On the Power of Expansion

More Efficient Constructions in the Random Probing Model

Sonia Belaïd [1], Matthieu Rivain [1]
and Abdul Rahman Taleb [1,2]

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Side-Channel Attacks & Masking

Security against **side-channel attacks**

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Masking countermeasure (sensitive variable x over field $\mathbb{K}$)

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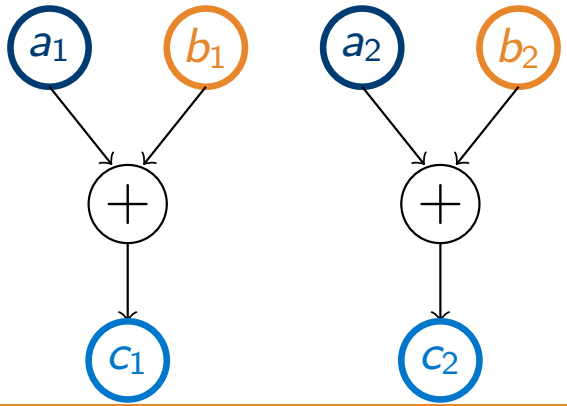

Leakage Models

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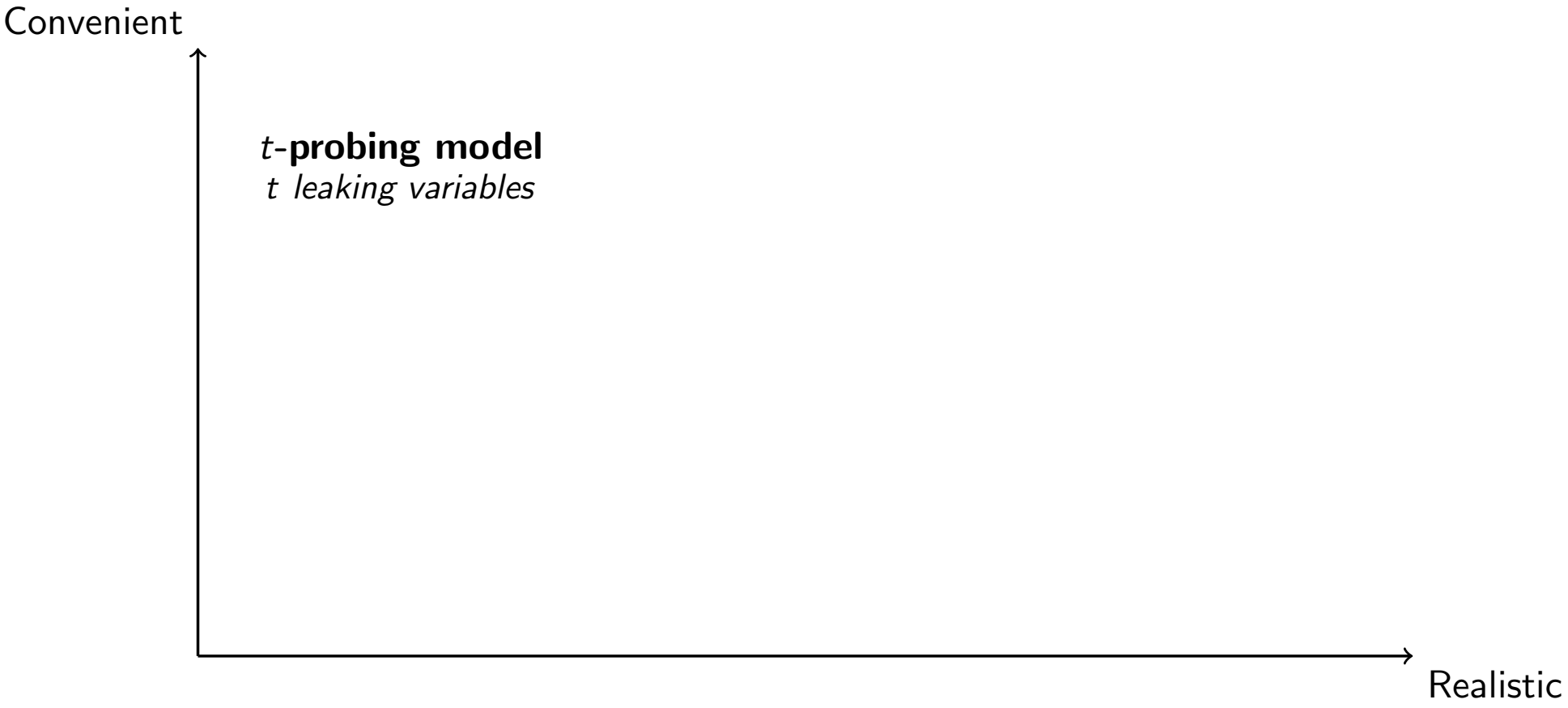

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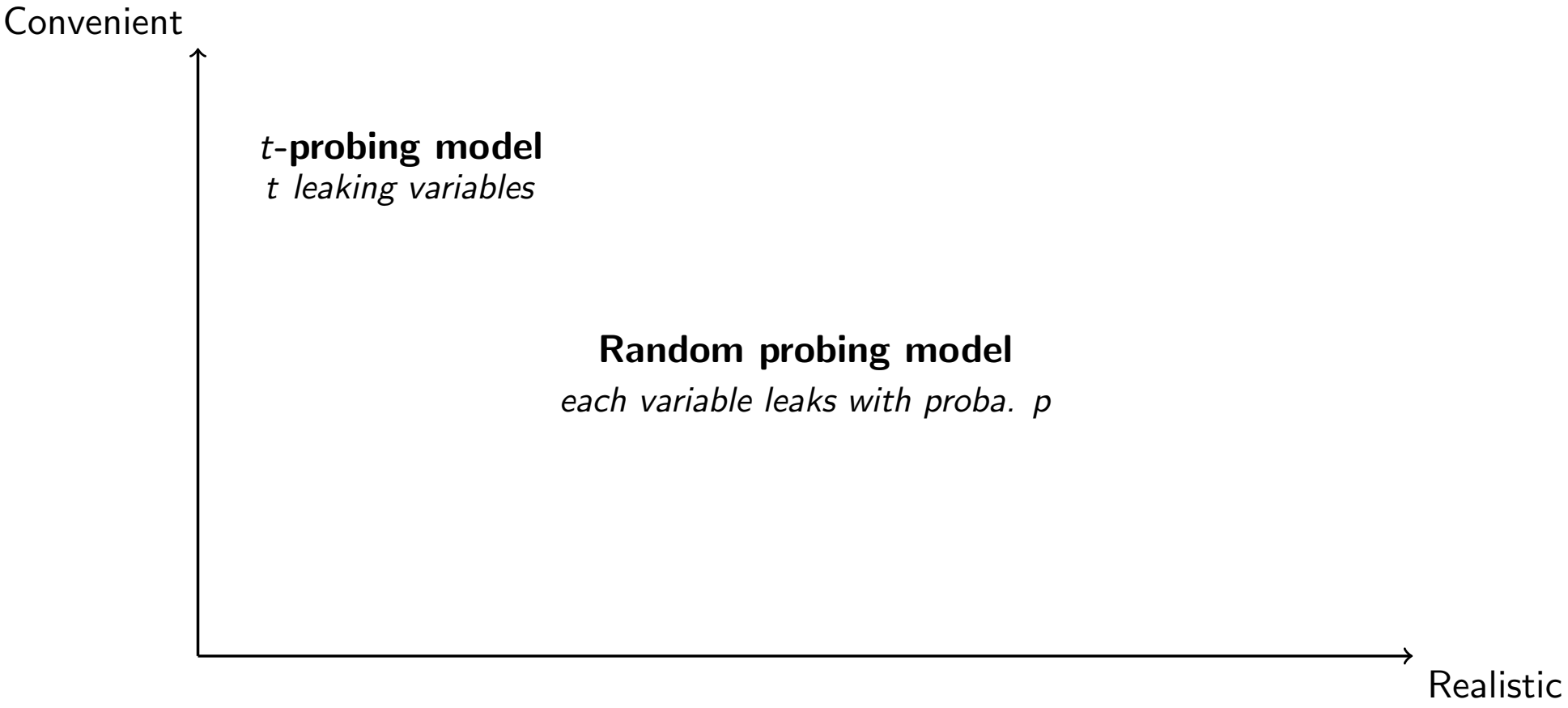

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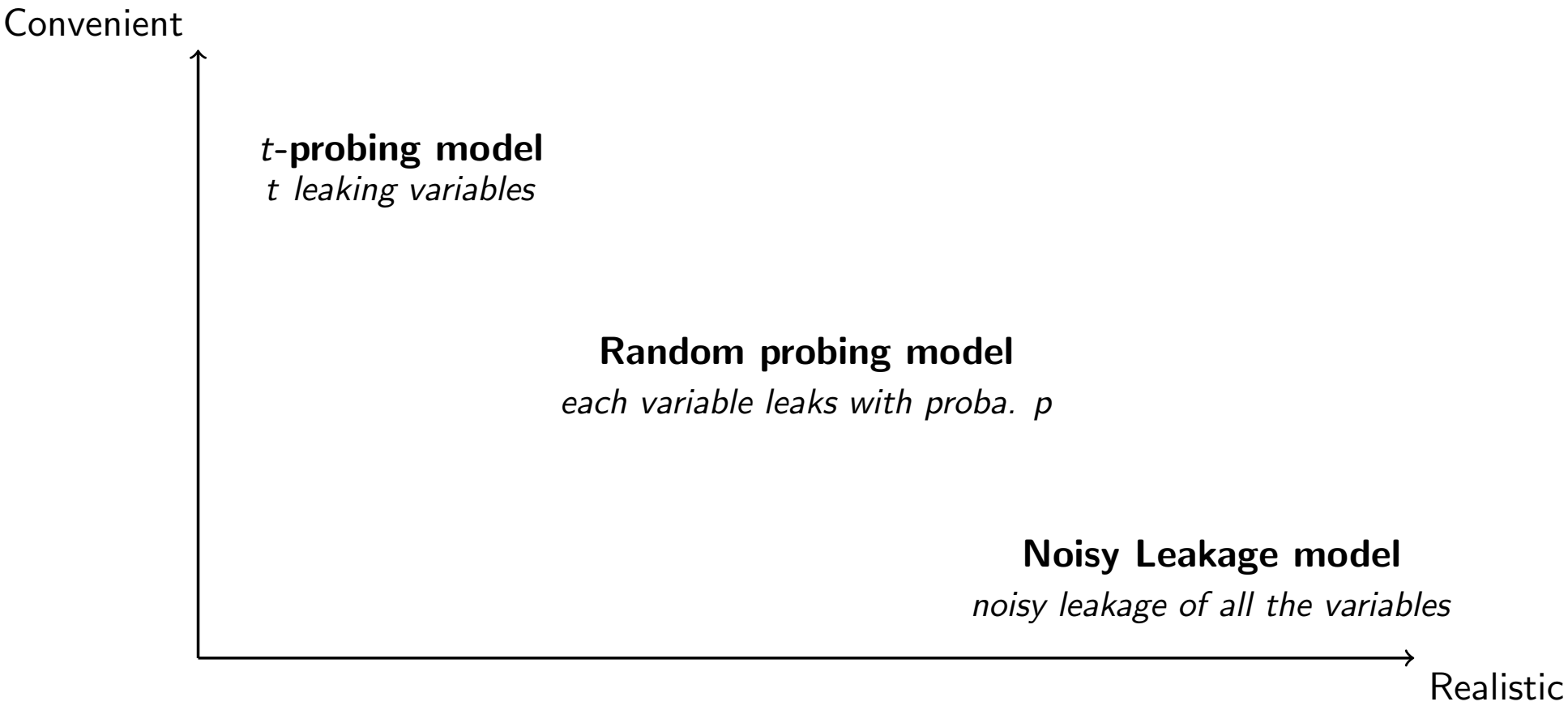

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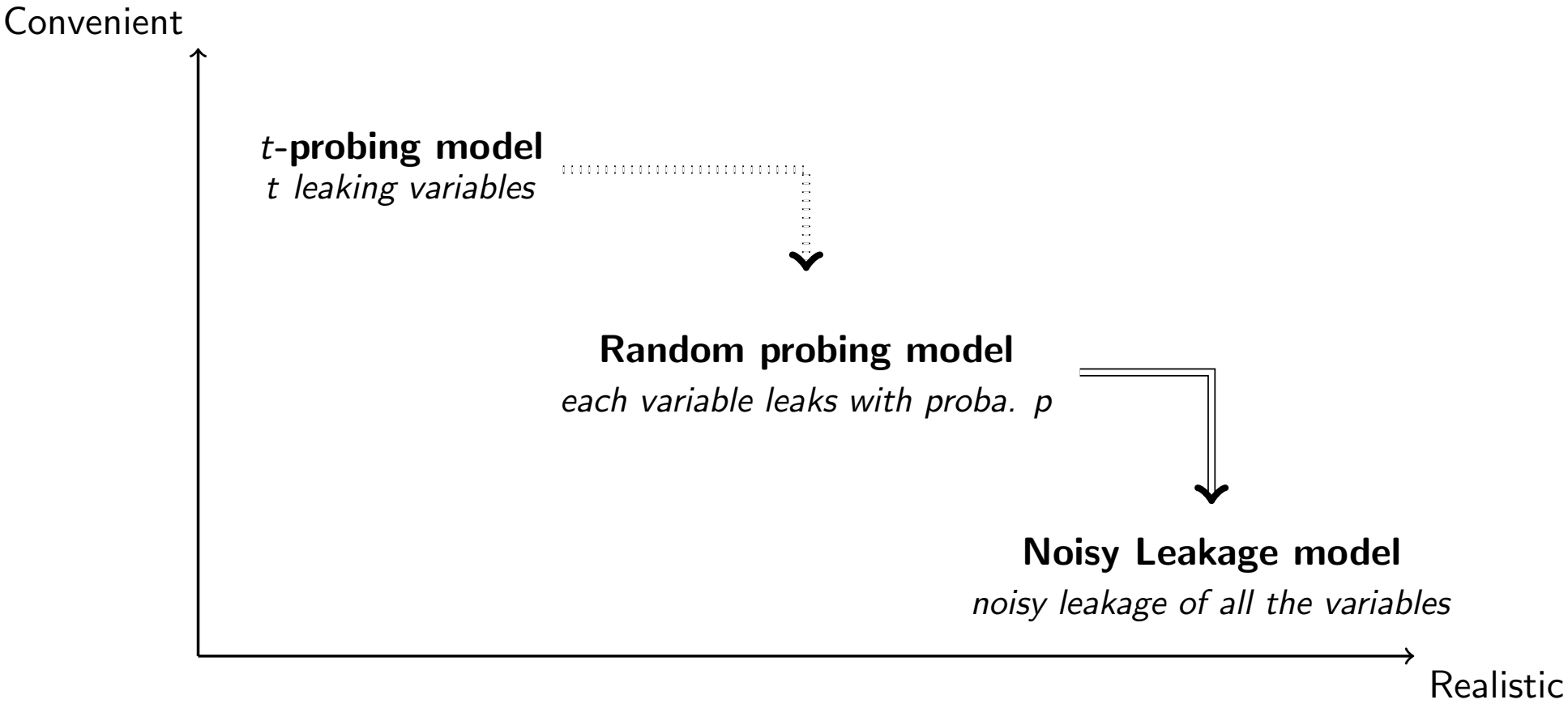

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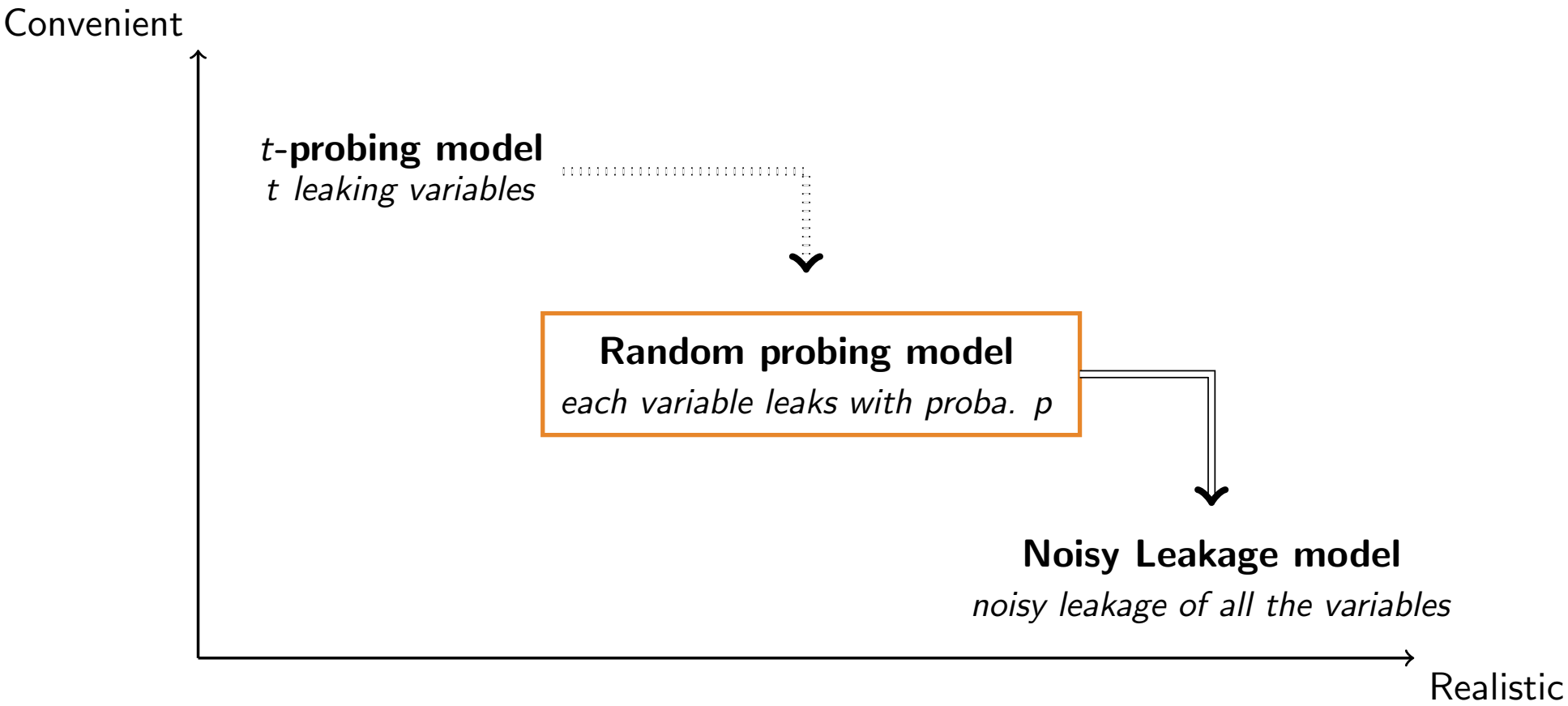

Prior Work

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- Security of masking in the **Random Probing (RP) Model**
- RP-secure gadgets composition (RP composition)
- RP-secure security level amplification (RP expansion)
- **VRAPS:** (V)erifier of (RA)ndom (P)robing (S)ecurity

Contributions

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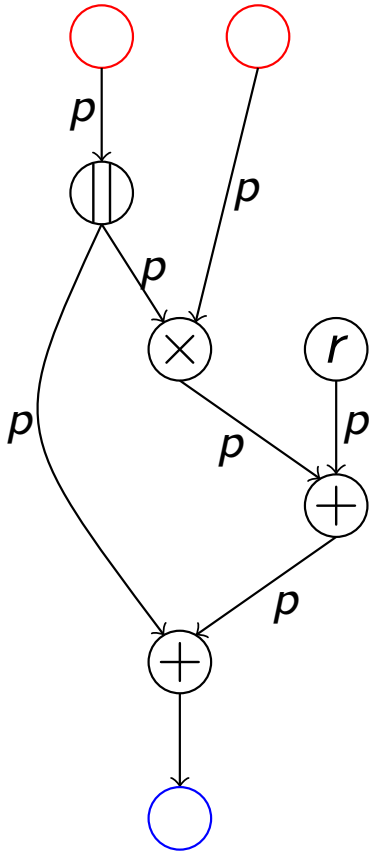
- Concrete instantiations for RP expansion (circuit C, security parameter κ)

Construction	Complexity	Tolerated Leakage rate
[AIS CRYPTO 18] MPC based	$\mathcal{O}(\lvert C\rvert \cdot \kappa^{7.87})$	$p = 2^{-25}$
[CRYPTO 20] 3-share	$\mathcal{O}(\lvert C\rvert \cdot \kappa^{7.5})$	$p = 2^{-8}$
New 3-share	$\mathcal{O}(\lvert C\rvert \cdot \kappa^{3.9})$	$p = 2^{-7.5}$
New 5-share	$\mathcal{O}(\lvert C\rvert \cdot \kappa^{3.2})$	$2^{-12} \leq p \leq 2^{-6}$

RP Security

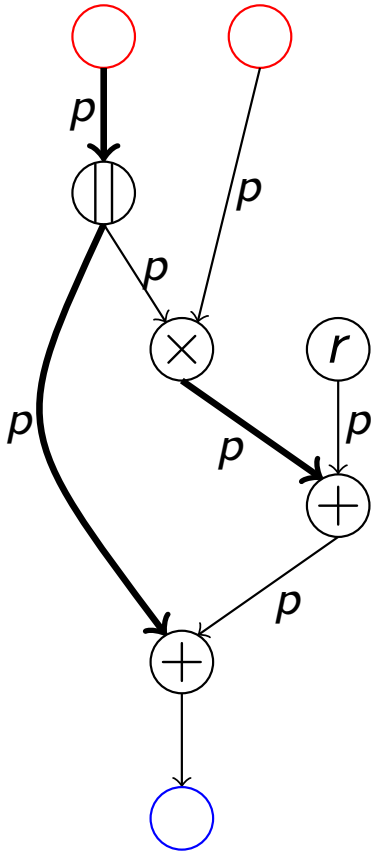


Add Mult.

Copy Random

(p, ε)-RP Security

RP Security



⊕ Add ⊗ Mult.

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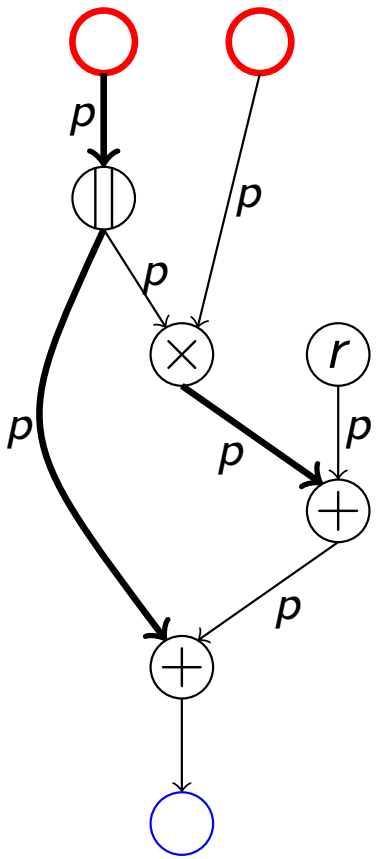
W set of wires

RP Security



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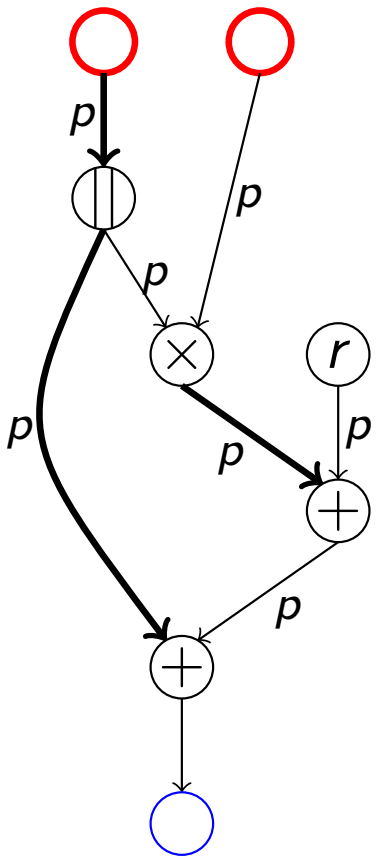
(p, ε)-RP Security

W set of wires

↓

Independent from secret inputs ?

RP Security



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W set of wires

Independent from secret inputs ?

yes no

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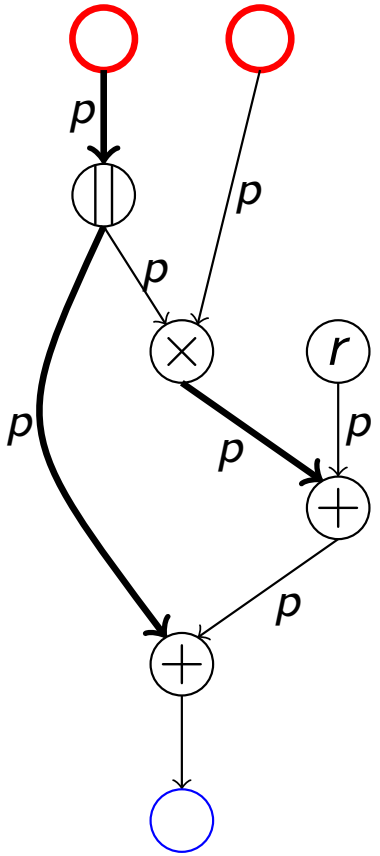

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(p, ε)-RP Security

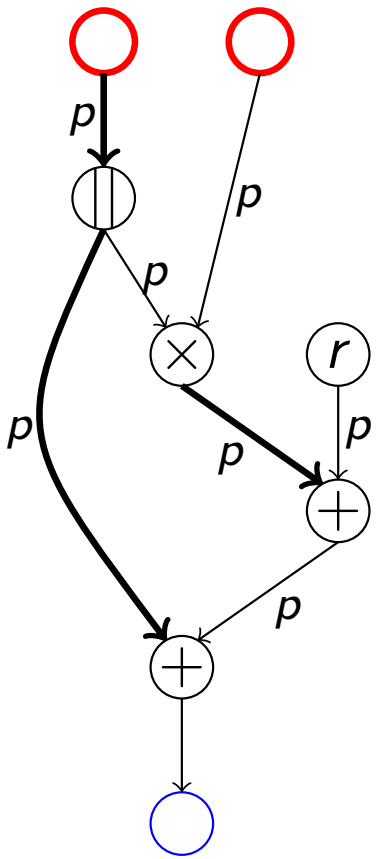
W set of wires

Independent from secret inputs ?

yes no

Simulation Success

RP Security



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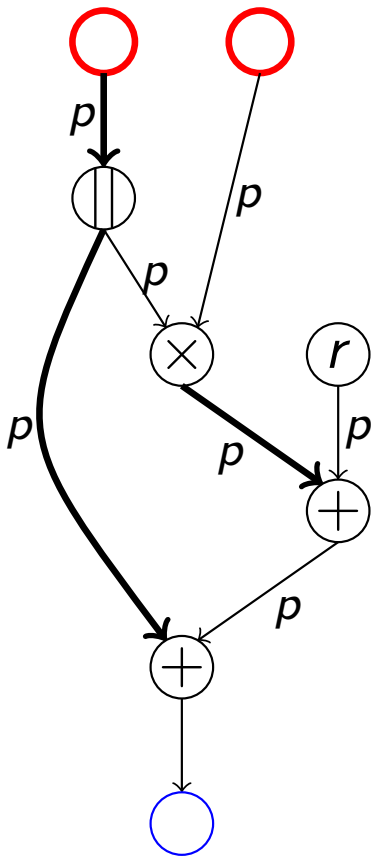
Independent from secret inputs ?

yes no

Simulation Success *Simulation Failure*

RP Security



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(p, ε)-RP Security

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Independent from secret inputs ?

yes — *Simulation Success*

no — *Simulation Failure*

↓

Failure Probability ε

RP Expansion

Illustration

Using n-share gadgets G_{add}, G_{copy}, G_{mult}

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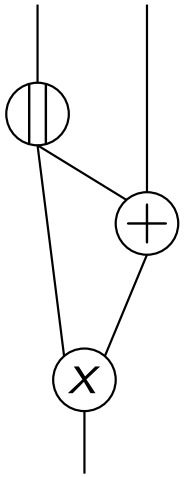

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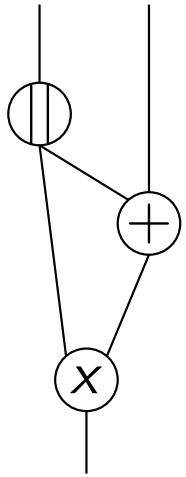

Leakage probability
p

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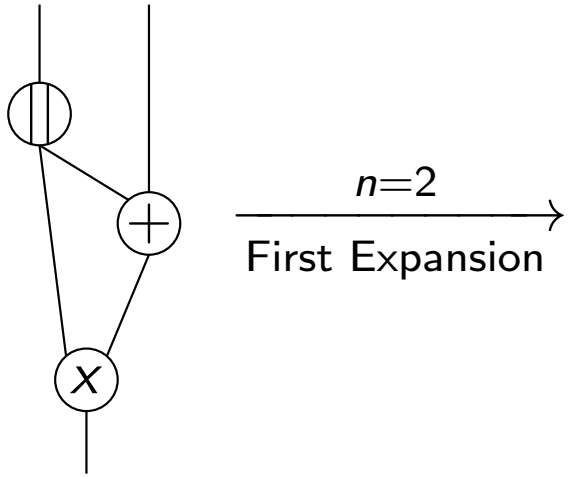

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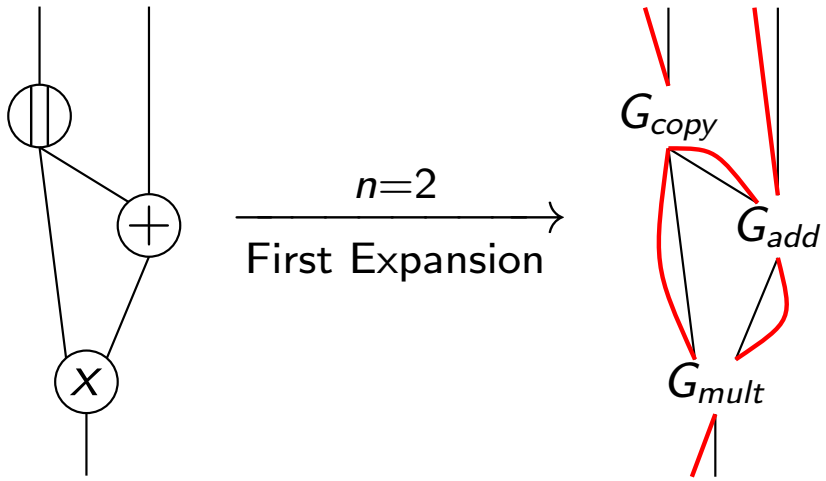

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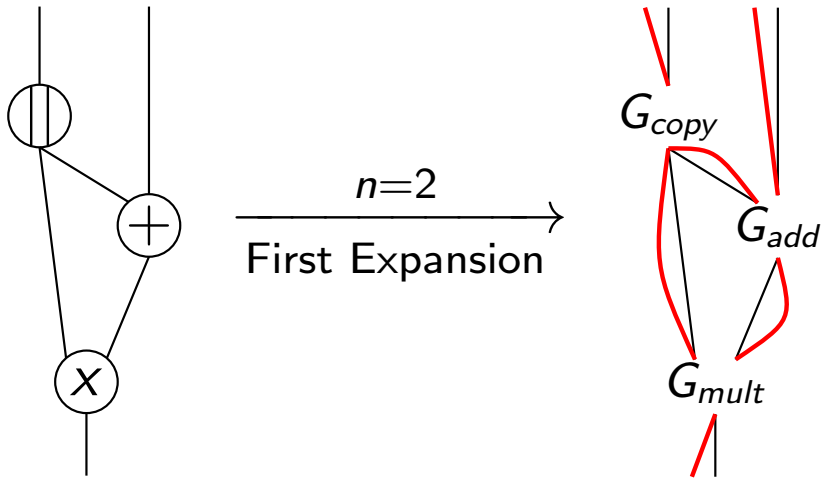

Leakage probability
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Simulation Failure
ε

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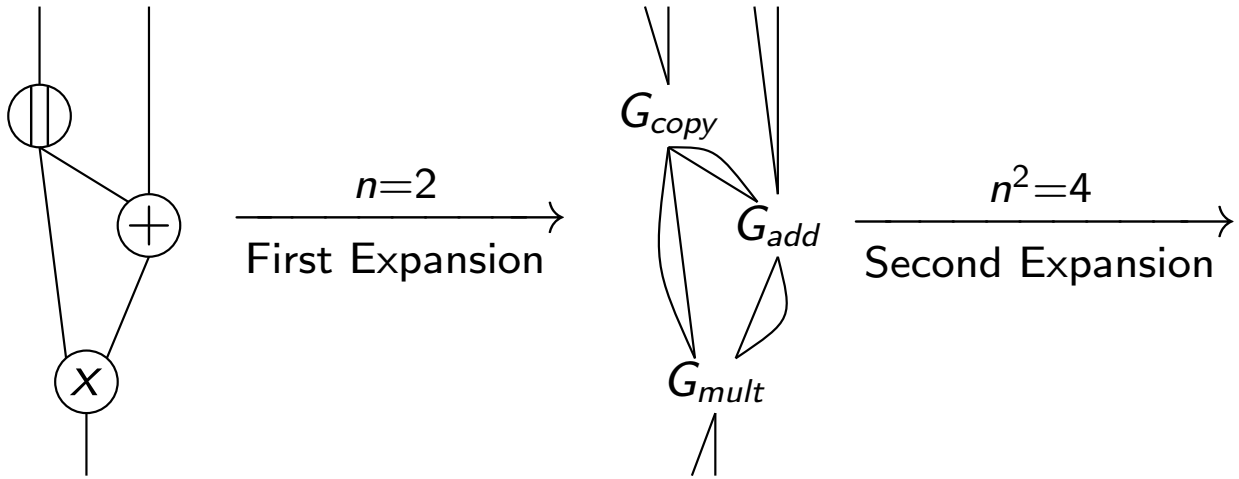

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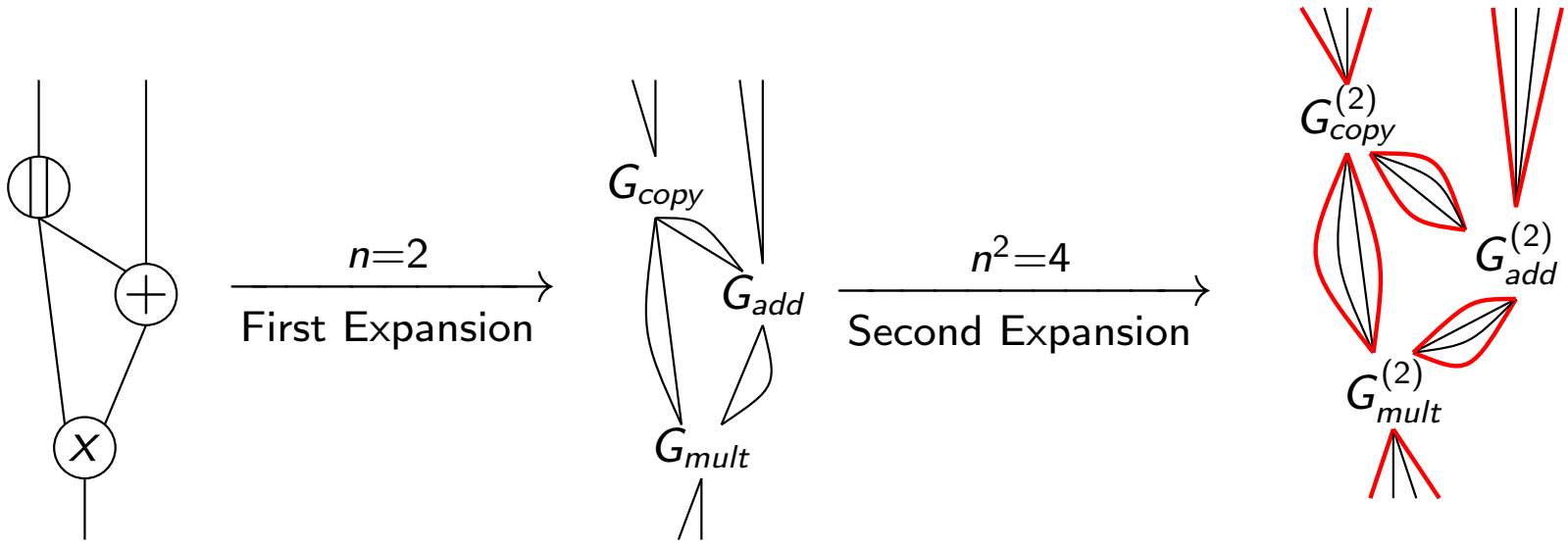

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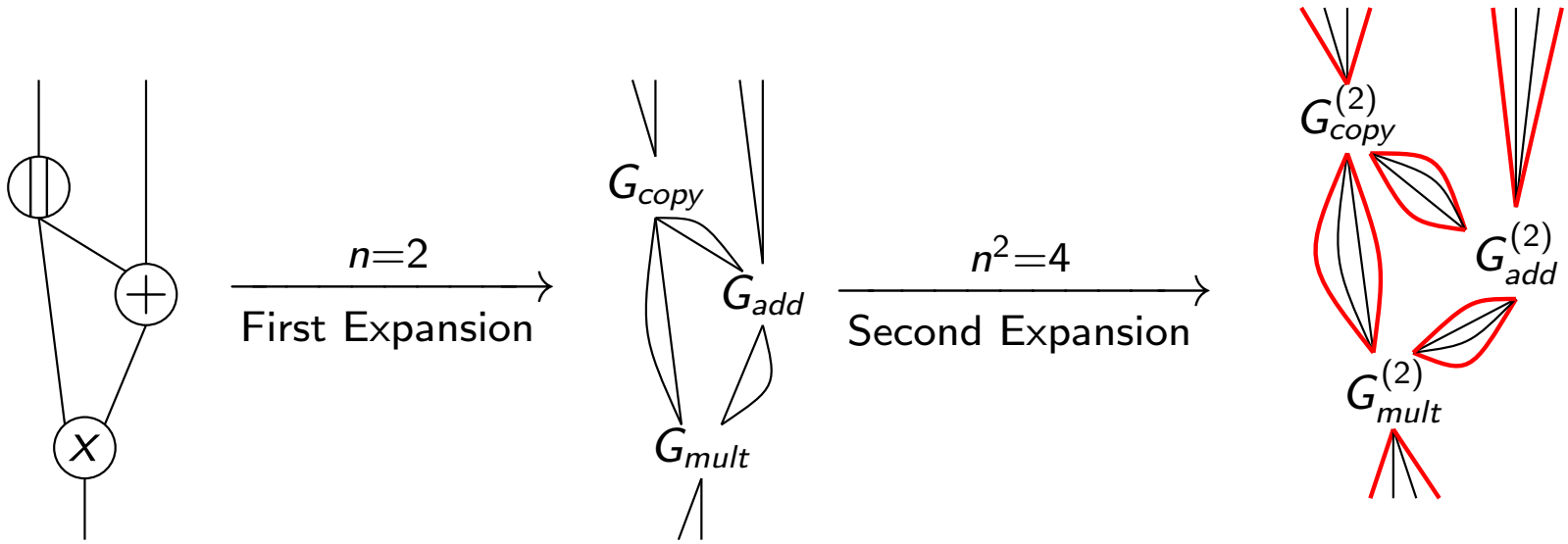

Leakage probability
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Simulation Failure
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ε^2

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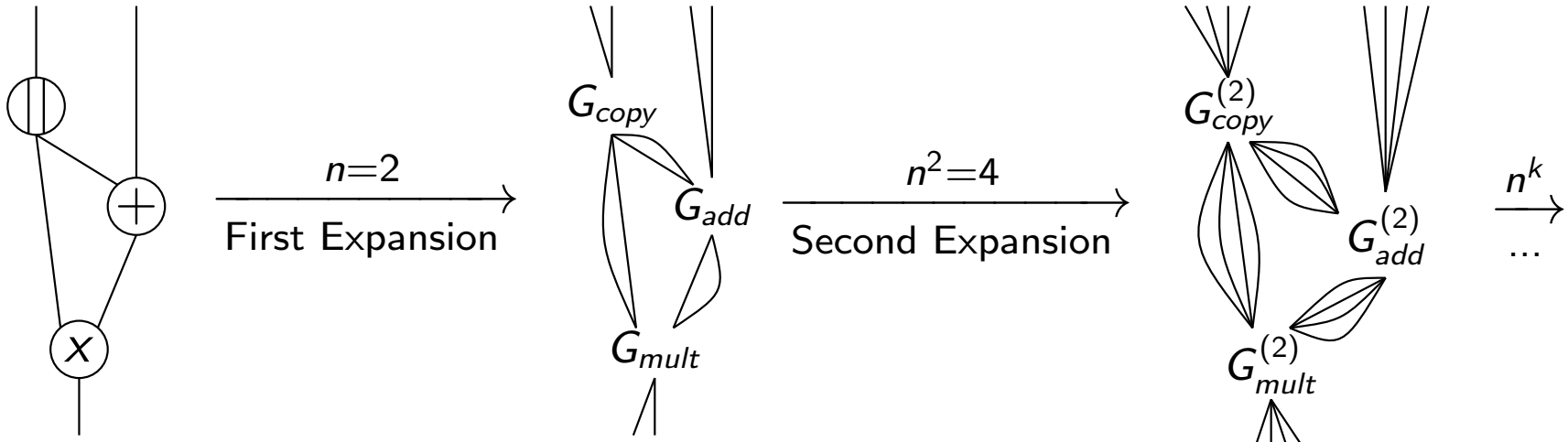

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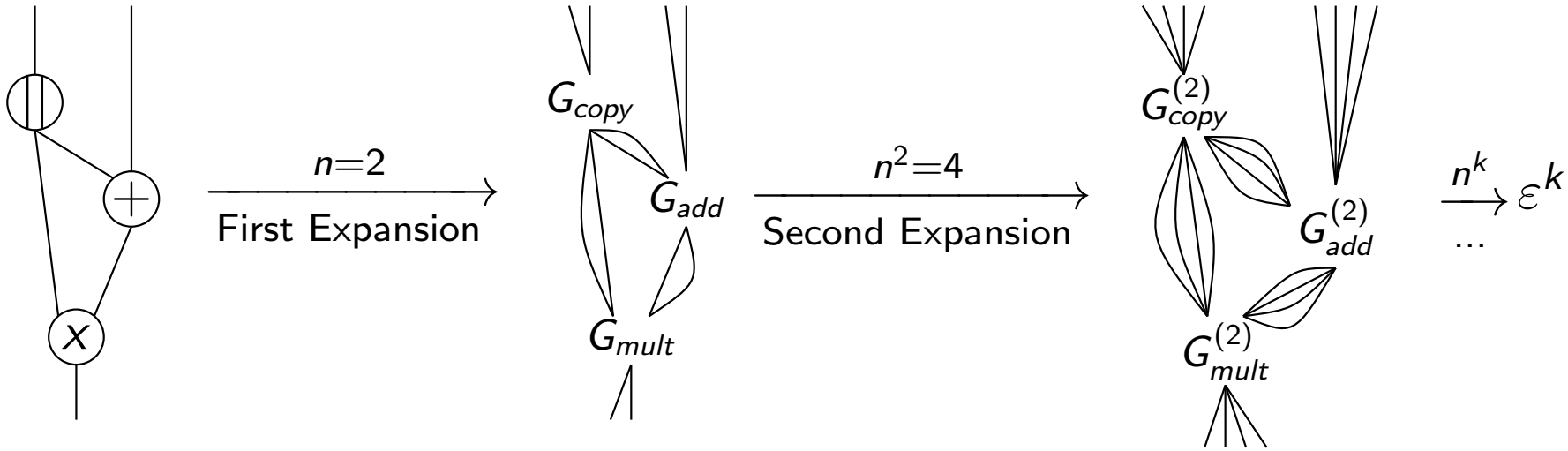

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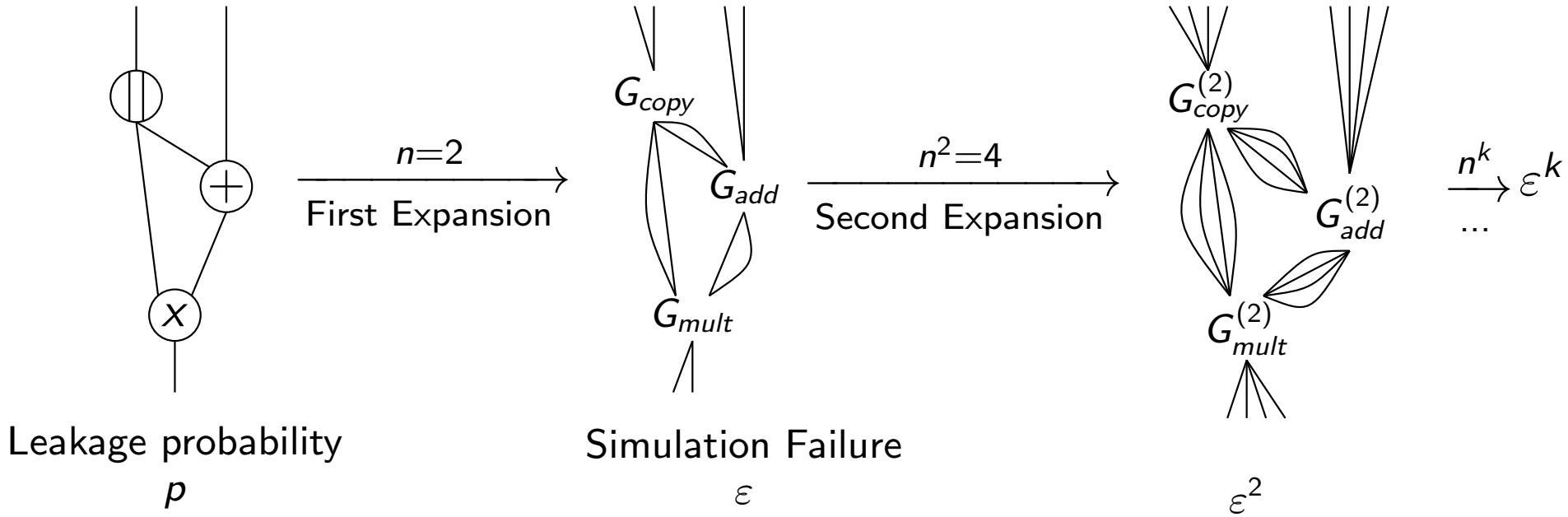

Condition : $\varepsilon < p$ (tolerated leakage rate)

RP Expansion

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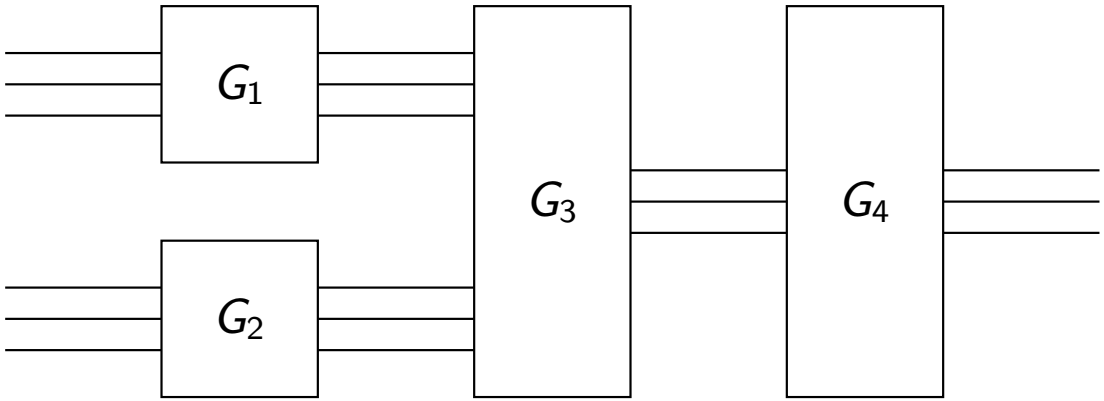

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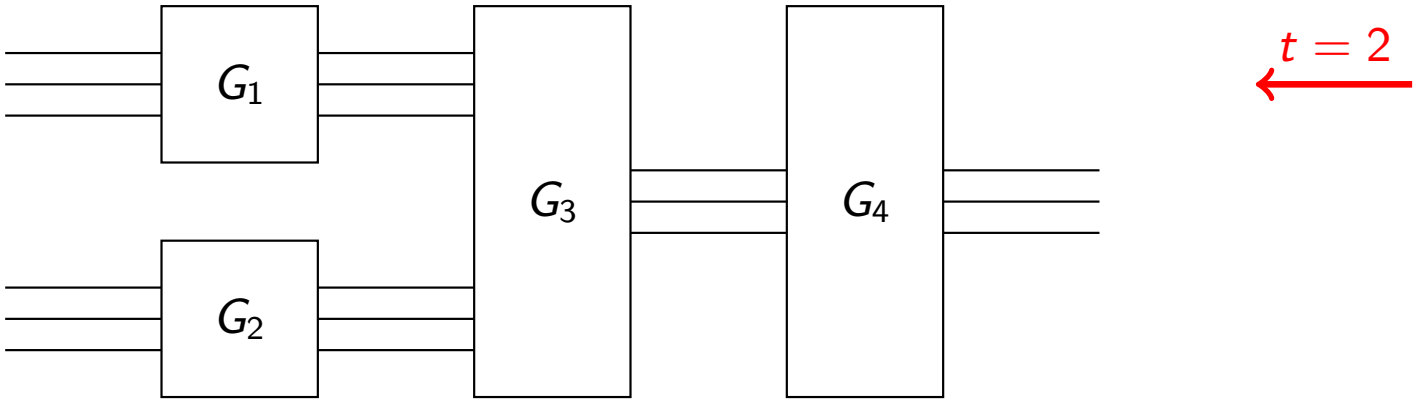

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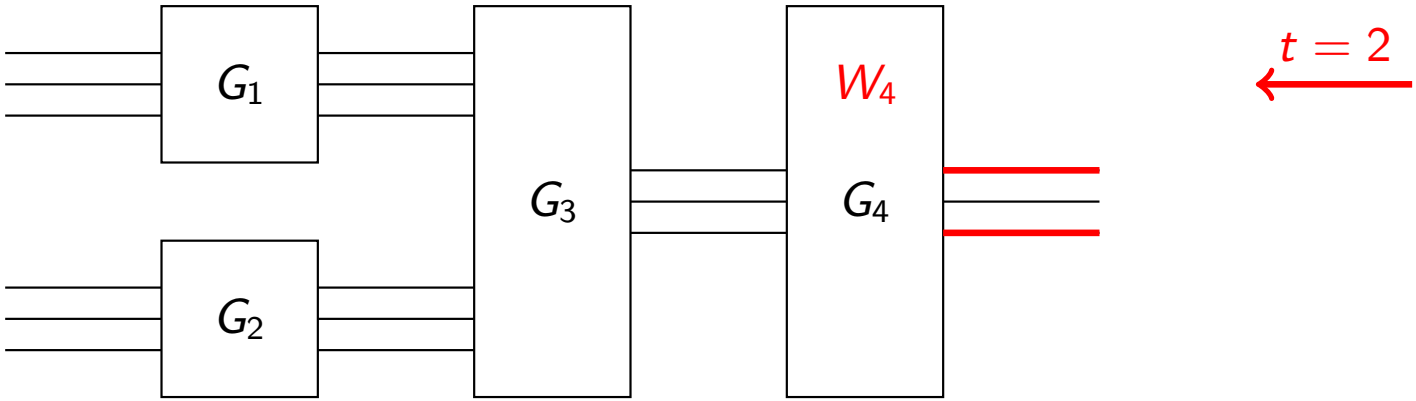

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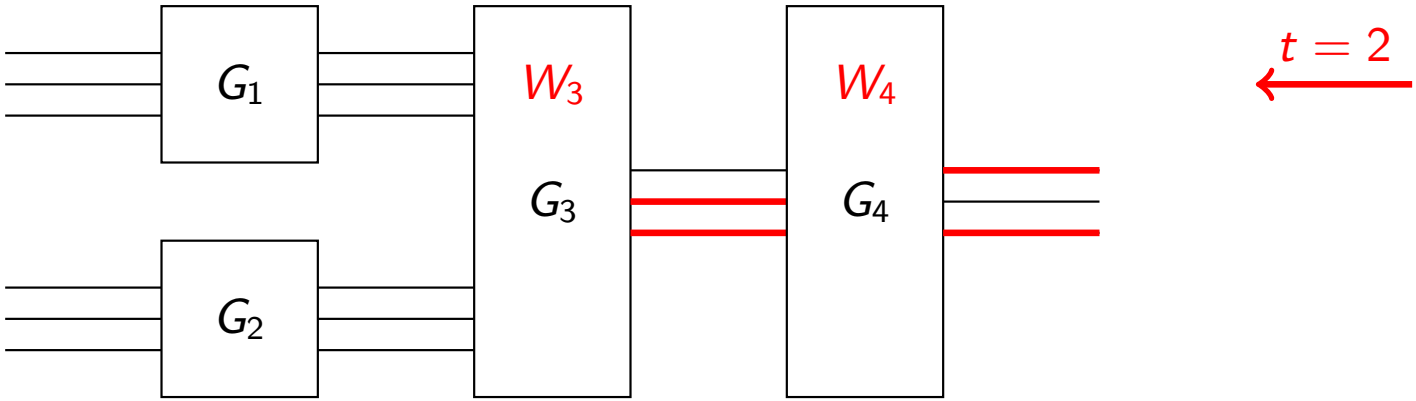

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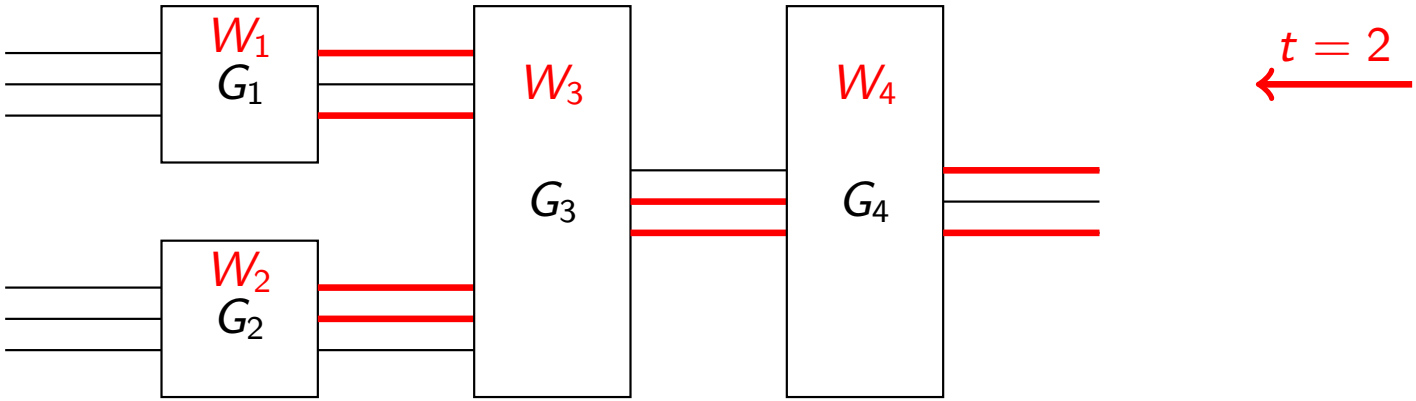

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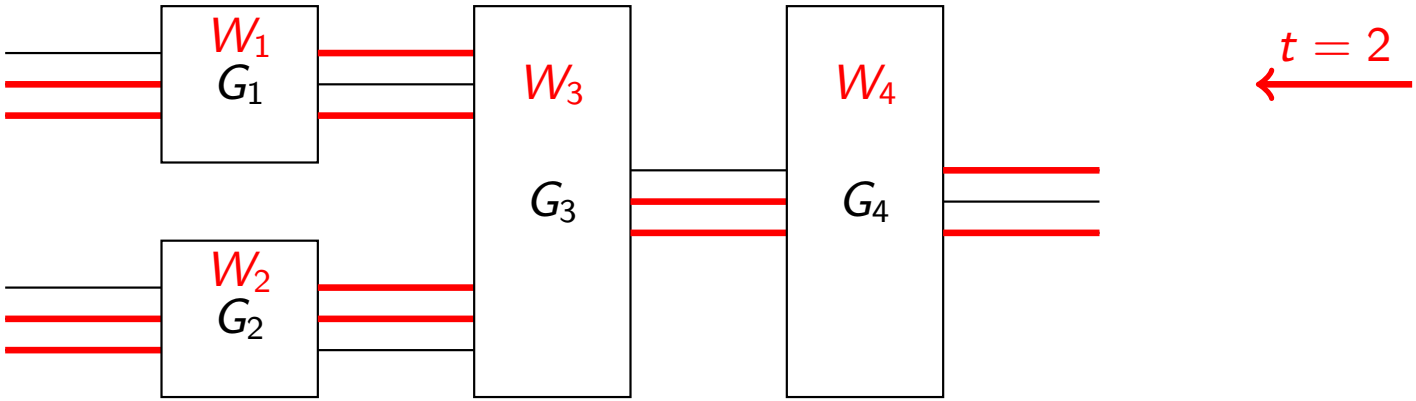

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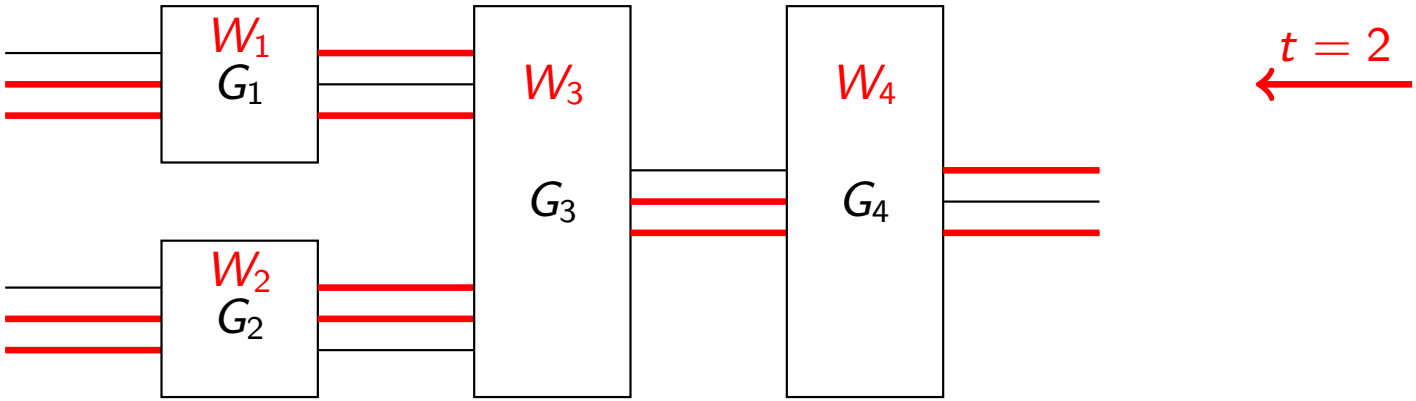

- Independent failure probability on each input sharing

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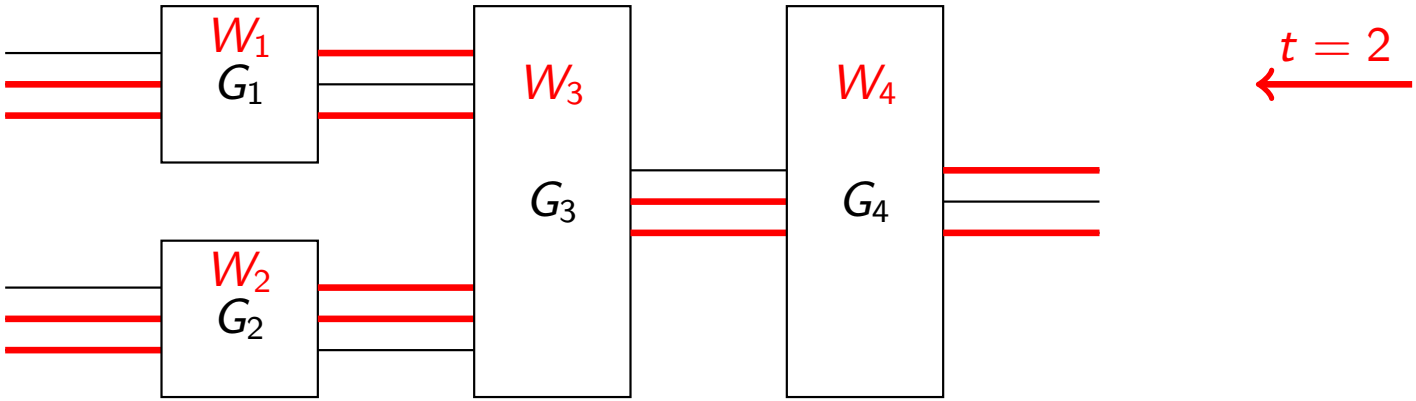

- Independent failure probability on each input sharing
- expansion of G with arbitrary level k

RP Expansion

Security

For an n-share gadget G:

RP Expansion

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$$G \text{ RP-Expandable, } \varepsilon \quad \Longrightarrow \quad G^{(k)} \text{ RP-Expandable, } \varepsilon^k$$

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For a circuit C, using G_{add}, G_{copy}, G_{mult}:

$$G_{add}, G_{copy}, G_{mult} \textbf{ RP-Expandable, } \varepsilon \quad \Longrightarrow \quad \textbf{Compiled circuit } (p, 2.|C|.\varepsilon^k)\textbf{-RP Secure}$$

RP Expansion

Parameters

Security parameter κ, (t, p, ε)-RP expandable gadgets $G_{\mathsf{add}}, G_{\mathsf{copy}}, G_{\mathsf{mult}}$, circuit C to expand, complexity:

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RP Expansion

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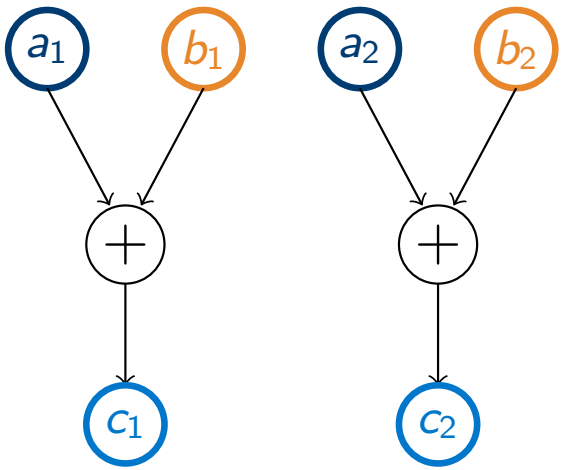

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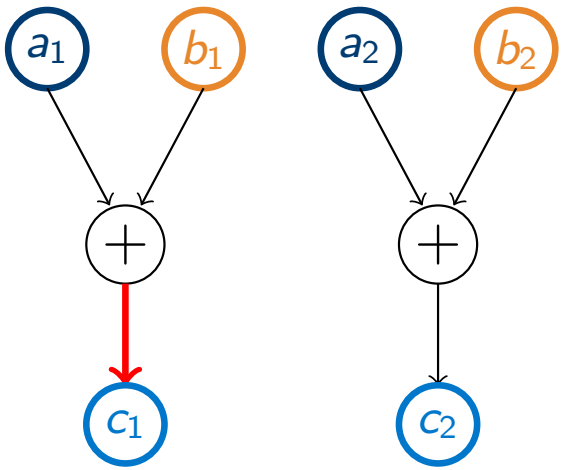

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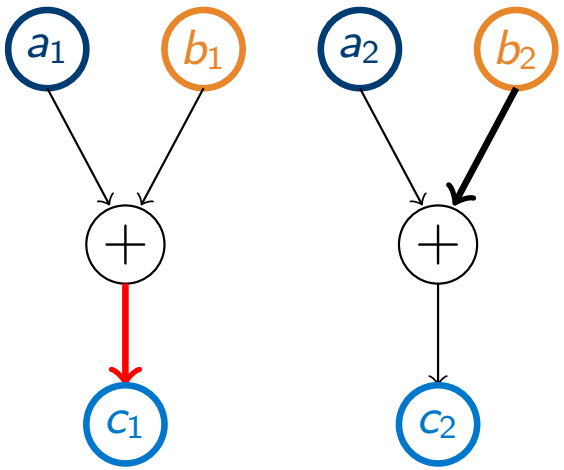

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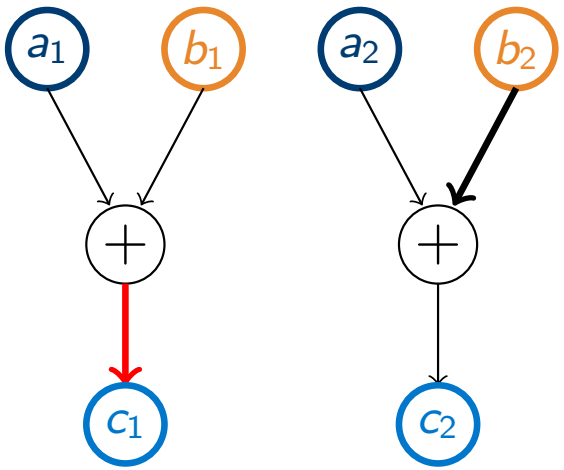

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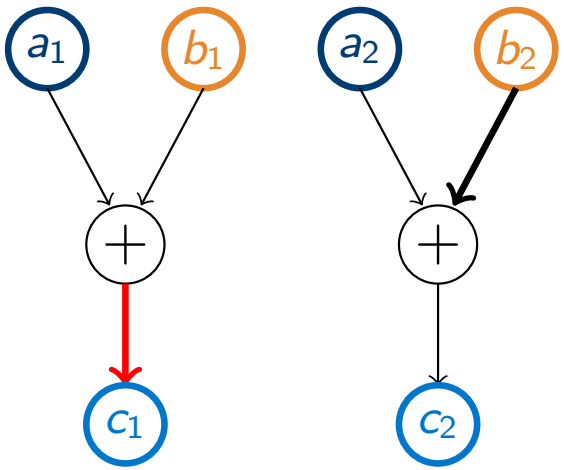

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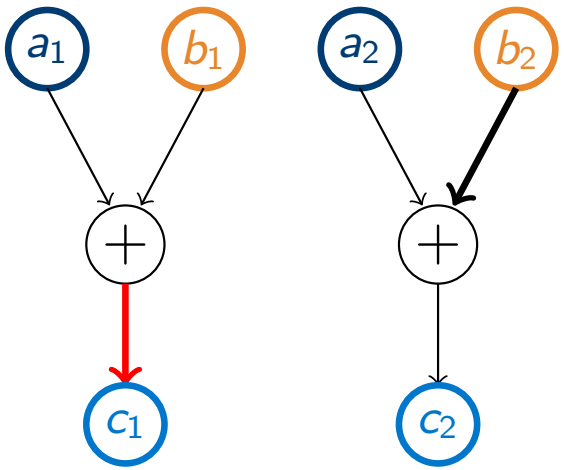

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Failure on $b \implies \mathbf{d} = |W| = \mathbf{1}$

RP Expansion

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- during expansion: $\varepsilon^k = f^{(k)}(p) = f(f(\ldots f(f(p))\ldots))$
- higher **d** $\implies$ faster decrease in failure probability

First Bound on d (with respect to t)

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n-share circuit with input a

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n-share circuit with input a

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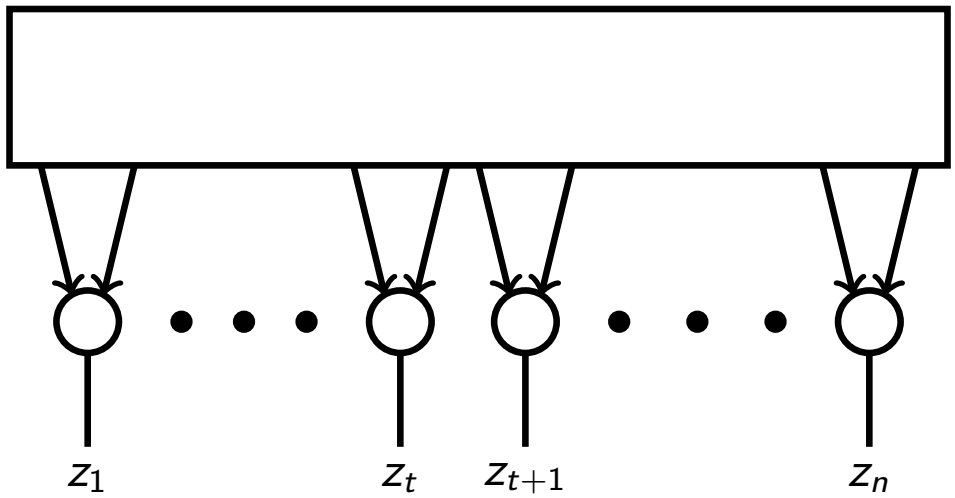
Need all $t+1$ shares to simulate $W \implies$ failure

$$\mathrm{d} \leq \mathrm{t} + 1$$

Second Bound on d (with respect to t)



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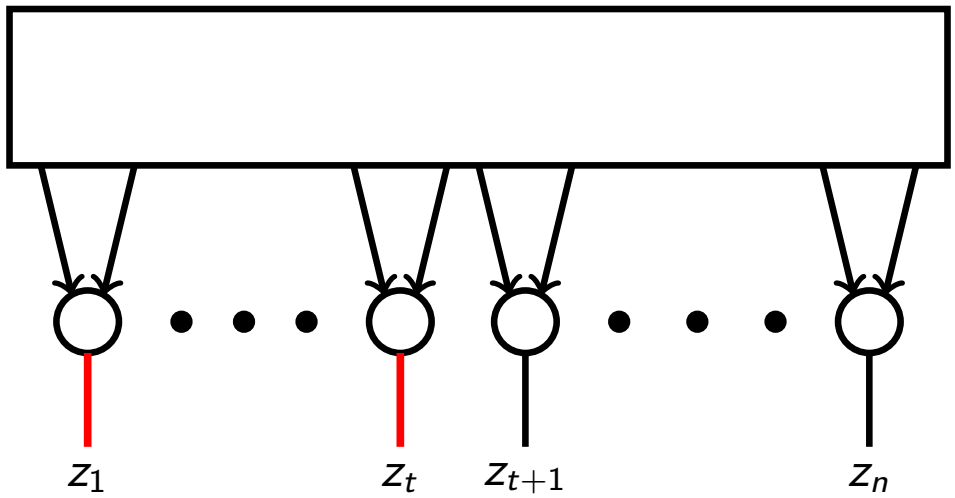

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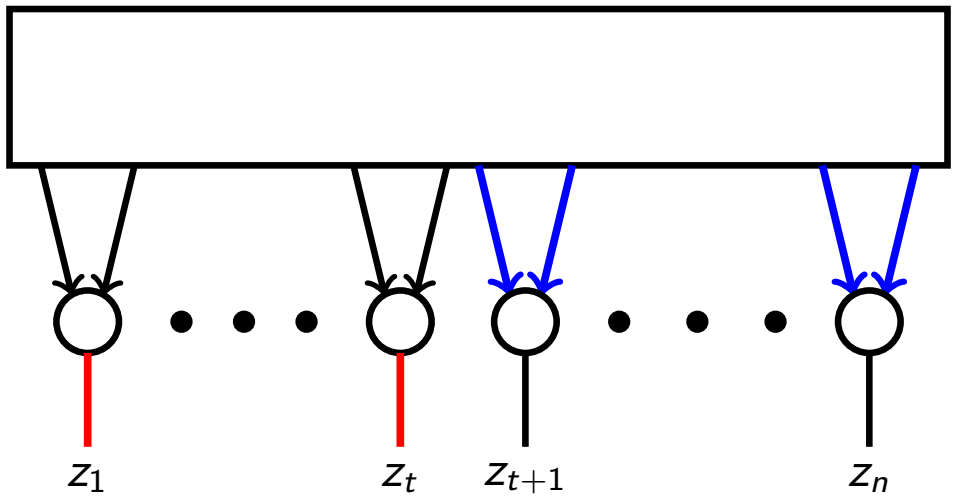

$J = \{1, \ldots, t\}$ $\quad |W| = 2(n - t)$

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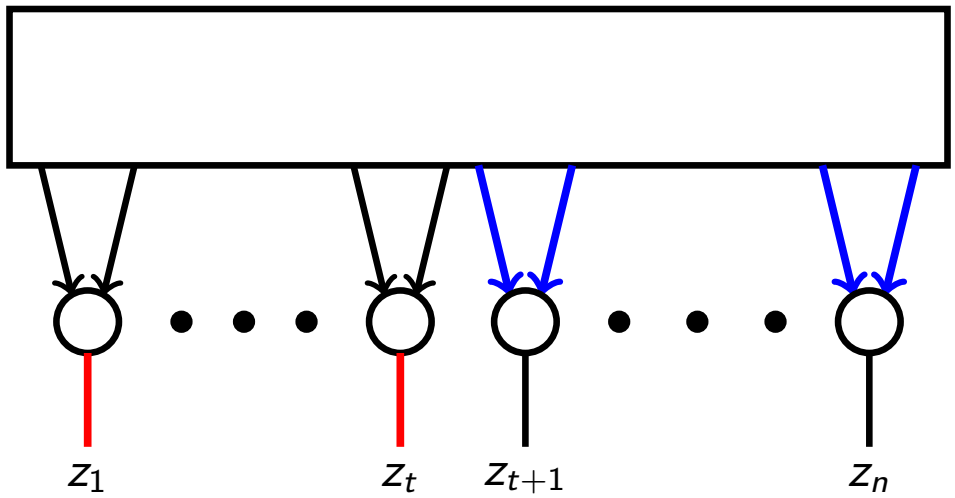

- 1-input gadgets $\longrightarrow$ $d \leq 2(n-t)$

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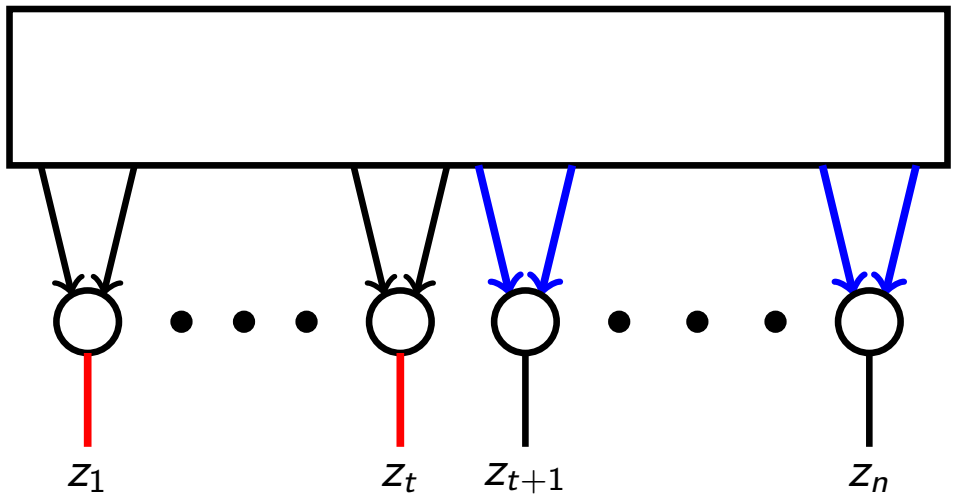

- 1-input gadgets $\longrightarrow$ $d \leq 2(n-t)$
- 2-input gadgets $\longrightarrow$ $d' \leq \frac{d}{2} \leq (n-t)$ (Failure on both inputs with independent failure events)

RP Expansion

Bounding the Amplification Order

(t, p, ε)-RPE compiler with $G_{\mathsf{add}}, G_{\mathsf{copy}}, G_{\mathsf{mult}}$

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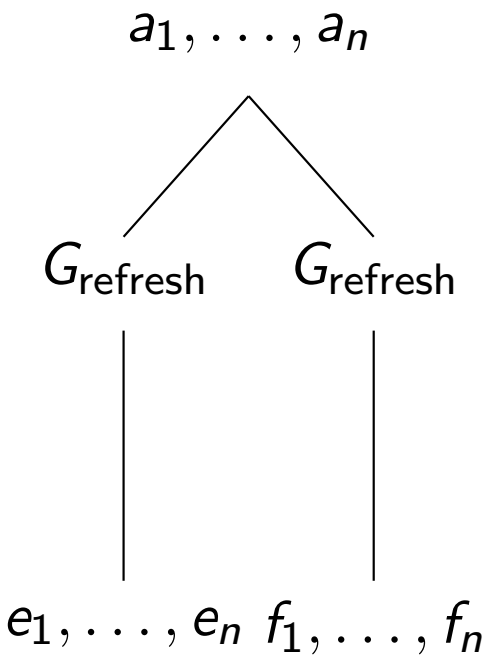
Idea:

- build G_{add}, G_{copy} from single building block G_{refresh} (easier conception)
- use G_{mult} from state of the art (*e.g.* ISW, ...)

Generic Constructions

Copy Gadget



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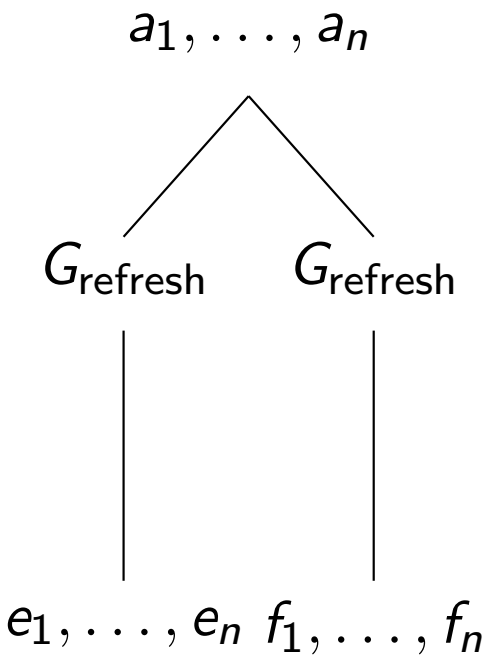

(t, p, ε)-RPE G_{refresh} achieves $d_{\max}$ $\Longrightarrow$
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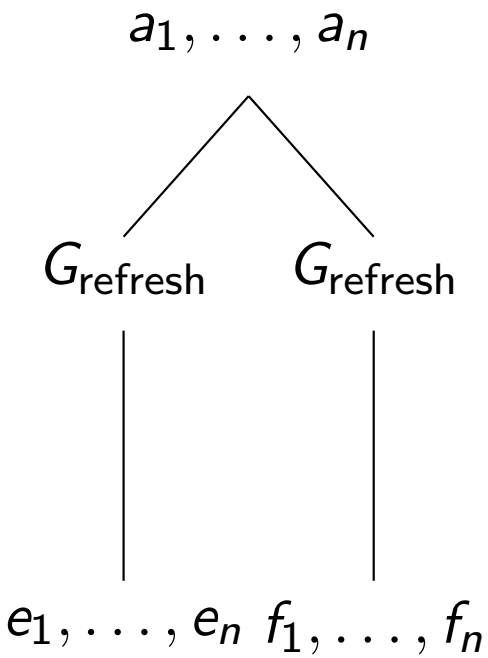

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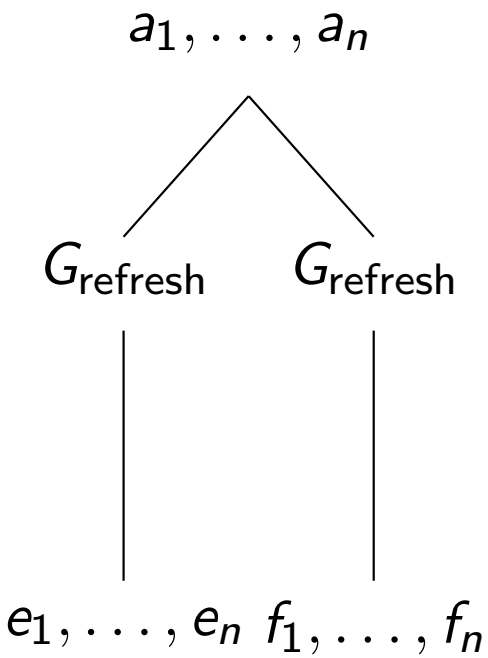

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Idea:

- probes on G_{copy} = **left** probes on G_{refresh} $\cup$ **right** probes on G_{refresh} (independent)

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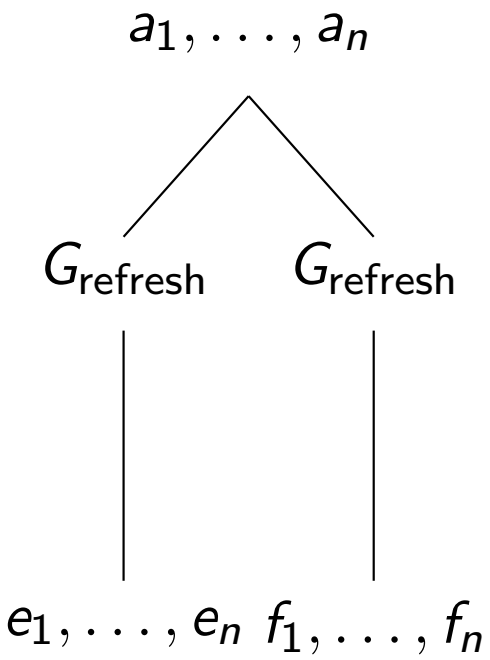

(t, p, ε)-RPE G_{refresh} achieves $d_{\max} \implies$
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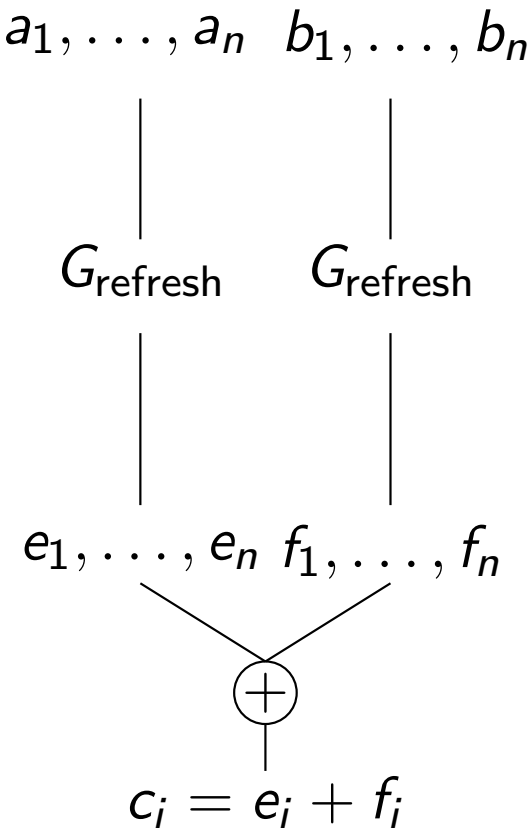
Idea:

- probes on $G_{\text{copy}} =$ **left** probes on $G_{\text{refresh}} \cup$ **right** probes on G_{refresh} (independent)
- **left** simulation success + **right** simulation success $\implies$ overall simulation success

Generic Constructions

Addition Gadget



Generic Constructions

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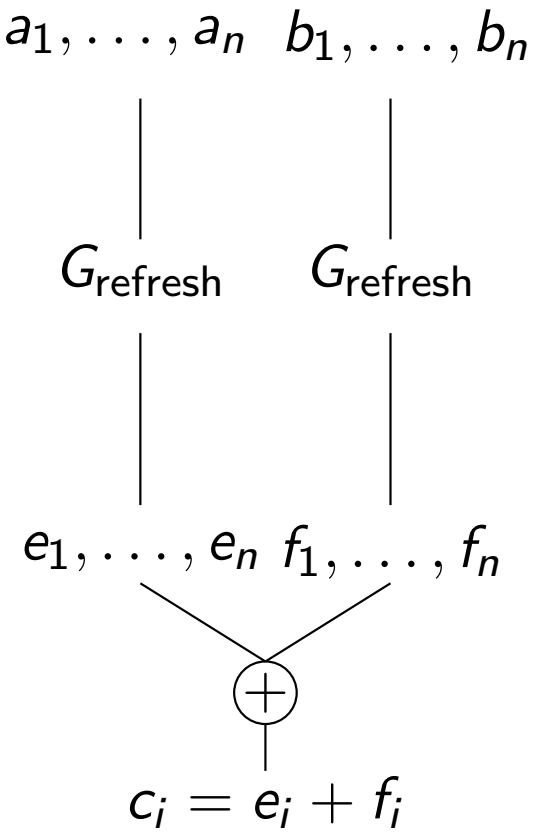

(t, p, ε)-RPE G_{refresh} achieves $d_{\max}$ $\Longrightarrow$

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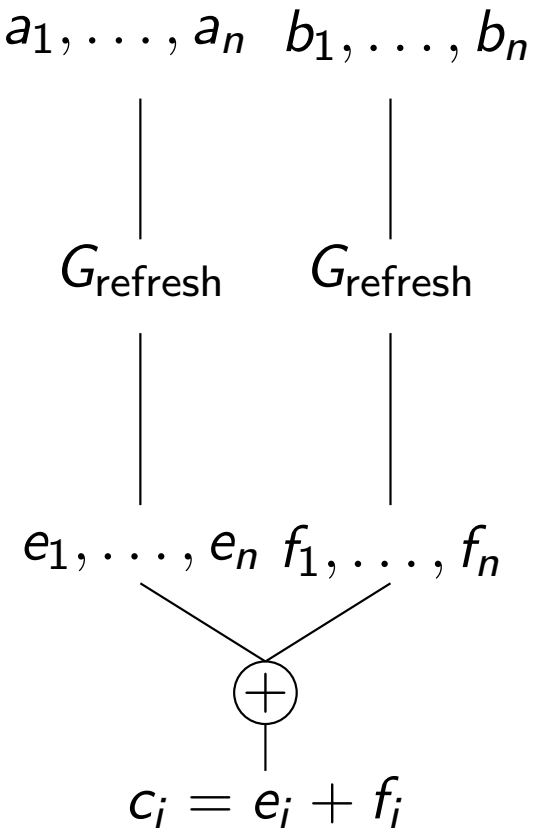

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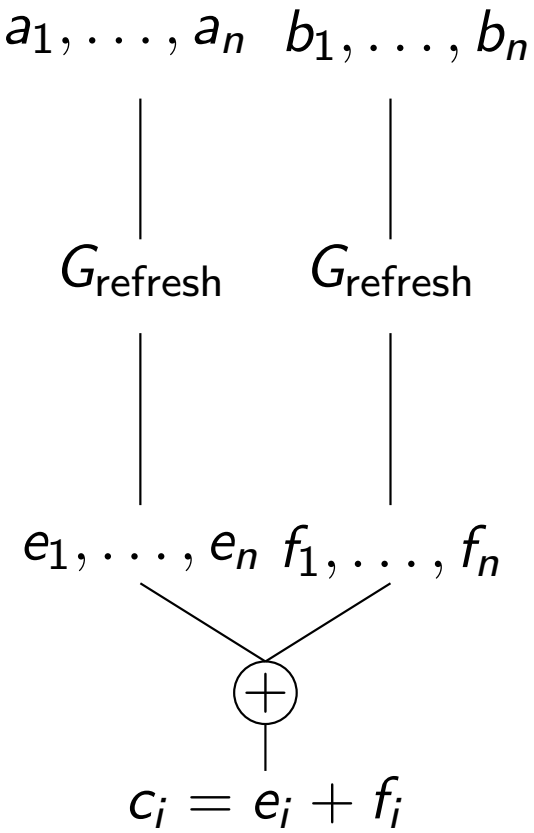

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Idea:

- similar to the case of G_{copy} (left probes + right probes)

Generic Constructions

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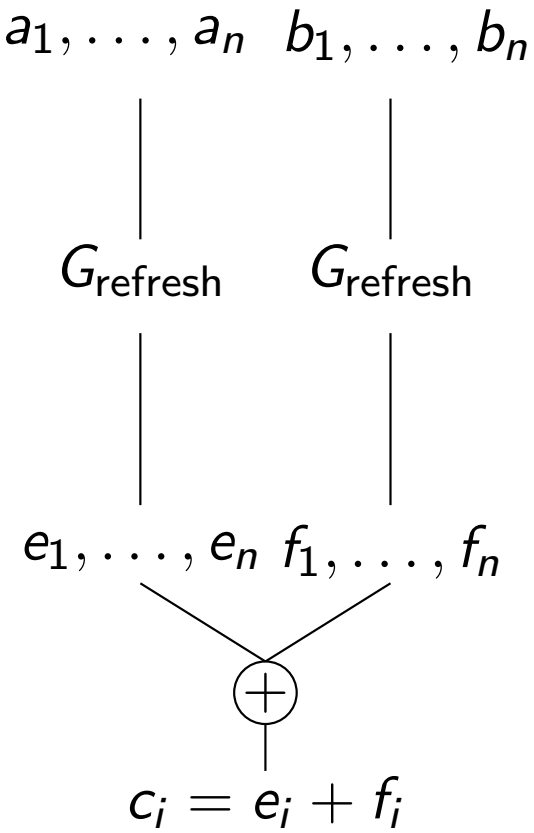

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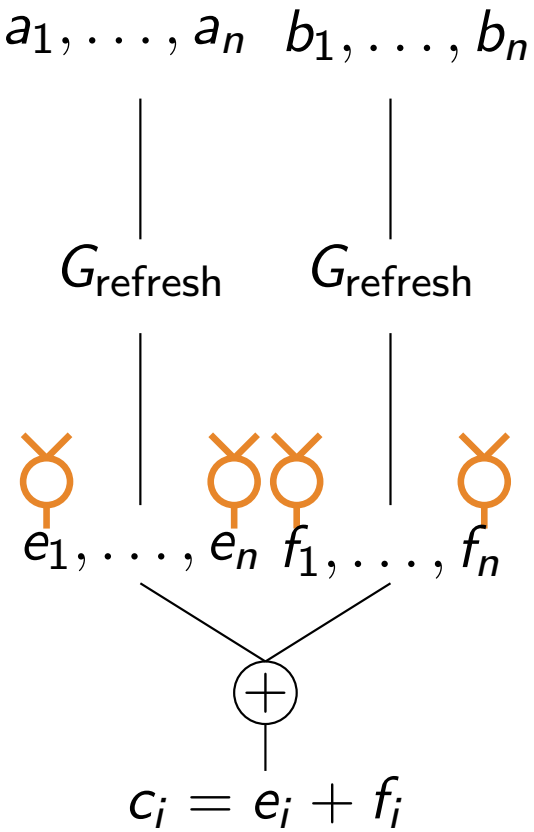

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Generic Constructions

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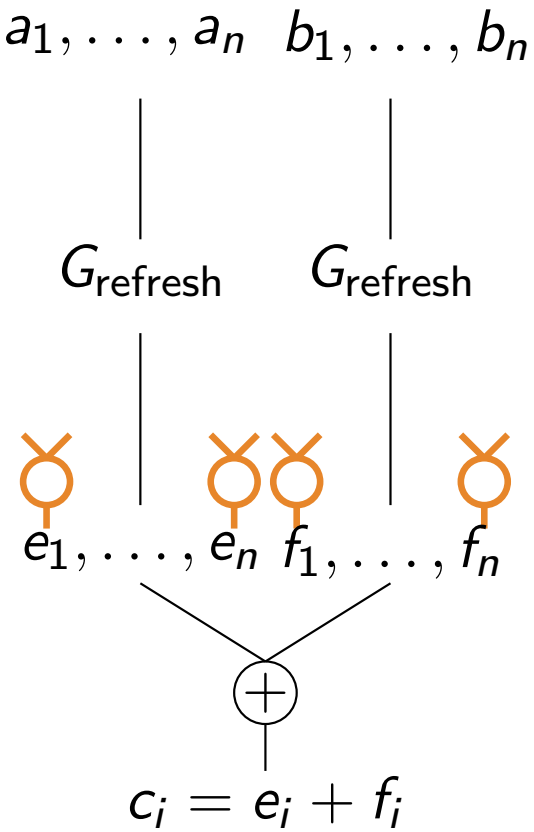

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- **exception:** potential internal probes on $\{e_i\}_{i\in[n]}$, $\{f_i\}_{i\in[n]}$ (output shares of G_{refresh}s)
- **solution:** replace each by 2 wires input of corresponding output gate
- double number of probes $\Longrightarrow$ at least $d_{\max}/2$

Generic Constructions

ISW-based construction

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ISW $G_{\mathsf{refresh}} \longrightarrow$ RPE achieving $d_{\mathsf{max}} = \dfrac{n+1}{2}$

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ISW $G_{\text{add}} \longrightarrow$ RPE achieving $d_{\max} = \dfrac{n+1}{2}$ (**Better than generic result**)

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ISW $G_{\text{mult}} \longrightarrow$ RPE achieving $\dfrac{d_{\max}}{2} = \dfrac{n+1}{4}$ **!!**

Generic Constructions

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Why?

Generic Constructions

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Why?

- direct product of input shares

Generic Constructions

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ISW $G_{\text{mult}} \longrightarrow$ RPE achieving $\dfrac{d_{\max}}{2} = \dfrac{n+1}{4}$ **!!**

Why?

- direct product of input shares
- $W = \{a_1 \times b_1, \ldots, a_{t+1} \times b_{t+1}\}$ simulation needs $t+1$ shares of a **and** b

Generic Constructions

ISW-based construction

ISW $G_{\text{refresh}} \longrightarrow$ RPE achieving $d_{\max} = \dfrac{n+1}{2}$

ISW $G_{\text{copy}} \longrightarrow$ RPE achieving $d_{\max} = \dfrac{n+1}{2}$

ISW $G_{\text{add}} \longrightarrow$ RPE achieving $d_{\max} = \dfrac{n+1}{2}$ (**Better than generic result**)

ISW $G_{\text{mult}} \longrightarrow$ RPE achieving $\dfrac{d_{\max}}{2} = \dfrac{n+1}{4}$ **!!**

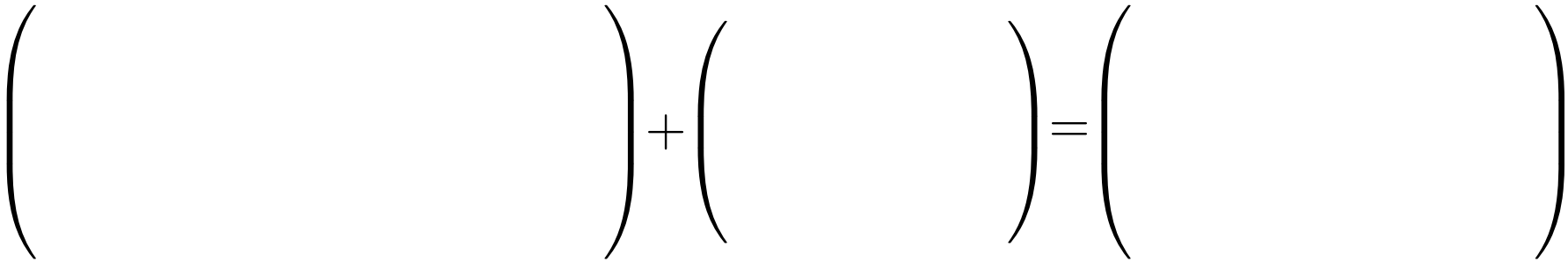
Why?

- direct product of input shares
- $W = \{a_1 \times b_1, \ldots, a_{t+1} \times b_{t+1}\}$ simulation needs $t+1$ shares of a **and** b
- Failure on both inputs with independent failure events

Generic Constructions

New Multiplication Gadget

Inputs a,b (illustration with 3 shares)



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$$\begin{pmatrix} a_1 \cdot b_1 & a_1 \cdot b_2 & a_1 \cdot b_3 \\ a_2 \cdot b_1 & a_2 \cdot b_2 & a_2 \cdot b_3 \\ a_3 \cdot b_1 & a_3 \cdot b_2 & a_3 \cdot b_3 \end{pmatrix} + \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

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$$\mathbf{v} = \begin{pmatrix} p_{1,1} + p_{1,2} + p_{1,3} \\ p_{2,1} + p_{2,2} + p_{2,3} \\ p_{3,1} + p_{3,2} + p_{3,3} \end{pmatrix}$$

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$$\mathbf{V} = \begin{pmatrix} p_{1,1} + p_{1,2} + p_{1,3} \\ p_{2,1} + p_{2,2} + p_{2,3} \\ p_{3,1} + p_{3,2} + p_{3,3} \end{pmatrix} \qquad \mathbf{X} = \begin{pmatrix} r_1 + r_4 + r_7 \\ r_2 + r_5 + r_8 \\ r_3 + r_6 + r_9 \end{pmatrix}$$

Output $C = V + X$

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New Multiplication Gadget

Non-standard G_{mult}

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$$\begin{pmatrix} a_1 \cdot b_1 & a_1 \cdot b_2 & a_1 \cdot b_3 \\ a_2 \cdot b_1 & a_2 \cdot b_2 & a_2 \cdot b_3 \\ a_3 \cdot b_1 & a_3 \cdot b_2 & a_3 \cdot b_3 \end{pmatrix}$$

where b, b, b are three independent fresh copies of b using G_{refresh}

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where b, b, b are three independent fresh copies of b using G_{refresh}

- G_{refresh} (t, p, ε)-RPE achieves d_{max} $\Longrightarrow$ G_{mult} (t, p, ε)-RPE achieves d_{max}

Non-standard G_{mult}

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- New G_{mult} achieves $d_{\mathsf{max}} = \dfrac{n+1}{2}$ (unlike ISW mult.)

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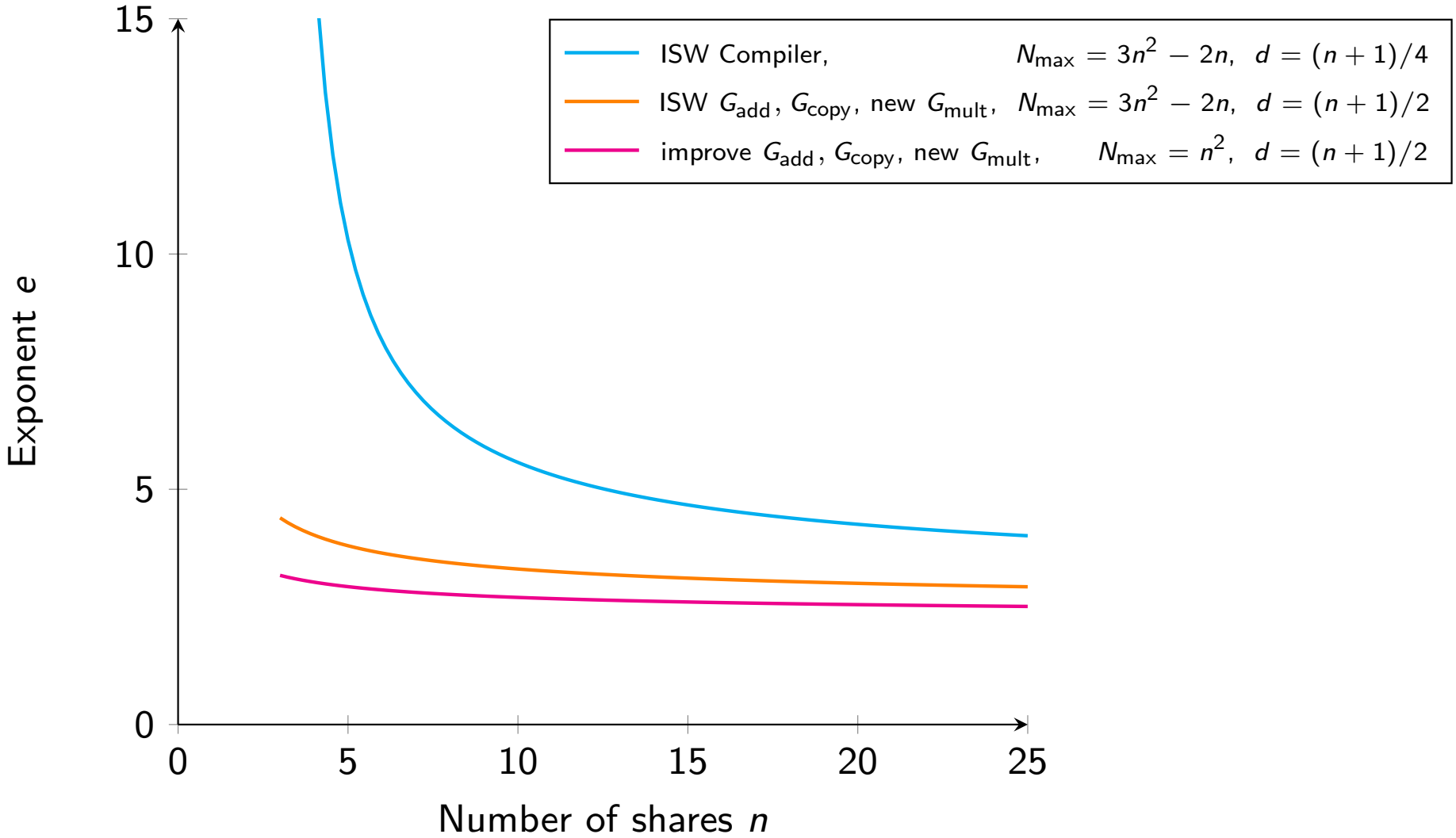
where b, b, b are three independent fresh copies of b using G_{refresh}

- G_{refresh} (t, p, ε)-RPE achieves $d_{\max}$ $\implies$ G_{mult} (t, p, ε)-RPE achieves $d_{\max}$
- New G_{mult} achieves $d_{\max} = \dfrac{n+1}{2}$ (unlike ISW mult.)
- The only multiplication gadget achieving $d_{\max}$ for any number of shares

Generic Constructions

Asymptotic Complexity

$$\mathcal{O}(|C|.\kappa^e), \quad e = \frac{log(N_{\max})}{d}$$



Concrete RPE instantiations

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3-share

G_{ref} : $c_1 \leftarrow a_1 + r_1$

$c_2 \leftarrow a_2 + r_2$

$c_3 \leftarrow (r_1 + r_2) + a_3$

Concrete RPE instantiations

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\end{aligned}
$$

5-share

$$
\begin{aligned}
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- G_{copy}, G_{add} based on G_{refresh}

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- G_{copy}, G_{add} based on G_{refresh}
- G_{mult} from new generic construction using G_{refresh}

Concrete RPE instantiations

3-share

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- G_{copy}, G_{add} based on G_{refresh}
- G_{mult} from new generic construction using G_{refresh}

Construction	$d_{\max}$	Complexity	Tolerated Leakage rate
[AIS 18] MPC based	-	$\mathcal{O}(\lvert C\rvert \cdot \kappa^{7.87})$	$p = 2^{-25}$
[CRYPTO 20] 3-share	3/2	$\mathcal{O}(\lvert C\rvert \cdot \kappa^{7.5})$	$p = 2^{-8}$
New 3-share	2	$\mathcal{O}(\lvert C\rvert \cdot \kappa^{3.9})$	$p = 2^{-7.5}$
New 5-share	3	$\mathcal{O}(\lvert C\rvert \cdot \kappa^{3.2})$	$2^{-12} \leq p \leq 2^{-6}$

Conclusion

- In-depth analysis of RP expansion (complexity, limitations, relation to other standard security notions)

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- In-depth analysis of RP expansion (complexity, limitations, relation to other standard security notions)
- New generic constructions for RPE gadgets achieving *near* optimal complexity
- Concrete instantiations with 3 and 5 shares improving results of the original work from [CRYPTO 20], providing explicit tolerated leakage rate as well as improved complexities