + b_3))$$

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$$c_7 \leftarrow -s_3 \cdot ((\gamma_{3,1} \cdot r_1 + a_1) + (\gamma_{3,2} \cdot r_2 + a_2) + (\gamma_{3,3} \cdot r_3 + a_3))$$

Multiplication Gadget

Extension of G_{submult} by *Belaïd et al. - Crypto 2017*

G_{submult}

Multiplication Gadget

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- uses $2n$ random values

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Multiplication Gadget

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Multiplication Gadget

Extension of G_{submult} by *Belaïd et al. - Crypto 2017*

G_{submult}

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- outputs $2n+1$ shares
- performs $2n+1$ multiplications operations
- performs $2n^2$ multiplications by a constant
- is proven to be secure for G_{mult} RPE secure construction, **for the right choice of constants in** γ (can be chosen uniformly at random if the field is large enough)

Multiplication Gadget

New Construction of G_{compress}

The $[m:n]$-compression gadget proposed by *Belaïd et al. - Crypto 2017* is not secure as claimed

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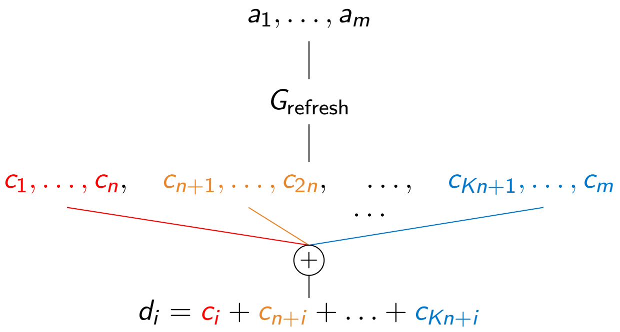
New Compression gadget



New G_{compress}

Multiplication Gadget

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Multiplication Gadget

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Multiplication Gadget
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- is RPE secure with amplification order $d = d_{\max} = \dfrac{n+1}{2}$

New RPE Compiler

With Quasi-Linear Asymptotic Complexity

New Linear gadgets $G_{\mathsf{add}}, G_{\mathsf{copy}}, G_{\mathsf{cmult}}$ with $\mathcal{O}(n \log n)$ complexity

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Complexity of expansion of a circuit C:

$$\mathcal{O}(|C|.\kappa^{e}), \quad e = \frac{log(\mathrm{N}_{\mathsf{max}})}{log(\mathrm{d})}$$

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$\mathbf{N_{max}} \approx \max(\# \times \text{ in } G_{\mathsf{mult}},\ \#(+, ||) \text{ in } G_{\mathsf{add}}, G_{\mathsf{copy}},\ \# \times_c \text{ in } G_{\mathsf{cmult}}) = \mathcal{O}(n \log n)$

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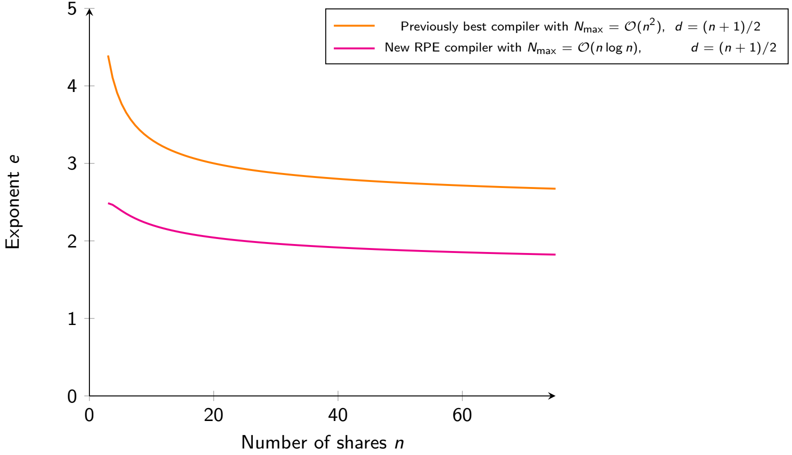

Conclusion

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- Future work: Find gadgets with small nb. of shares (*e.g.* 3 shares) which tolerate the **best possible** leakage rate